Lesson 9

Glencoe Geometry Chapter 4.3, 4.4, 4.5

Exploring Congruent Triangles

By the end of this lesson, you should be able to

1. Name and Label corresponding parts of congruent triangles.
2. Understand three types of congruence transformations.
3. Determine if two triangles are congruent.

This week we are sticking with our topic from last week:

TRIANGLES

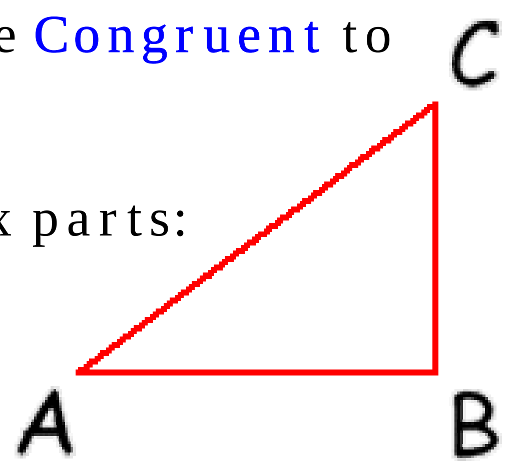
(They are by far the most popular geometric figures, go figure . . .)

When working with multiple triangles, it is important to recognize if any of them are the same size and the same shape.

If so, these triangles are said to be Congruent to each other.

Remember that triangles have six parts:

3 angles and 3 sides.



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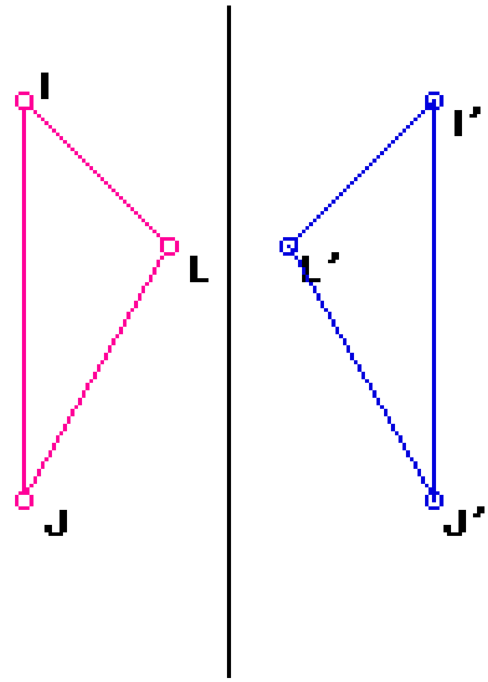
A triangle that is created from another by means an ISOMETRY (or congruence transformation), will be congruent to the first.

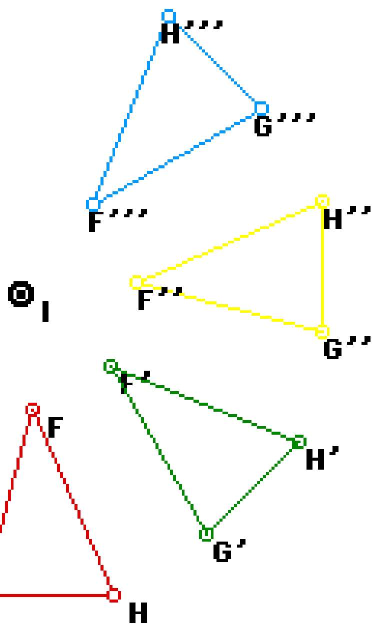


We will look at three types:

1. A Reflection across a line.

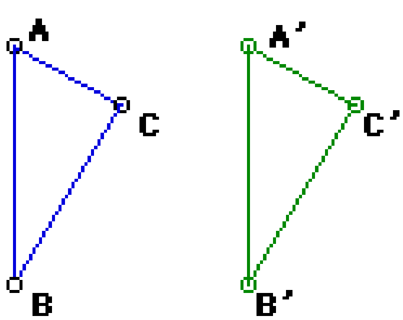
$$\triangle ILJ \cong \triangle I'L'J'$$

2. A Rotation about a point.

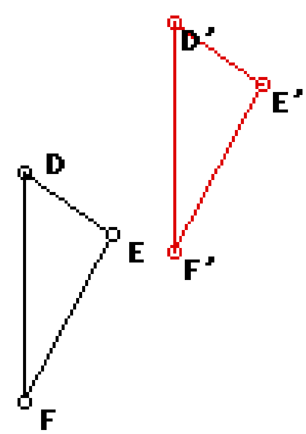


$$\triangle FGH \cong \triangle F'G'H' \cong$$
$$\triangle F''G''H'' \cong \triangle F'''G'''H'''$$

3. A shift, slide or translation.

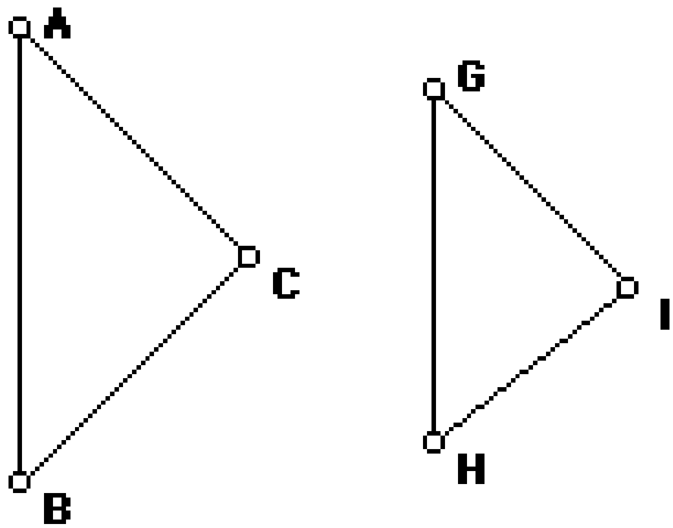


$$\triangle ABC \cong \triangle A'B'C'$$
and
$$\triangle DEF \cong \triangle D'E'F$$



Example:

Is this transformation an isometry? Why or why not?

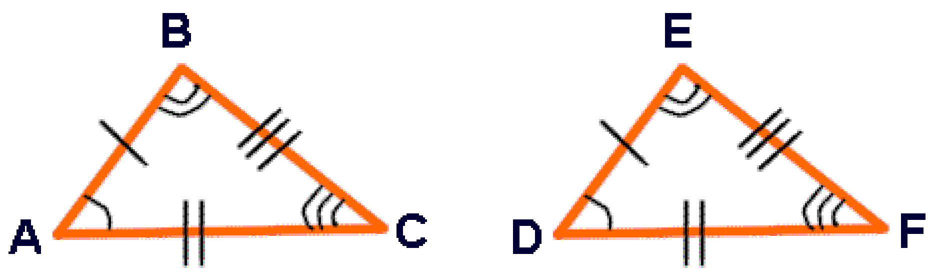


No, the triangles are not only translated from each other (a legal isometry), but they are also DILATIONS of one another (not an isometry).

A dilation changes the size and/or shape, either "stretching" it to make it proportionally bigger, or "compressing" it to make it proportionally smaller.

But what if we start with two different triangles, rather than creating one from an isometry? How do we determine if they are congruent?

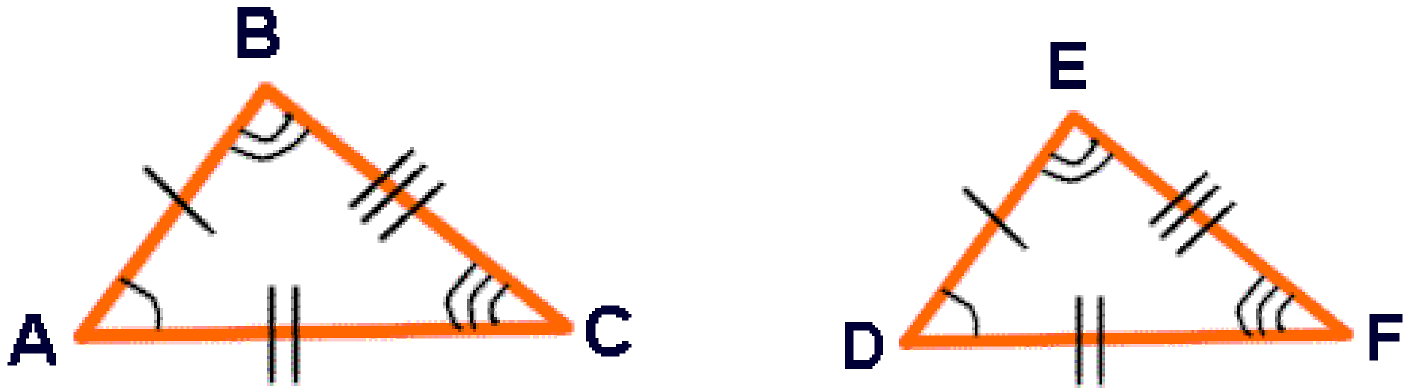
If all the corresponding parts of two triangles are congruent (equal measures), then they are congruent. (This is actually an IFF statement!)



$$\triangle ABC \cong \triangle DEF$$

The corresponding congruent sides are marked with small straight-line segments called tick marks.

The corresponding congruent angles are marked with arcs.



So given any two congruent triangles, there are 6 facts:

$\overline{AB} \cong \overline{DE}$

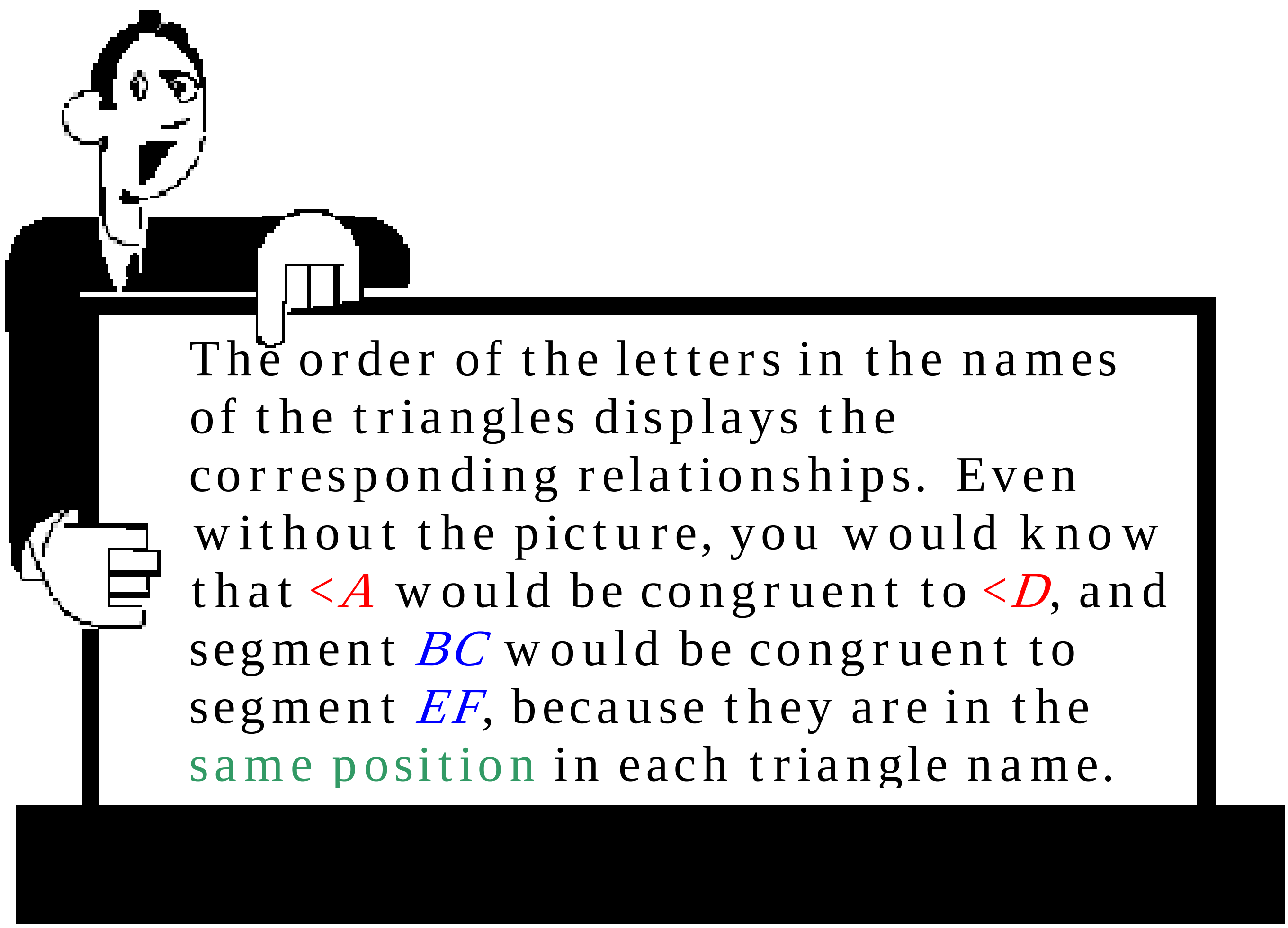
$\overline{BC} \cong \overline{EF}$

$\overline{AC} \cong \overline{DF}$

$\angle A \cong \angle D$

$\angle B \cong \angle E$

$\angle C \cong \angle F$



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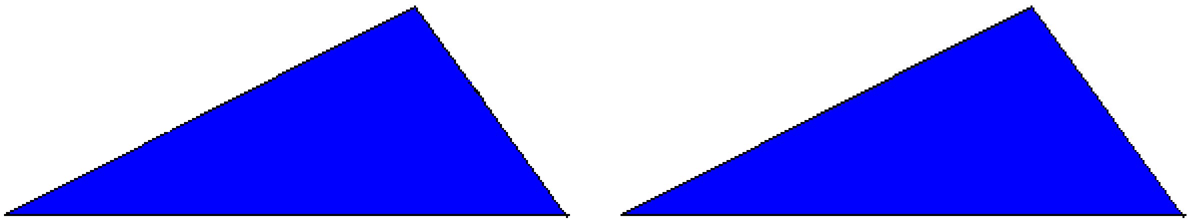
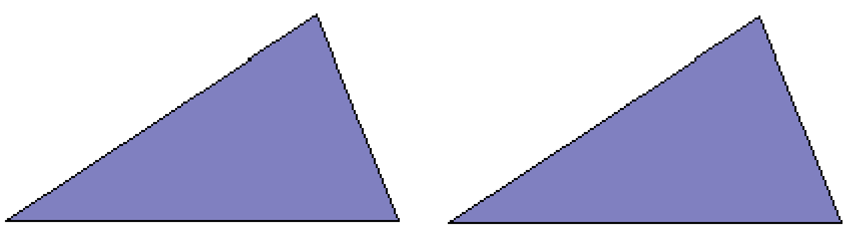
Luckily for us, when we need to show (or PROVE) that triangles are congruent, we don't need to prove all six facts. There are certain combinations of the facts that are sufficient to prove that triangles are congruent.

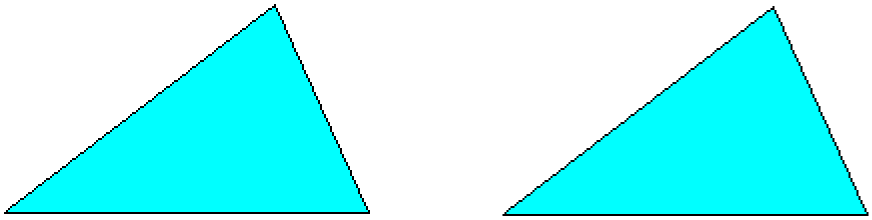
1. SSS: If three sides of one triangle are congruent to three sides of another triangle, the triangles are congruent.



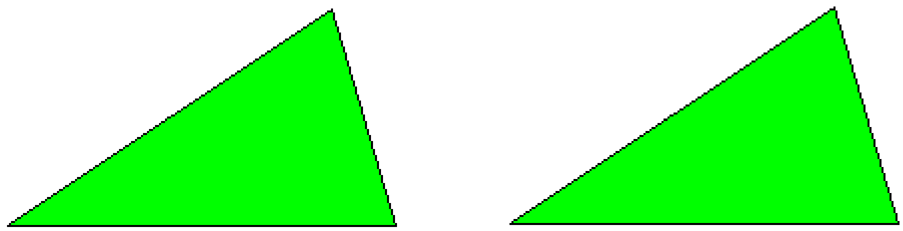
2. SAS: If two sides and the included angle of one triangle are congruent to the corresponding parts of another triangle, the triangles are congruent.



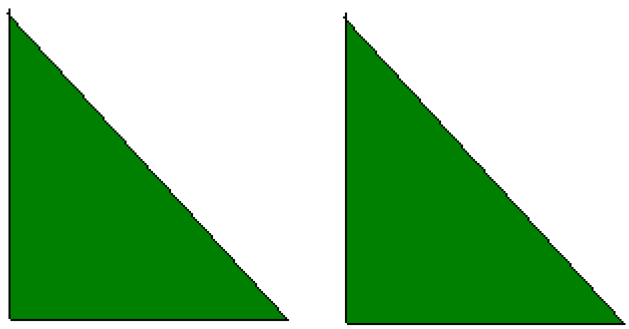
3. ASA: If two angles and the included side of one triangle are congruent to the corresponding parts of another triangle, the triangles are congruent.



4. **AAS = SAA**: If two angles and the non-included side of one triangle are congruent to the corresponding parts of another triangle, the triangles are congruent.



5. **HL**: If the hypotenuse and leg of one right triangle are congruent to the corresponding parts of another right triangle, the right triangles are congruent.

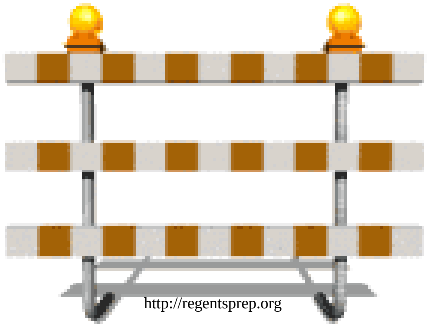


Say What??!!

Can we make a statement about congruent triangles under any other circumstances?

What if we knew two sides and the non-included angle of two triangles were congruent. Would the two triangles necessarily be congruent??????

SSA = ASS: This does NOT work, so be careful!!



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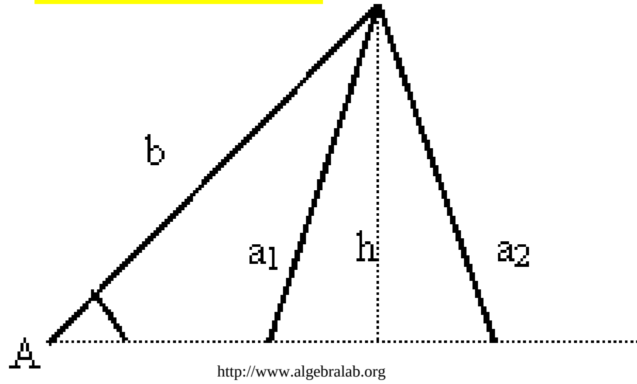

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There are two different possible configurations. Without more information, we cannot make a claim of congruence.

What if we only knew that all three angles of two triangles were congruent?

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AAA: This also does not work. We need at least one side length. We can, however, conclude that the triangles are similar (or proportional)