Lesson 6

Glencoe Geometry Chapter 3.1 and 3.2

Slope

By the end of this lesson, you should be able to

1. Find the slopes of lines.
2. Know the slopes of two special lines (and why they are so!!).
3. Determine if two lines are parallel, perpendicular or neither.
4. Build a ramp in accordance with the *American with Disabilities Act* of 1990.

If you've ever snow skied, you've probably thought about the steepness of a slope. In math, we use the word ***slope*** to describe the steepness of a line in the coordinate plane.

http://www.teacherfiles.com/clip_sports.htm

Slope can be described as a **ratio** describing the steepness of a graph, measuring how fast the graph "rises" with respect to how fast it "runs." We can also think of it as the average rate of change between two points. Let's call the two points (x_1, y_1) and (x_2, y_2). Here is a precise, mathematical definition:

$$slope = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{rise}{run} = \frac{\Delta y}{\Delta x}, \Delta x \neq 0$$

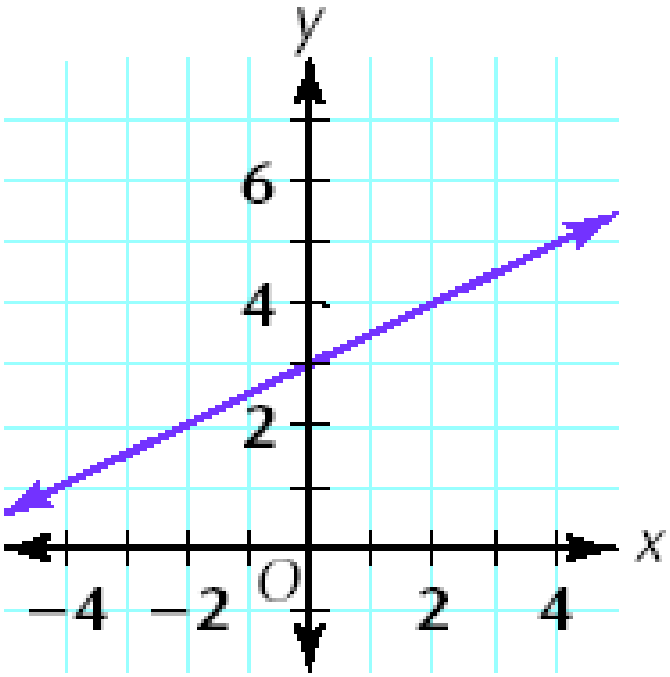
We are basically measuring the change in the vertical (rise) with respect to the change in the horizontal (run).

BUT WHY THE LETTER *m*? Great question!!

The French verb for "to climb," or "to ***M***ount," is "MONTAR," which starts with the letter *m*. Thanks, France.

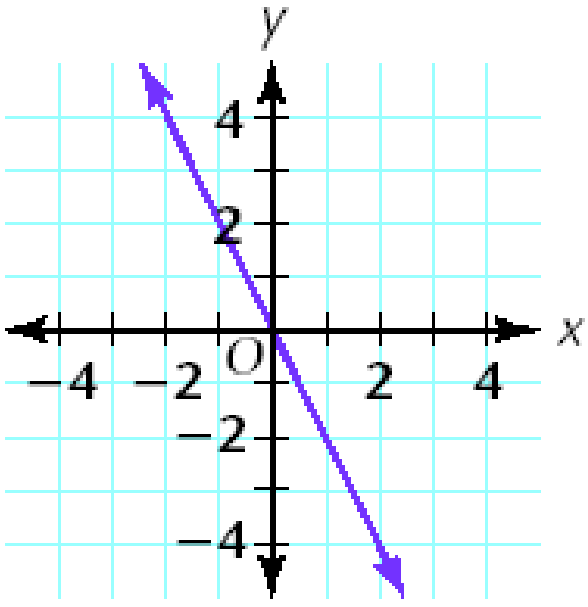
Let's hit the slopes . . . . WoooHooo!

Calculate the slope of the given line:



Using (0,3),(4,5)
m=(4-0)/(5-3)=4/2= 2 or 2/1
Or
Using (-4,1) and (4,5)
m=(4—4)/(5-1)=8/4= 2 or 2/1

Let's do it again:



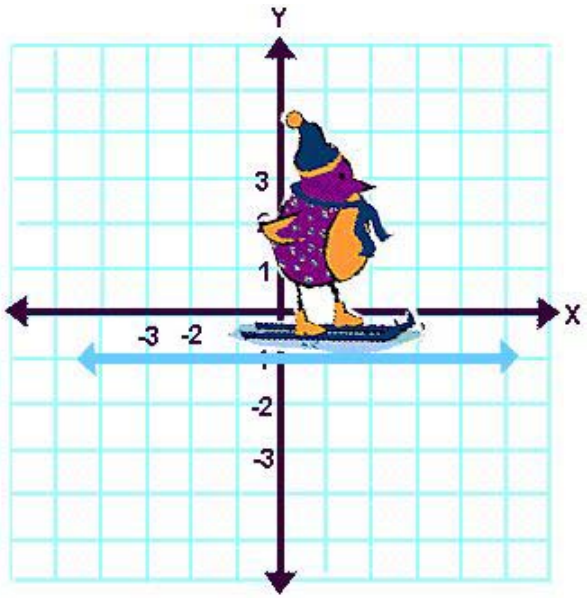
Using (-2,4) and (0,0)
m=(0-4)/(0- -2)= -4/2= -2
Or
Using (2,-4) and (1,-2)
m=(-2- -4)/(1-2)=(2/-1)= -2

What did you notice about the sign in the last two examples?
Well, we always read a graph from LEFT to RIGHT, like a book.

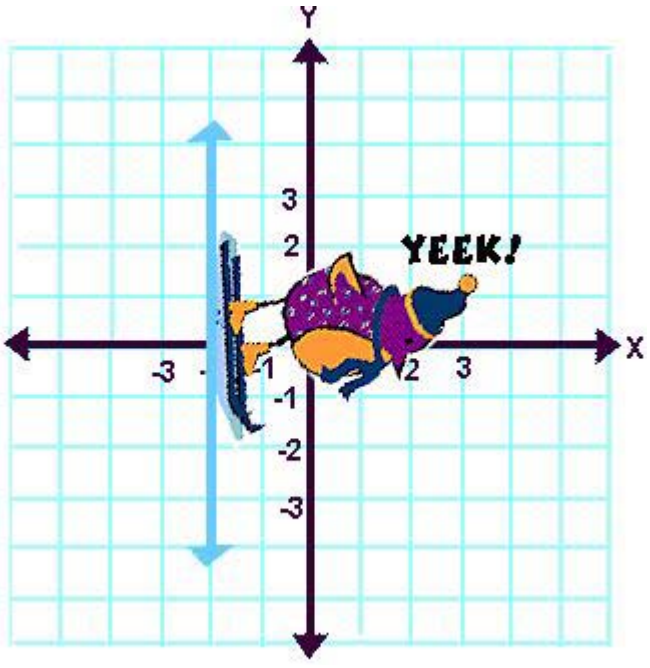
A line with a POSITIVE slope will rise from left to right.
A line with NEGATIVE slope will fall from left to right.

So when we talk about STEEPNESS, we are talking about the ABSOLUTE VALUE of the slope. $\boxed{steepness = |slope|}$

Now what if you are a novice skier (or a purple penguin)? What slope would you prefer????
It's easiest to learn on a flat slope or a HORIZONTAL line. Let's calculate it.



Using any two points on the line, (-3,-1) and (4,-1)
m=dy/dx=(-1 - -1)/(-3 – 4)=0/-7=0



What about VERTICAL lines
Well, you surely cannot ski down it . . . . . .it's called Free-Falling!!

Let's calculate the slope:

Using (-2,3) and (-2,-3)

m=dy/dx=(-3 – 3)/(-2- -2)= -6/0 = undefined

So a Horizontal Line has ZERO slope
And a Vertical Line has NO slope (or undefined slope)


René Descartes
(1596-1650)
http://microscopy.fsu.edu

So where does Geometry come into this?? I did this way back in Algebra I!! Well, we have actually been doing what's called **ANALYTIC GEOMETRY** (thanks Rene Descartes!)

Here are some VIPs (Very Important Postulates)

—Two non-vertical lines have the same slope *IFF* they are parallel.

—Two non-vertical lines are perpendicular *IFF* the product of their slopes is -1.

Example:
Points A (n, -5) and B (2, 4) are on a line. If the slope of the line is $\frac{3}{2}$ find the value of n.

m=dy/dx=(4 - -5)/(2-n)=3/2 so 9/(2-n)=3/2 cross multiplying gives 9(2)=3(2-n) so 18=6-3n so 12=3n so n=4

Example:
Find the slope of a line perpendicular to the line passing through points at (1, -2) and (3, 2).

m=(2- -2)/(3-1)=4/2=2 so the perpendicular slope is the negative reciprocal of m, which is –1/2

Example:

Find the value of x so the line that passes through (x, 6) and (10,-3) is parallel to the line that passes through (7, 2) and (6, 11).

If the lines are parallel, their slopes are equal so
(-3 – 6)/(10-x) = (11-2)/(6-7)
-9/(10-x) = 9/(-1)
cross multiplying gives
9 = 9(10-x)
9=90-9x
9x=81
x=9

Example:

Given the points A (-3, -2), B (1, 4), C (-4, 3) and D (5, -3), determine if $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are parallel, perpendicular or neither.

Slope of line AB is (4- -2)/(1- -3)= 6/4 = 3/2
Slope of line CE is (-3 –3)/(5- -4)=6/9 = 2/3

Since 3/2 and 2/3 are not equal to each other, nor are they the negative reciprocals of each other (the negative reciprocal of 3/2 is –2/3), the two lines are neither parallel nor perpendicular. They must then intersect, but at an angle different than 90 degrees.

Say What??!!

From the *American with Disabilities Act* (ADA) of 1990

4.7 Curb Ramps.

4.8.2. The slope shall be measured as shown in Fig. 11. Transitions from ramps to walks, gutters, or streets shall be flush and free of abrupt changes. Maximum slopes of adjoining gutters, road surface immediately adjacent to the curb ramp, or accessible route shall not exceed 1:20.

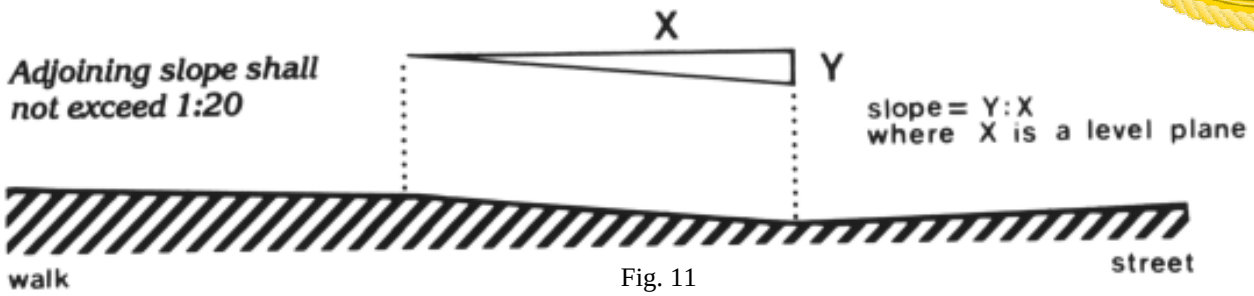


Fig. 11
Measurement of Curb Ramp Slopes
www.usdoj.gov

If we need to build a ramp to obtain access to a doorway 50 inches (4ft 2in) above grade, **how long must our ramp be**? **How should we design it**? If another ramp happened to be parallel with it, **what would its slope be**? **What color is the ramp**?

The ration 1:20 means a rise of one unit for at least 20 units of run, where the units can be anything: feet, inches, miles, furlongs, etc.. For example, a rise in one foot must have a ramp of at least 20 feet. For our example, we can set up a proportion. Our rise is 50 inches, we need to have a run of :

$$\frac{1}{20}=\frac{50}{x} \quad so \quad x=(50)(20)=1000 \ inches$$

$$or \quad 1000/12=83\frac{1}{3} \ ft \quad or \quad 83 \ ft \ 4in$$

so the ramp must be at leas 83feet 4inches long to comply with federal regulations. We usually build a ramp with a switch-back so we don't need to run out this long distance, but turn around have their own federal regulations . . . . .

Fortunately, there is no federal mandate on the color, so your favorite will do, safety orange will always keep OSHA off your back.

*unless otherwise noted, all images are from www.Glencoe.com or http://regentsprep.org/Regents/math/glines/Llines.htm