Lesson 7

Glencoe Geometry Chapter 3.5

Distance Between Parallel Lines

By the end of this lesson, you should be able to

1. Draw a perpendicular line from a vertex to a line.
2. Construct a line perpendicular to a line through a point using a compass.
3. Determine the distance between a point and a line.
4. Determine the distance between two lines.

You've always heard that the shortest distance between two points is a straight line. This is true, but what is the shortest distance between a line and a point not on the line???

When we say distance, we mean the __________________ distance.

The **distance from a line to a point not on the line** is the length of the segment ___________________ to the line from the point. It is important to be able to draw these lines from given geometric figures.

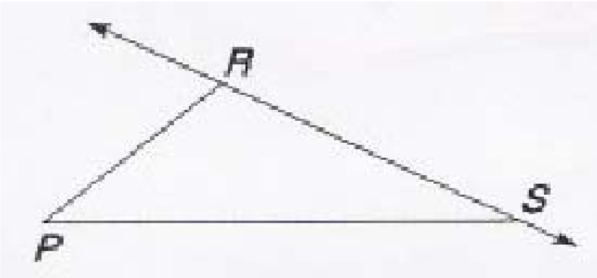
We usually draw this perpendicular line from a point, called the **vertex**, to a side opposite the vertex. We call these lines _________________.

Let's try some examples . . .

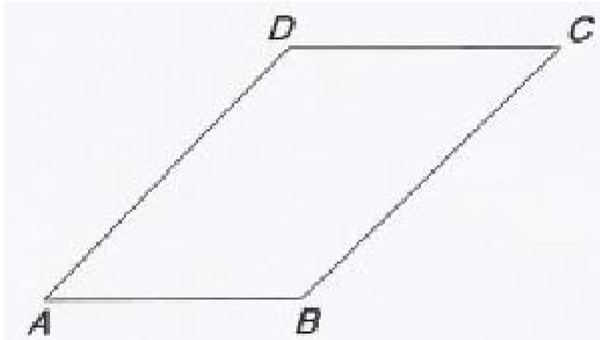
Examples:

Draw the segment that represents the distance indicated.

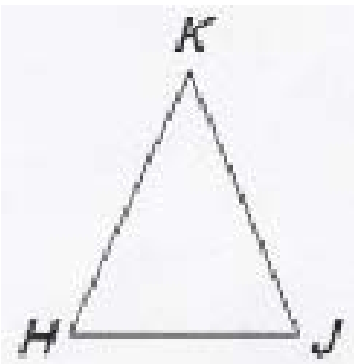
1. P to $\overleftrightarrow{RS}$



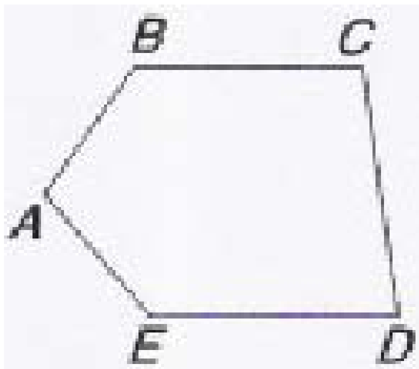
2. B to $\overline{AD}$



3. K to $\overline{HJ}$



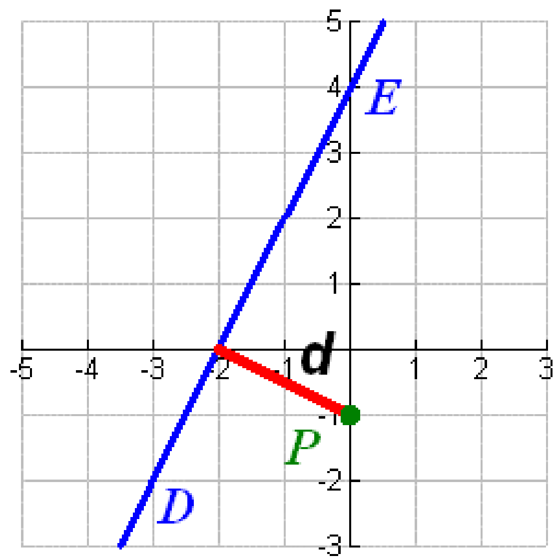
4. A to $\overline{BC}$

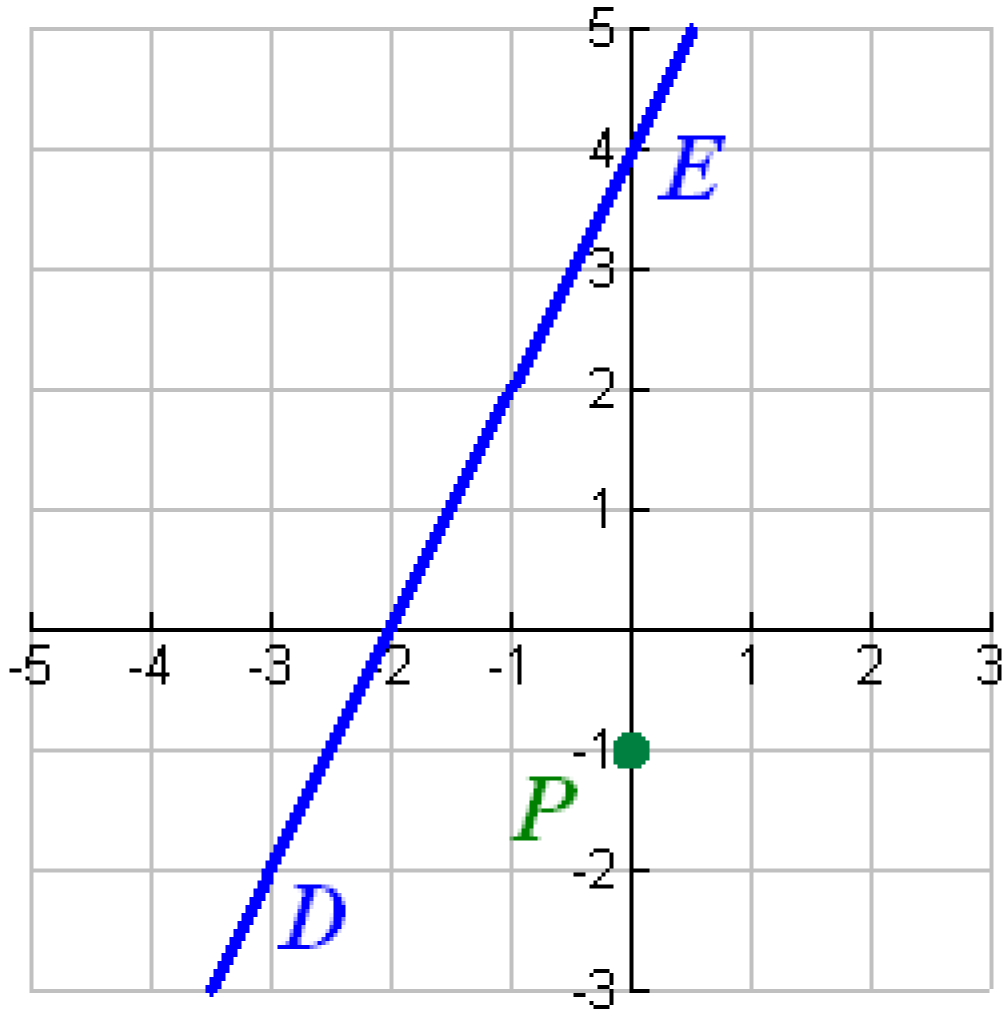


Now let's look at a point and a line on the coordinate plane. In the diagram below, we want to find the measure from point P to $\overleftrightarrow{DE}$. The perpendicular line segment is shown in red.

We want to measure this perpendicular distance, ***d***, but how can we be sure that the two lines are actually perpendicular?

We can construct this perpendicular line, called the _______________ line, by using a ruler and a compass.





Now let's verify that $\overleftrightarrow{PQ}$ is an orthogonal line. Remember, perpendicular slopes are <u>opposite reciprocals</u> of each other.

Slope of $\overleftrightarrow{DE} =$

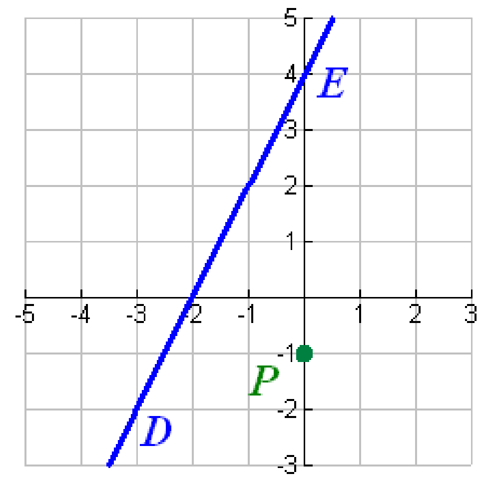
Slope of $\overleftrightarrow{PQ} =$

Now we can approximate the distance between P and $\overleftrightarrow{DE}$:

Recall the distance formula, $d = \sqrt{(\Delta x)^2 + (\Delta y)^2}$. We need two points. Since our two lines intersect at $Q(-2,0)$, we will use this point and our point $P(0,-1)$.

So what if we don't have our handy-dandy compass? We can get the same answer by a little cleverness, mental grease, and algebra.

We can first find the equation of the given line. Then using the opposite reciprocal slope and the given point, write the equation of the orthogonal line. We then find the point of intersection *Q*, and then finally use our old distance formula to find the distance between *Q* and *P* . . . . whew . . . It's easier done than said.



1. The slope between $D(-3,-2)$ and $E(0,4)$

2. Find the equation of the line in slope-intercept form: $y = mx + b$

3. Find the equation of the orthogonal line using the opposite reciprocal slope and point *P*.

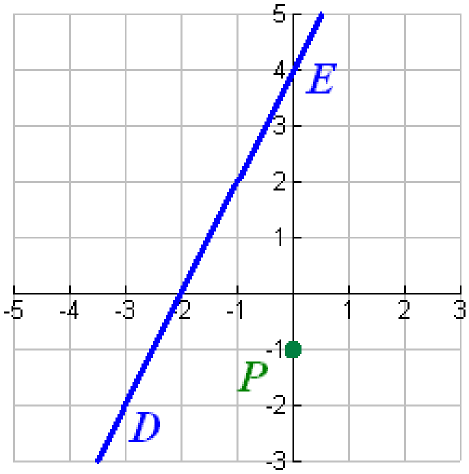
4. Find the point of intersection.

5. Find the distance between *P* and *Q*

   From our previous calculation using $d = \sqrt{(\Delta x)^2 + (\Delta y)^2}$, we know this to be $\sqrt{5} \approx 2.236$

There is yet a third way to calculate this distance, and that is by way of a formula, the derivation of which is similar to the previous method, but using variables instead of numbers.

To use this formula, we need to review another form of an equation of a line:



Standard Form: $Ax + By + C = 0$

For our given line $\overleftrightarrow{DE}$, we can just modify our equation we found in slope-intercept form.

$$y = 2x + 4 \rightarrow 2x - y + 4 = 0$$

where A= B= C=

We are now ready for the formula . . .
The distance from a point $P(x_1, y_1)$ to a line ℓ with an equation $Ax + By + C = 0$ is given by

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

Say What??!!

Now that we know several ways to find the distance between line and a point not on the line, let's put it to good use.

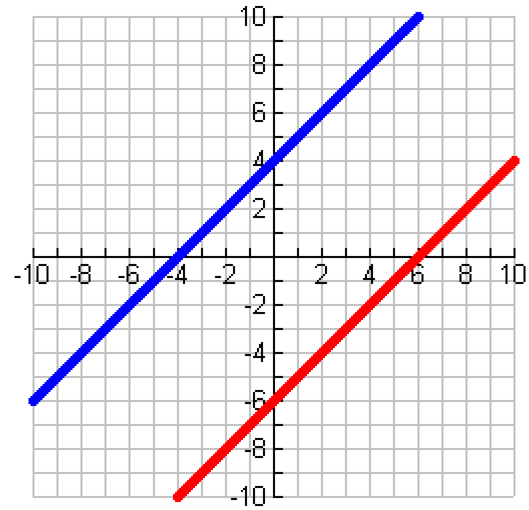
Remember when we said that parallel lines have the same slope? It's still true, but there is another way to define parallel lines . . .

Two parallel lines in a plane are parallel if they are everywhere equidistant.

To measure the distance between two parallel lines, we can measure the distance between one of the lines and any point on the other.

Example:

Find the distance between the parallel lines *p* and *q* whose equations are $y = x + 4$ and $y = x - 6$, respectively.



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