Lesson 5

Glencoe Geometry Chapter 3.1 and 3.2

Parallell Lines, Transversals, & Angles

By the end of this lesson, you should be able to

1. Solve problems by drawing diagrams.
2. Identify relationships between two lines.
3. Name angles formed by a pair of lines and a transversal.
4. Use the properties of parallel lines to determine angle measures.

There are four types of lines you need to know about for this lesson.

Definitions

Intersecting lines have a point in common.

Skew lines do not intersect and are on different planes.

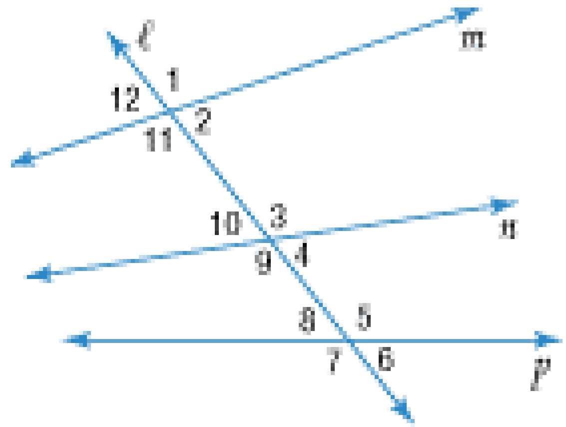
Parallel lines are coplanar lines that do not intersect. We use the symbol || to denote parallel lines, such as $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$.

A transversal is a single line that intersects two or more coplanar lines, each at a different point.

A transversal is a very useful line in geometry. Transversals tell us a great deal about angles. Let's take a look at some angles formed by a transversal and some coplanar lines.

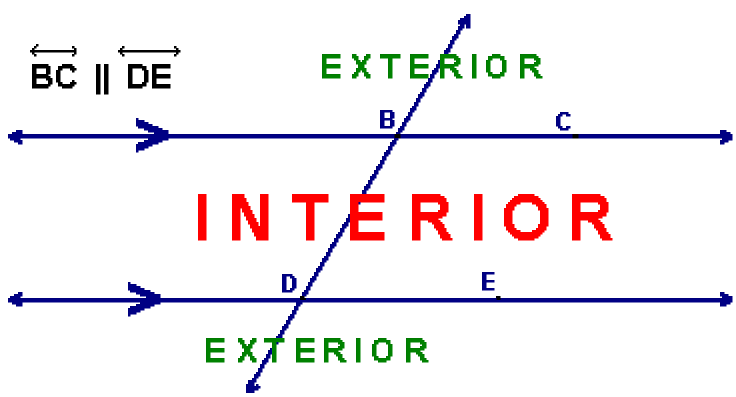
In the diagram at the right, line *l* is a transversal intersecting the three lines *m*, *n*, and *p*, forming 12 angles. These have special names in relation to the others.

- Exterior angles: 1,6,7,12
- Interior angles: 2,3,10,11 & 4,5,8,9
- Consecutive interior angles: 2,3&8,9
- Alternate exterior angles: 12,4 & 3,7 & 1,9
- Alternate interior angles: 2,10 & 5,9
- Corresponding angles: 10,12 & 2,4



Notice how important the diagram is here in helping us "see" the relationships.

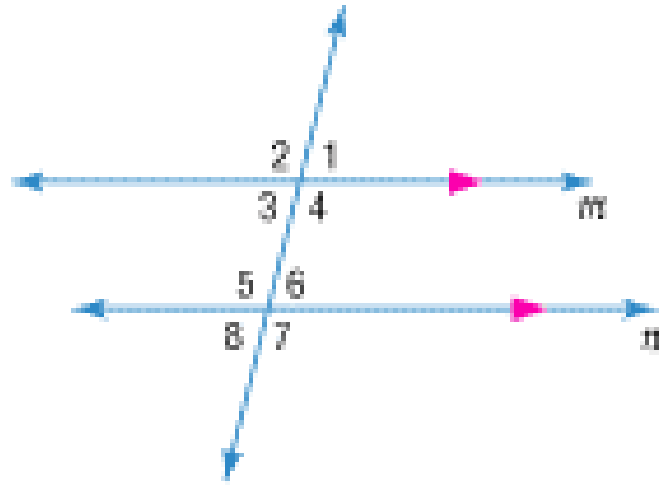
Certain angle "names" describe "where" the angles are located. Remember that:
- the word **INTERIOR** means **BETWEEN** the lines.
- the word **EXTERIOR** means **OUTSIDE** the lines.
- the word **ALTERNATE** means **"alternating sides"** of the transversal



http://regentsprep.org/Regents/Math/angles/Lparallel.htm

Now, when angles are formed by *PARALLEL* lines (noted by arrows on the lines) and a transversal, the angles have the same names, but also some *very*, *very* interesting relationships. The following are some Very Important Postulates: VIPs

1. Alternate Interior angles are congruent. 3.6 & 4.5
2. Alternate Exterior angles are congruent. 1.8 & 2.7
3. Corresponding angles are congruent. 1.6 & 4.7 & 2.5 & 3.8
4. Vertical angles are congruent (of course!) 2.4 & 1.3
5. Consecutive Interior angles are supplementary. 4.6 & 3.5
6. Adjacent angles are supplementary. 1.2 & 1.4



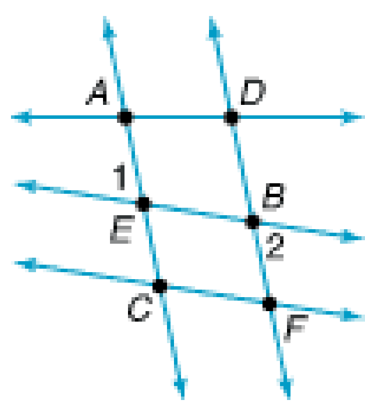
This is a lot to remember, and drawing an accurate picture with labels, and knowing the names of the angle relationships is critical!!
It's important to clearly identify relationships when there are more than one set of parallel lines and/or more than one transversal.

Example:

If $\angle 1 \cong \angle 2$ which lines *must be* parallel?

Remember that if two lines in a plane are cut by a transversal so that a pair of alternate exterior angles are congruent, then the two lines are parallel.

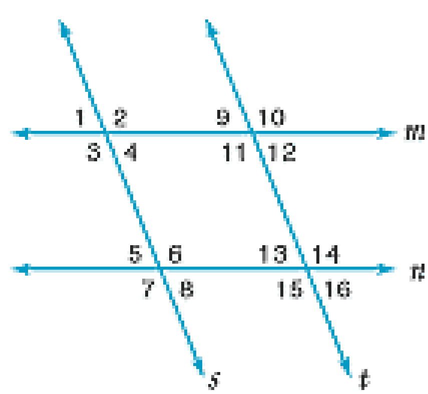
$\overleftrightarrow{DF} \parallel \overleftrightarrow{AC}$



Example:

$\angle 8$ and $\angle 13$ are C________.

A. consecutive interior angles
B. alternate exterior angles
C. alternate interior angles

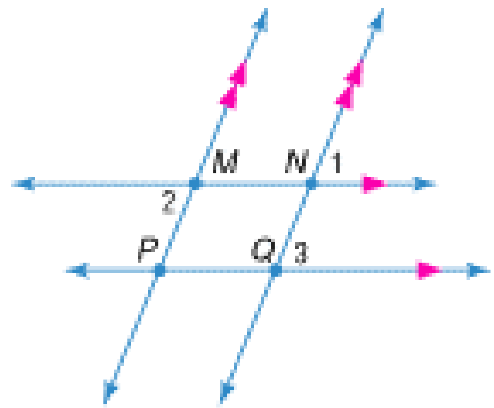


D. corresponding angles

Example: ALGEBRA

Find Values of Variables

If $m\angle 1 = 16x - 8$, $m\angle 2 = 4(y + 8)$, and $m\angle 3 = 14x + 2$, find x and y.



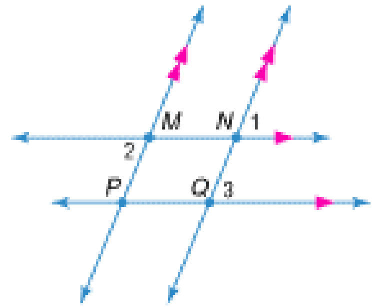
- Find x.

  Since $\overleftrightarrow{MN} \parallel \overleftrightarrow{PQ}$, $\angle 1 \cong \angle 3$ by the Corresponding Angles Postulate.

$m\angle 1 = m\angle 3$	Definition of congruent angles
$16x - 8 = 14x + 2$	Substitution
$2x - 8 = 2$	Subtract $14x$ from each side.
$2x = 10$	Add 8 to each side.
$x = 5$	Divide each side by 2.

- Find y.

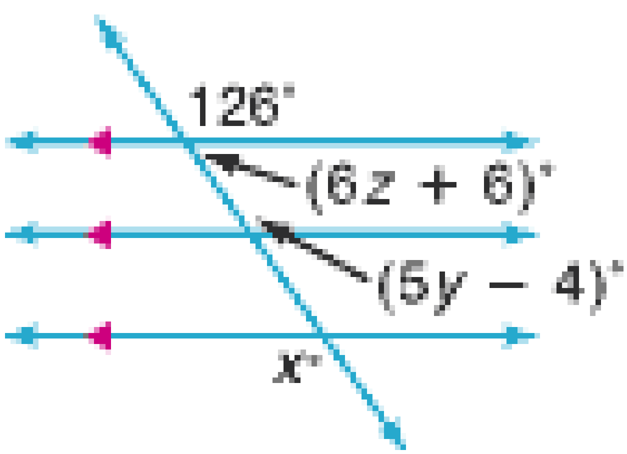
Since $\overleftrightarrow{MP} \parallel \overleftrightarrow{NQ}$, $\angle 1 \cong \angle 2$ by the Alternate Exterior Angles Theorem.



$m\angle 1 = m\angle 2$	Definition of congruent angles
$16x - 8 = 4(y + 8)$	Substitution
$16(5) - 8 = 4(y + 8)$	$x = 5$
$72 = 4y + 32$	Simplify.
$40 = 4y$	Subtract 32 from each side.
$10 = y$	Divide each side by 4.

Example:

Determine the value of x, y, and z.



x: $126^\circ = x^\circ$ by Alternate Exterior Angles

y: $126^\circ = (5y - 4)^\circ$ by corresponding angles so,

$126^\circ + 4^\circ = (5y)^\circ$

$y = \dfrac{130^\circ}{5} = 26^\circ$

z: $126^\circ + (6z + 6)^\circ = 180^\circ$

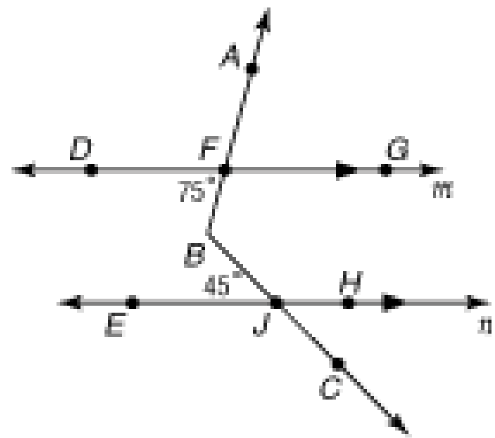
$132^\circ + 6z = 180^\circ$

$6z = 180^\circ - 132^\circ = 48^\circ$

Say What??!!

Sometimes drawing something extra (or auxiliary) on a given graph will help us solve problems whose answers do no seem immediately evident. THAT'S OK!! A problem worthy of attack will prove it's worth by hitting back. In the following example we will draw and use an Auxiliary Line

What is the measure of $\angle ABC$?



Draw $\overleftrightarrow{QP}$ through *B* parallel to *m* and *n*.

$\angle ABP \cong \angle DFB$	**Alternate Interior Angles Theorem**
$m\angle ABP = m\angle DFB$	**Definition of congruent angles**
$m\angle ABP = 75$	**Substitution**
$\angle JBP \cong \angle EJB$	**Alternate Interior Angles Theorem**
$m\angle JBP = m\angle EJB$	**Definition of congruent angles**
$m\angle JBP = 45$	**Substitution**
$m\angle ABC = m\angle ABP + m\angle JBP$	**Angle Addition Postulate**
$= 75 + 45 = 120$	$m\angle ABP = 75$, $m\angle JBP = 45$

*unless otherwise noted, all images are from www.Glencoe.com