Lesson 4

Glencoe Geometry Chapter 2.2 and 2.3

If-Then Statements & Deductive Reasoning

By the end of this lesson, you should be able to

1. Write a statement in if-then Form.
2. To write the converse, inverse, and contrapositive of an if-then statement.
3. Understand The Laws of Detachment and Syllogism
4. Appreciate a simple Ham Sandwich.

If-Then Statements: An if-then statement is a form of _________ reasoning that is just what the name says it is.

We assume that if the "if" part is true, then, by the ***Law of Detachment***, it automatically follows that the "then" part is always true. The previous statement is one such statement!!

For example,

"*If* You are watching this show, *then* you will learn something about geometry."

Or

"*If* D is between C and E, *then* $CD + DE = CE$"

(segment addition postulate)

These kinds of if-then statements are called _______________ **statements**, or just ____________________.

An “if-then” statement contains two parts:

- The “if” part is called the __________________.
- The “then” part is called the _________________.

To symbolize an if-then statement, then let ***p*** represent the hypothesis, and ***q*** represent the conclusion. Here are some other ways to write an if-then statement:

- If *p*, then *q*
- *p* implies *q*
- $p \to q$
- *p* only if *q*
- *q* if *p*
- *q* is necessary for *p*
- *p* is sufficient for *q*

Deductive reasoning, unlike inductive reasoning, is a valid form of ___________. It is, in fact, the way in which geometric proofs are written. Here’s an example:

> A person knows that all the men in a certain room are bakers, that all bakers get up early to bake bread in the morning, and that Jim is in that specific room.

Now this statement is not written in “if-then” form. Sometimes rewriting a statement in this fashion can help clarify:

If a person is in a certain room, then he is a baker.

If a person is a baker, then he gets up early to bake that delicious bread.

So, what can we say about Jim, if Jim is in that specific room???

Knowing these statements to be true, a person could deductively reason that Jim gets up early in the morning and has a recipe for delicious bread!! This is due to the ***Law of Syllogism***, the Transitive property of Logical statements.

$$\text{If } p \rightarrow q \text{ and } q \rightarrow r\text{, then } p \rightarrow r$$

Deductive reasoning in geometry is much like the situation described above, except it relates to geometric terms.

Example:

Statement: Adjacent angles have a common side

Rewriting: *If* two angles are adjacent, *then* they have a common side.

Now, every statement has some relatives:

The _______________ of the statement is formed when the hypothesis and the conclusion are switched. $q \rightarrow p$

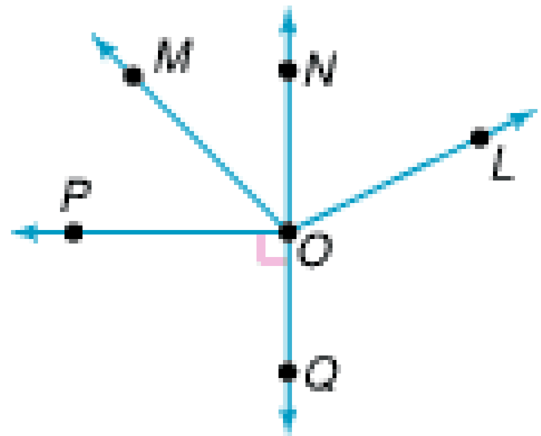
Example:

Converse: *If* two angles have common side, *then* they are adjacent.

Is this a true statement???? If we can find an example that disproves this statement, then we have found a ______________, and we say the statement is false or not true (even though it might be true sometimes).

To find a counterexample, we must find an example that satisfies the hypothesis, but does not satisfy the conclusion.

In the diagram at right, $\angle POM$ and $\angle PON$ have common side $\overrightarrow{OP}$, yet they are NOT adjacent.



In general, the Converse is **NOT** TRUE!

The _______________ of a statement is formed by negating both the hypothesis and the conclusion by adding the word "NOT" in front of them. $\sim p \to \sim q$ or $\neg p \to \neg q$

Example:

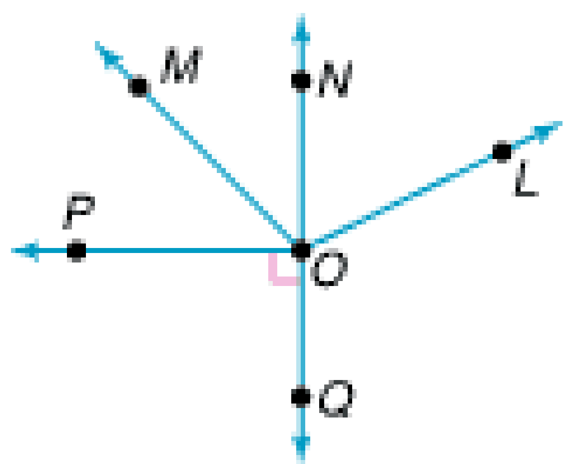
Statement: Vertical angles are congruent

Rewriting: *If* two angles are vertical, *then* they are congruent.

Converse: *If* two angles are congruent, *then* they are vertical.

Inverse: *If* two angles are ***not*** vertical, *then* they are ***not*** congruent.

Is the Inverse true???



To disprove it, we must find two angles are **not** vertical, but that ARE congruent. The same counterexample we found for the converse works here too!!

We can again disprove the converse with the following counterexample.

$\angle QOP \cong \angle NOP$, yet they are ***not*** vertical angles.

There are many examples where two angles are congruent, yet not vertical, so in general, the Inverse statement is NOT true.

The ____________________ of a statement is the combination of the converse and inverse: it is the negation of the switching the hypothesis and the conclusion. $\sim q \rightarrow \sim p$ or $\neg q \rightarrow \neg p$

Example:
Statement: Vertical angles are congruent
Rewriting: *If* two angles are vertical, *then* they are congruent.
Contrapositive: *If* two angle are not congruent, *then* they are not vertical.

We have already shown that the Converse and the Inverse of this statement are not necessarily true, and therefore false. But is the Contrapositive true???

To disprove the contrapositive, we must find two vertical angles that are NOT congruent . . . . . .

Did you find them yet ???? . . . . Me neither!

That's because we know that ALL vertical angles are congruent, so you will never find two that are NOT!!

Therefore, the contrapositive of a statement is ____________ true if the statement is true.

If the original statement is TRUE, the contrapositive is TRUE.
If the original statement is FALSE, the contrapositive is FALSE.
They are said to be *logically equivalent*.

The contrapositive is often more useful than the original statement when testing conditional statements.

Example:
Statement: *If* *n* is an even number, *then* it is divisible by two.
Contrapositive: *If* *n* is not divisible by two, *then* it is not an even number.

We have now established a criteria by which all even numbers must pass in order to be bestowed the glorious title of "even number." *Is 17 an even number?*

What about those *if and only if* (abbreviated ***iff***) statements? These include your ***postulates*** and ***definitions***. Is this stated this way merely for dramatic effect? Are we just trying to be adamantly emphatic?

Example:
Statement: Two angles are complementary *if and only if* their degree measures add to 90°.

If and only if, sometimes denoted by the bi-directional arrow $\Leftrightarrow$, means that each side of the argument implies the other. In other words, the hypothesis and the conclusion are interchangeable. The hypothesis is both necessary AND sufficient for the conclusion.

We can rewrite the previous *iff* statement two ways:

1. *If* two angles are complementary, *then* their measures add up to 90°.
2. *If* two angles' measures add up to 90°, *then* they are complementary.

This automatically means that the Converse of either statement is true. We also know the Contrapositive of both statements is true, which then implies that the Inverse of either statement is true. Isn't Logic fun??!!

Say What??!!

"Let's obey the Laws!!"—Korpi

Example:

The Law of Detachment and other laws of logic can be used to provide a system for reaching logical conclusions, called _____________.

A. inductive reasoning
B. reasonable doubt
C. detachment reasoning
D. deductive reasoning

Example:

"If two numbers are even, then their sum is even" is a true conditional, and 8 and 24 are even numbers. Use the Law of Detachment to reach a logical conclusion.

A. The sum of 8 and 24 must be even.
B. If the numbers 8 and 24 are odd, then their sum is 32.
C. The sum of 8 and 24 must be odd.
D. If the numbers 8 and 24 are even, then their sum is 32.

Example:

"If two angles are vertical, then they are congruent." is a true conditional, and $\angle 1$ and $\angle 2$ are vertical. Which is not true *based on the Law of Detachment*?

A. $\angle 1$ and $\angle 2$ both have a measure of 45.
B. Both $\angle 1$ and $\angle 2$ are obtuse.
C. $\angle 1$ and $\angle 2$ are not congruent.
D. $\angle 1$ and $\angle 2$ are congruent.

Example:

Which statement follows from statements (1) and (2) by the Law of Syllogism?

(1) If an object is a square, then it is a rhombus.
(2) If an object is a rhombus, then it is an equilateral.

A. An object is a rhombus.
B. If an object is a square, then it is an equilateral.
C. An object is a square.
D. If an object is an equilateral, then it is a square.

Example:

Which statement follows from (1) and (2) by the Law of Syllogism?

(1) If two angles form a linear pair, then they are supplementary.
(2) If two angles are supplementary, then the sum of their measures is 180.

A. The measures of supplementary angles add up to 180.
B. The sum of the measures of the angles in a linear pair is 180.
C. Supplementary angles form a linear pair.
D. A linear pair is formed by supplementary angles.

Here's one more use (or misuse) of the Laws of Detachment and Syllogism. Can you find the error? Is there one?

Nothing is better than eternal bliss.
A Ham Sandwich is better than nothing.

A Ham Sandwich is better than eternal bliss!

(you heard it hear first!)

http://i23.photobucket.com/albums/b372/DonSurber/ham_sandwich.jpg

*unless otherwise noted, all images are from www.Glencoe.com