



# LESSON 8

## Glencoe Geometry Chapter 4.1, 4.2 Classifying Triangles & Angle Measure

**BY THE END OF THIS LESSON, YOU SHOULD BE ABLE TO**

1. Identify the different parts of a triangle
2. Classify triangles by their angle measures
3. Classify triangles by their side lengths
4. Find the measure of different types of angles of a triangle.

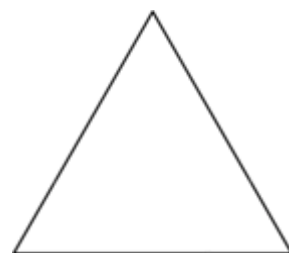
We encounter triangles everyday, in all shapes and sizes. As you know, all triangles have 3 **sides** and 3 **angles**.

We can classify them by the measure of the **lengths** of their sides.



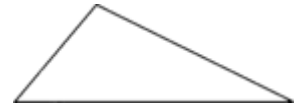
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In an **EQUILATERAL TRIANGLE**, all three sides are the same length. An equilateral triangle is always equiangular.



In an **ISOSCELES TRIANGLE**, two sides are the same length. An isosceles triangle may be right, obtuse, or acute.

In a **SCALENE TRIANGLE**, none of the sides are the same length. A scalene triangle may be right, obtuse, or acute.

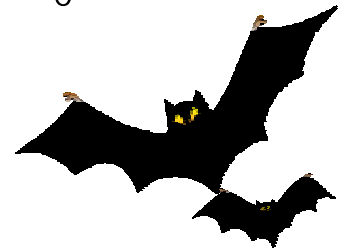
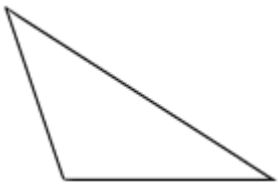


We can also classify angles by the **measure** of their interior angles.

In an **ACUTE TRIANGLE**, all angles are less than right angles—each one is less than 90 degrees. An acute triangle may be equilateral, isosceles, or scalene.

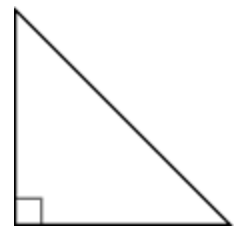


In an **OBTUSE TRIANGLE**, one angle is greater than a right angle—it is more than 90 degrees. An obtuse triangle may be isosceles or scalene.

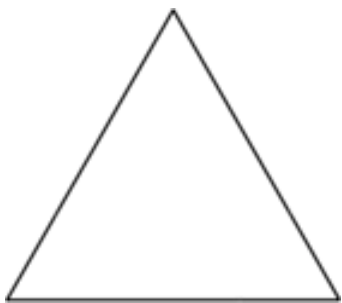


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In a **RIGHT TRIANGLE**, one of the angles is a right angle—an angle of 90 degrees. A right triangle may be isosceles or scalene.



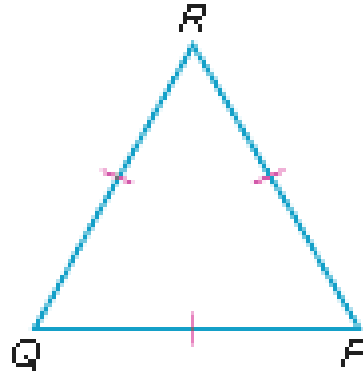
In an **EQUIANGULAR TRIANGLE**, all the angles are congruent—each one measures 60 degrees. An equiangular triangle is a kind of acute triangle, and is always equilateral. All equilateral triangles are also isosceles triangles, too! **BUWAHAAHAAAAAAA!!**



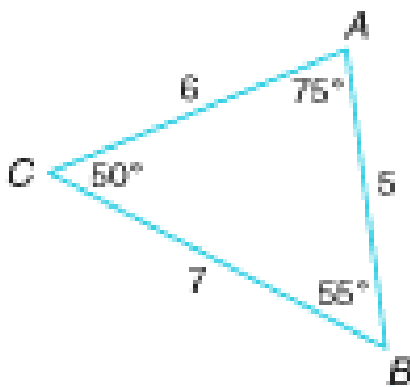
**EXAMPLES:**

1. Which does not describe  $\triangle PQR$

- A. acute                      B. isosceles  
C. equilateral              D. obtuse



2.  $\triangle ABC$  is \_\_\_\_\_.



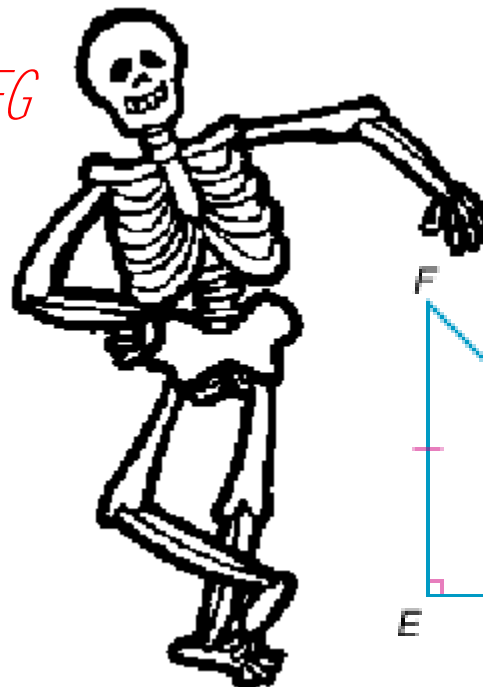
- A. isosceles and scalene  
B. equilateral  
C. scalene but not acute  
D. scalene and acute



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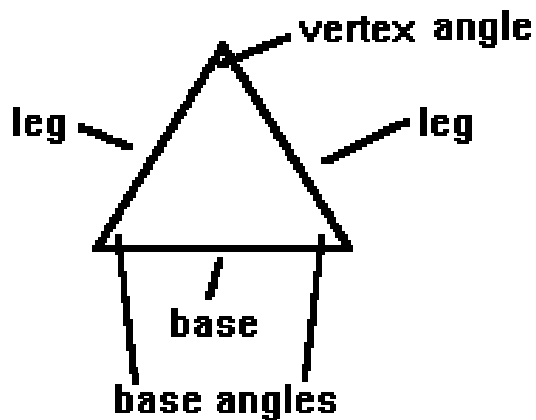
3. What type of triangle is  $\triangle EFG$

- A. right isosceles  
B. acute equilateral  
C. acute isosceles  
D. right equilateral



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The parts of an isosceles triangle also have special names.



<http://library.thinkquest.org/2609/13s1.htm>

**VERTEX ANGLE** - the angle opposite the base.

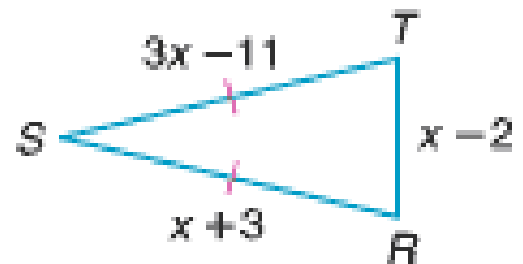
**BASE** - the side opposite the vertex angle.

**LEG** - either of the 2 congruent sides.

**BASE ANGLES** - the two congruent angles opposite each leg.

### EXAMPLE:

Triangle  $RST$  is isosceles with  $\angle S$  as the vertex angle. If  $ST = 3x - 11$ ,  $SR = x + 3$ , and  $RT = x - 2$ , find  $RT$ .



First find  $x$ . In this isosceles triangle,  $\overline{ST} \cong \overline{SR}$  so

$$3x - 11 = x + 3$$

$$3x - x = 3 + 11$$

$$2x = 14$$

$$x = 7$$

Plug this into the measure of  $\overline{RT}$ :  $x - 2 \rightarrow 7 - 2 = 5$

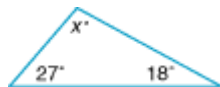


Another very important property of triangles is that the sum of the measures of all the interior angles is **180 DEGREES**. If the sum is anything other, then we don't have a triangle. This is called the **Angle Sum Theorem**.

## EXAMPLE:



Find the value of  $x$ .



By the angle sum theorem, the sum must be 180.

$$\text{So, } 27 + 18 + x = 180$$

$$45 + x = 180$$

$$x = 180 - 45$$

$$x = 135$$

so this is an OBTUSE triangle. Since all the angles are different, it is also scalene.

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A. 125

B. 135

C. 45

D. 145

## EXAMPLE:

What is  $m\angle D$  in  $\triangle DEF$

Since angle F is 90, the sum of the other two must be  $180 - 90$  or 90.

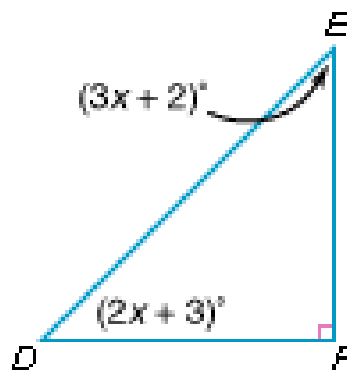
$$(3x+2) + (2x+3) = 90$$

$$5x + 5 = 90$$

$$5x = 85$$

$$x = 17$$

so measure of angle D is  $2(17)+3 = 34 + 3 = 37$



## EXAMPLE:

Which statement is not true?

A. A triangle cannot be scalene and isosceles.

B. In an isosceles triangle, the base is congruent to one of the legs.

C. A triangle cannot be obtuse and contain a  $90^\circ$  angle.

D. A triangle can be obtuse and isosceles.



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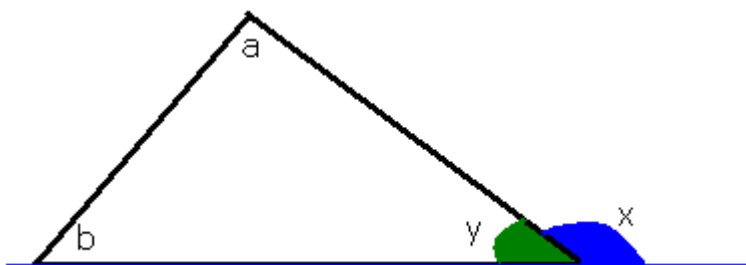
# SAY WHAT??!!

Triangles can also have

**EXTERIOR ANGLES!**

That's right, angles **OUTSIDE** the triangle.

Angle  $x$  is an exterior angle of the triangle:



<http://www.mathsrevision.net/gcse/pages.php?page=17>

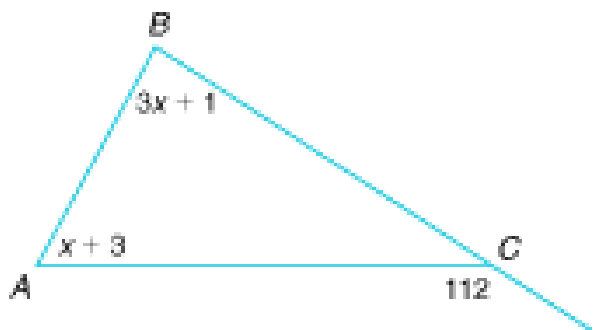


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The exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices. In other words,  $x = a + b$  in the diagram. This is because  $x$  and  $y$  are supplementary! Angles  $a$  and  $b$  are called remote interior angles. Their sum is also supplementary with the measure of angle  $y$ ! **BUWAHAAHAAAAAAA!!**

## EXAMPLE:

Find  $m\angle B$  if  $m\angle A = x + 3$  and  $m\angle C = 3x + 1$ .



$$\begin{aligned} m\angle ACB &= 180 - 112 = 68 \\ (3x + 1) + (x + 3) &= 180 - 68 = 112 \\ 4x + 4 &= 112 \\ 4x &= 108 \\ x &= 27 \\ m\angle B &= 3(27) + 1 = 81 + 1 = 82 \end{aligned}$$



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