Lesson 3

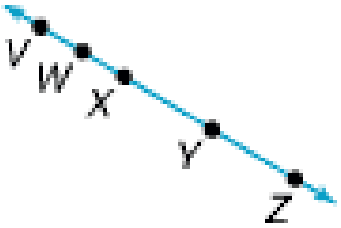
Glencoe Geometry Chapter 1.6 & 1.7

Angles: Exploration & Relationships

By the end of this lesson, you should be able to

1. Identify angles and classify angles.
2. Use the Angle Addition Postulate to find the measure of angles.
3. Identify and use congruent angles and the bisector of an angle.
4. Identify and use special pairs of angles.
5. Identify your favorite Math television program host.

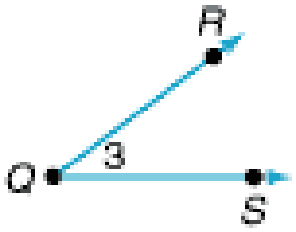
Remember from Lesson 1 that a ray has one fixed end and extends indefinitely in one direction. For example $\overrightarrow{YV}$ in the figure at right. Since direction matters, $\overrightarrow{YV}$ and $\overrightarrow{YZ}$ are called opposite rays, but they share a common endpoint. Opposite rays are always collinear.



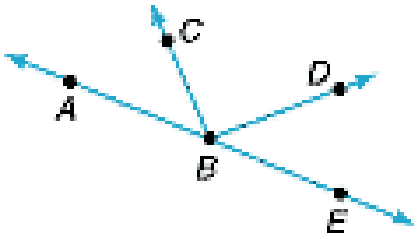
An angle is usually formed by two non-collinear rays with a common endpoint. The common endpoint is called the vertex.

Give some names for the angle at right:

$\angle Q, \angle RQS, \angle SQR,$ or $\angle 3,$ But NOT $\angle RS$!!



Notice in the last diagram, there was only one angle. You must be more careful when naming different angles that share a common vertex. In the diagram below, you CANNOT name either of the angle as just $\angle B$!!! What *are* some names?



$\angle ABC, \angle CBA$

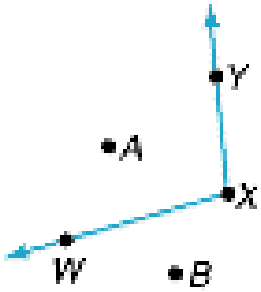
$\angle CBD, \angle DBC$

$\angle DBE, \angle EBD$

Angle $\angle ABE$ or $\angle EBA$ is called a straight angle, since $\overrightarrow{BA}$ and $\overrightarrow{BE}$ are opposites.

An angle separates a plane into three distinct parts:

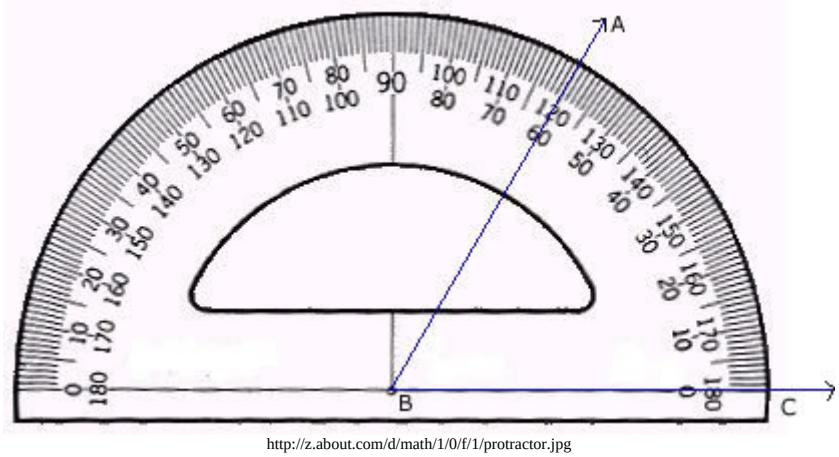
1. The interior of the angle. *A*
2. The exterior of the angle. *B*
3. and the angle itself. $\angle X$



We typically measure angles in degrees using a protractor.

*All angles this year will be in degrees. The degree symbol is sometimes used, but without it, we infer that the measure is still in degrees:

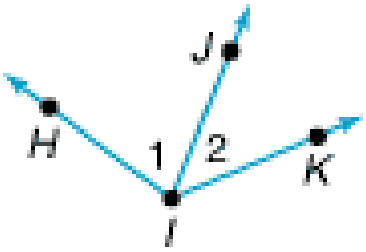
$85^\circ = 85$



http://z.about.com/d/math/1/0/f/1/protractor.jpg

Using the inner scale, we can say that the degree measure of $\angle ABC$ is 60, or equivalently, $m\angle ABC = 60$

By the Angle Addition Postulate, in the figure below, $m\angle HIJ + m\angle JIK = m\angle HIK$ . . . . . Duhhhh!!



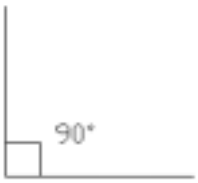
So, what is $m\angle HIJ$ if $m\angle JIK = 45^\circ$ and $m\angle HIK = 100^\circ$?

$$m\angle HIJ + m\angle JIK = m\angle HIK$$

$$m\angle HIJ + 45^\circ = 100^\circ$$

$$m\angle HIJ = 100^\circ - 45^\circ = 55^\circ$$

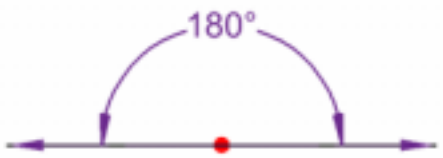
We can also classify individual angles by their measures:



right angle = 90

acute angle < 90

obtuse angle > 90



straight angle = 180

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Congruent angles have the same measure. Which of the angles above are congruent to all others in the same class?

right angles and straight angles

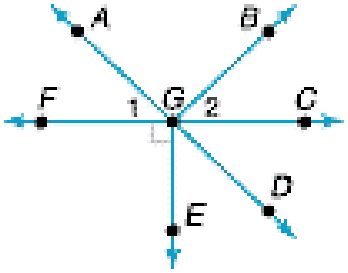
Two angles that add to 180 are said to be supplementary angles.

Two angles that add to 90 are said to be complementary angles.

An angle bisector is a **ray** that divides and angle into two congruent angles.

Example:

If $\overrightarrow{GD}$ bisects $\angle CGE$, which angle is congruent to $\angle CGD$?

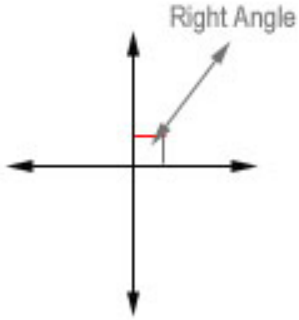


$\angle EGD$

What other angle is congruent to $\angle CGE$?

$\angle FGE$

When two lines intersect, they form four angles. When they intersect to form four right angles, we say the lines are perpendicular, and denoted by the $\perp$ symbol Not all lines are perpendicular to each other, though.



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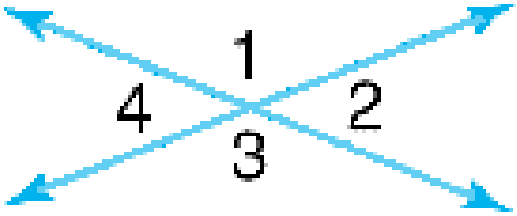


When two lines intersect, it is useful to classify angles by their relationship to other angles.

Adjacent Angles—have a common vertex and a common side with no common interior points

Ex) $\angle 1 \& \angle 2,\ \angle 2 \& \angle 3,\ \angle 3 \& \angle 4,\ \angle 4 \& \angle 1$

Vertical Angles—non-adjacent angles across from each other. Vertical angles are congruent!!!

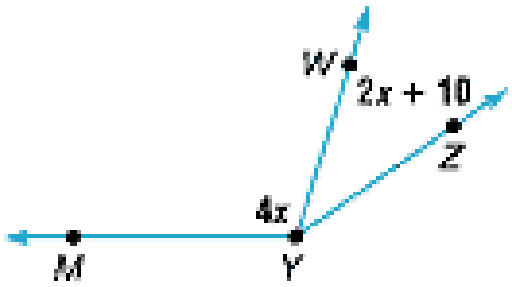
Ex) $\angle 1 \& \angle 3,\ \angle 2 \& \angle 4$

Linear Pair—adjacent angles formed by opposite rays. Linear pairs will always be supplements of each other. Which angle above are linear pairs?

All the adjacent angles

Example:

If $m\angle MYZ = 160$, what is $m\angle MYW$?



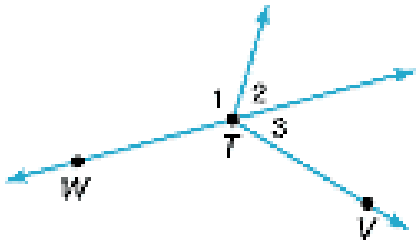
$$\angle MYW + \angle WYZ = \angle MYZ$$
$$(4x) + (2x + 10) = 160$$
$$6x + 10 = 160$$
$$6x = 150$$
$$x = \frac{150}{6} = 25$$

Example:

Name two angles that are adjacent to $\angle WTV$.

A. $\angle 1$ and $\angle 2$ B. $\angle 2$ and $\angle 3$

C. $\angle WTV$ and $\angle 3$ D. $\angle 1$ and $\angle 3$



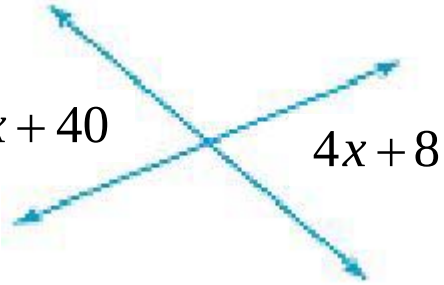
Example:

If $m\angle 1 = 2x$ and $m\angle 2 = 4x$. Find the value of x if $\angle 1$ and $\angle 2$ are complementary.

$$m\angle 1 + m\angle 2 = 90$$
$$2x + 4x = 90$$
$$6x = 90$$
$$x = \frac{90}{6} = 15$$

Example:

Find the value of x.



These are vertical angles so they are congruent.

$$2x+40=4x+8$$

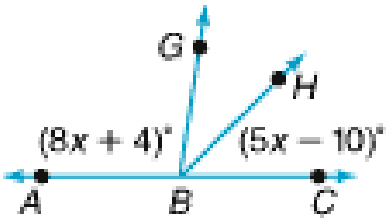
$$2x-4x=8-40$$

$$-2x=-32$$

$$x=16$$

Example:

If $m\angle ABC = 180$ and $x = 12$, what is $m\angle GBH$?



$$m\angle ABG + m\angle GBH + m\angle HBC = 180$$

$$(8x+4) + m\angle GBH + (5x-10) = 180$$

$$100 + m\angle GBH + 50 = 180$$

$$150 + m\angle GBH = 180$$

$$m\angle GBH = 180 - 150 = 30$$

Say What??!!

Circle the right Answer:

1. Angles are measured in units called (sides) or (degrees).
2. In Figure 1, $\angle 2$ and $\angle 3$ are (complementary) or (supplementary) angles.
3. A (compass) or (protractor) is used to find the measure of an angle.
4. In Figure 2, the two angles shown are (supplementary) or (congruent) angles
5. In Figure 3, $\angle 5$ and $\angle 6$ are (vertical) or (adjacent) angles.
6. Perpendicular lines intersect to form (obtuse) or (right) angles.
7. In Figure 3, *A* is called (a side) or (the vertex) of $\angle 6$.
8. In Figure 1, $\angle 1$ and $\angle 4$ form a _____ or (right angle).
9. In Figure 4, $\overrightarrow{KM}$ is the (vertex) or (bisector) of $\angle JKL$.

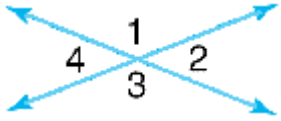


Figure 1

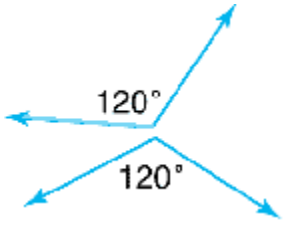


Figure 2

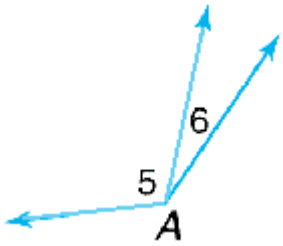


Figure 3

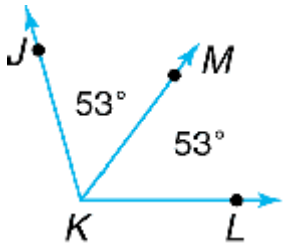


Figure 4

*unless otherwise noted, all images are created using TI-Interactive Software or from www.Glencoe.com