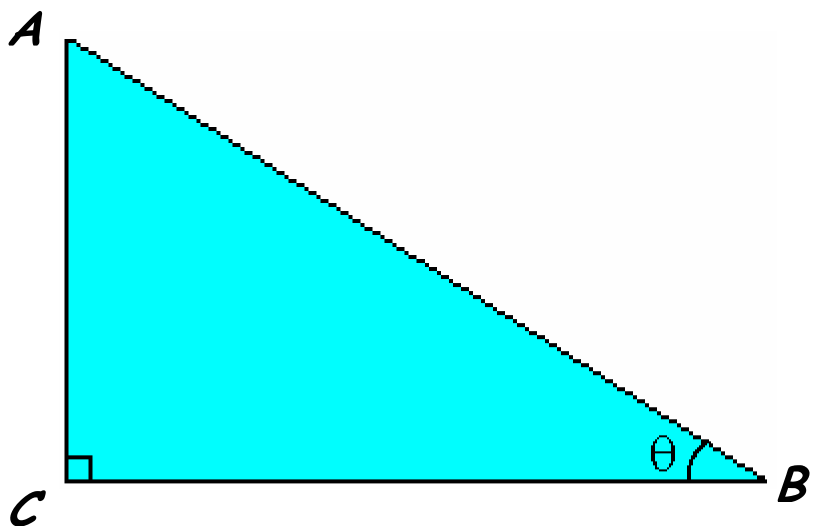
Lesson 29

Glencoe Geometry Chapter 8.2

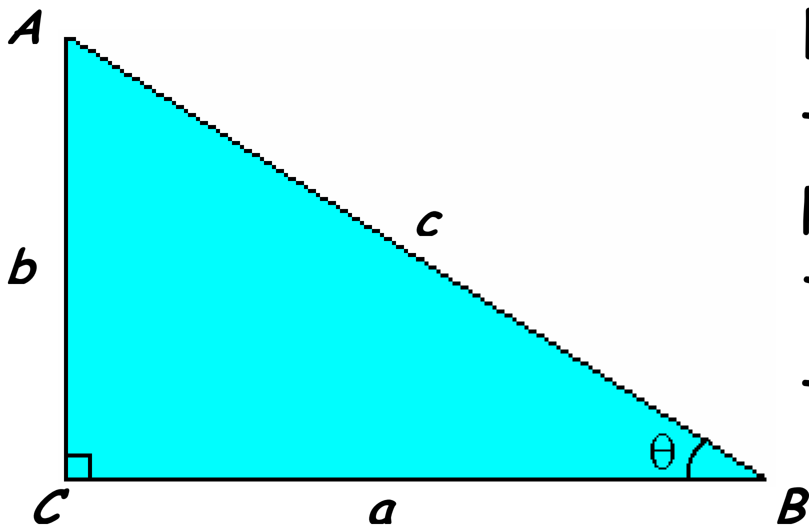
Special Right Triangles

Today we will look at two special right triangles, their side lengths, and their angle measure. These two triangles are not only very important in geometry, but the form the basis of the study of coordinate geometry and trigonometry.

Remember a right triangle is a three-sided polygon with exactly one angle of 90°. The other two angles are complementary, meaning the sum of their measures is also 90°.



We can label the vertices as capital letters, and the line segments opposite the angles as the corresponding lower-case letters.



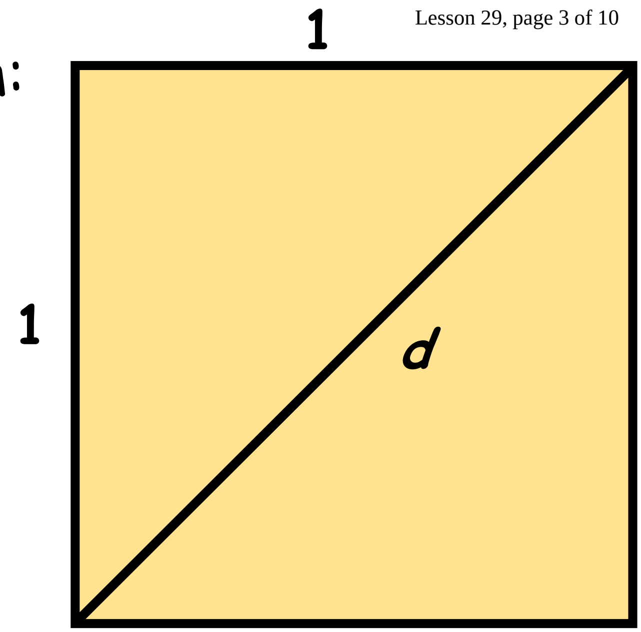
For all right triangles, the Pythagorean Theorem relates the three sides:

$$a^2 + b^2 = c^2$$

We donate an angle (and its measure) by using θ, the capital Greek letter "theta."

When specifically referring to a one of the two non-right angles, like above, we call that angle the ____________________ angle.

Let's start our study by looking at a unit square (each side of one unit in length) and its diagonal, ***d***.



By the Pythagorean theorem:

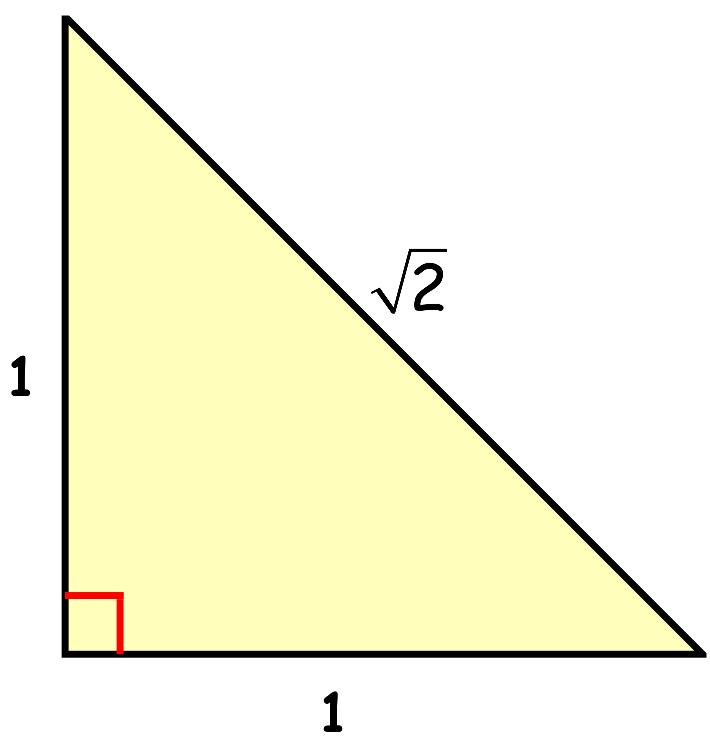
$$d^2 = 1^2 + 1^2$$

$$d^2 = 2$$

$$d = \sqrt{2}$$

We now have the unit lengths of an isosceles right triangle: $1 - 1 - \sqrt{2}$

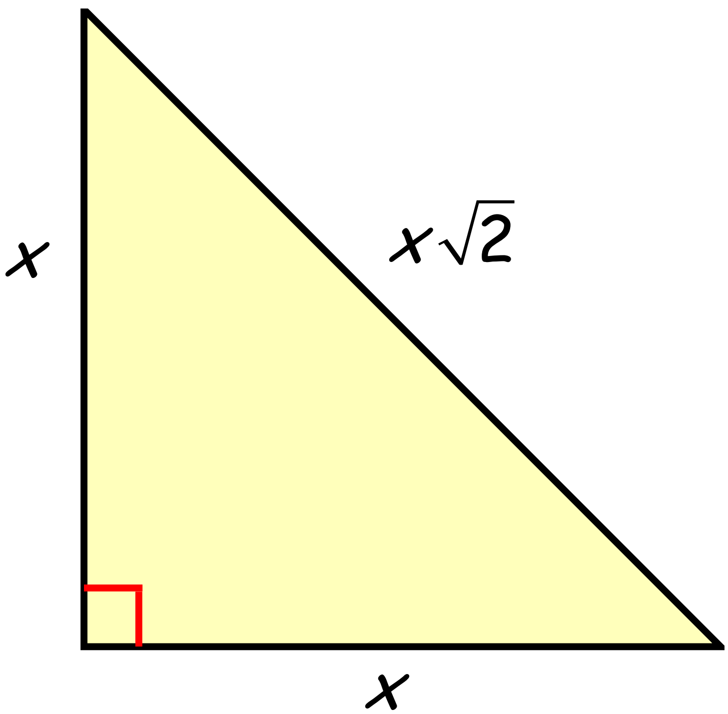
Now we take notice of the angle measures. Since the diagonal of a square bisects the angles (which were originally 90°), each of the angles in the triangle is 45°.



Using the idea of proportion and similar triangles, we can multiply all three sides by any non-negative number.

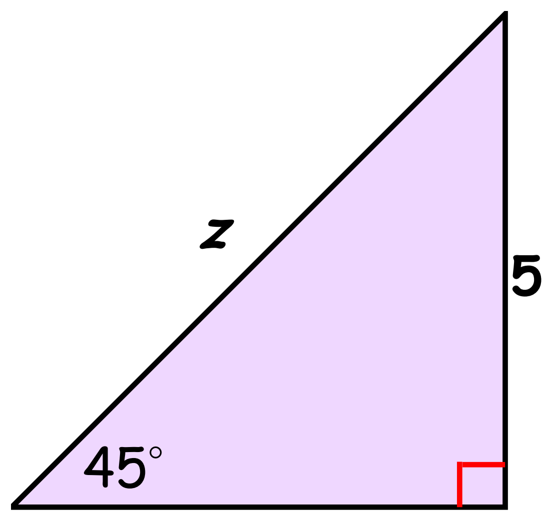
Special Triangle #1

<u>Theorem:</u> In a $45^\circ - 45^\circ - 90^\circ$ triangle, the hypotenuse is $\sqrt{2}$ times as long as a leg.



Example:

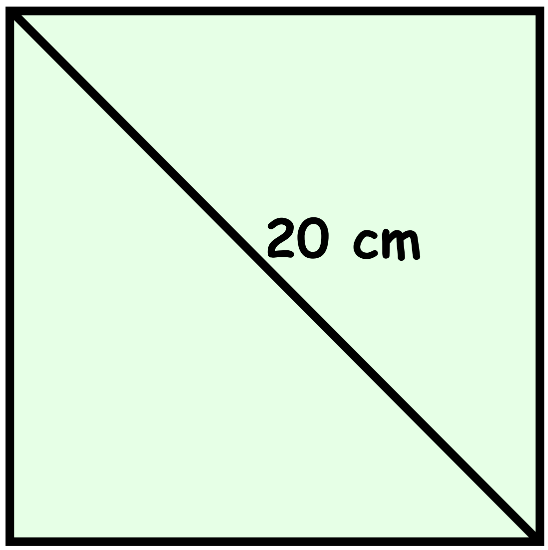
Find the value of z.



$45^\circ - 45^\circ - 90^\circ$ Rule of Thumb #1:
When going from a leg to a hypotenuse, we ____________ by $\sqrt{2}$.

Example:

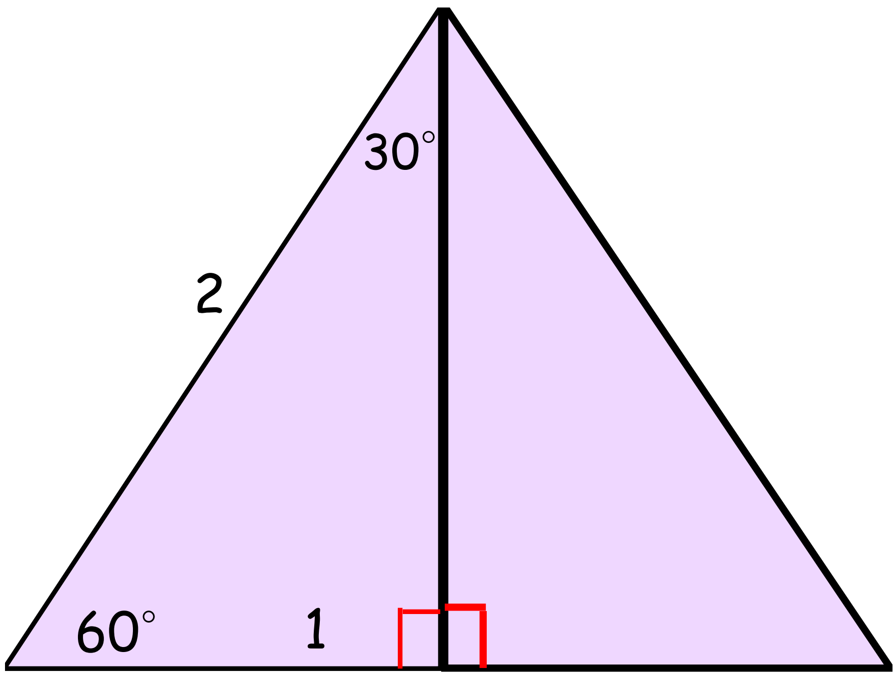
The length of a diagonal of a square is 20 centimeters. Find the length of a side of a square.



$45^\circ - 45^\circ - 90^\circ$ Rule of Thumb #2:
When going from a hypotenuse to a leg, we __________ by $\sqrt{2}$.

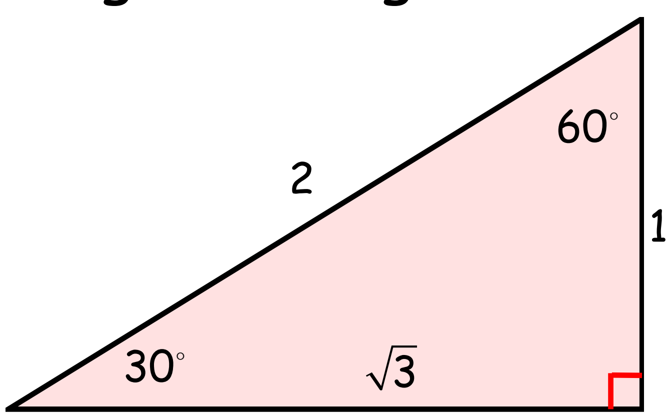
Now let's take a look at our second special right triangle.

This one is derived from an equilateral triangle with a side length of 2 units.



Special Triangle #2

<u>Theorem:</u> In a $30^\circ - 60^\circ - 90^\circ$ triangle, the corresponding side lengths are $1-\sqrt{3}-2$.

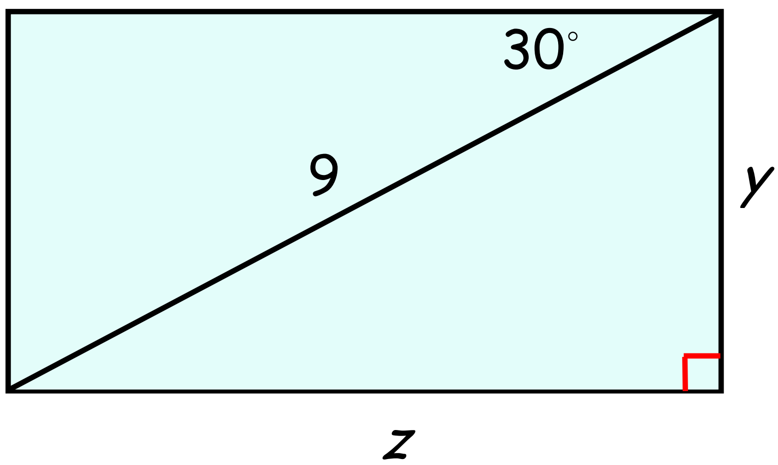


Remember that the relative sizes of the angles and side lengths correspond.

For example, the smallest angle is 30°, so it is opposite the shortest side, the unit length of 1.

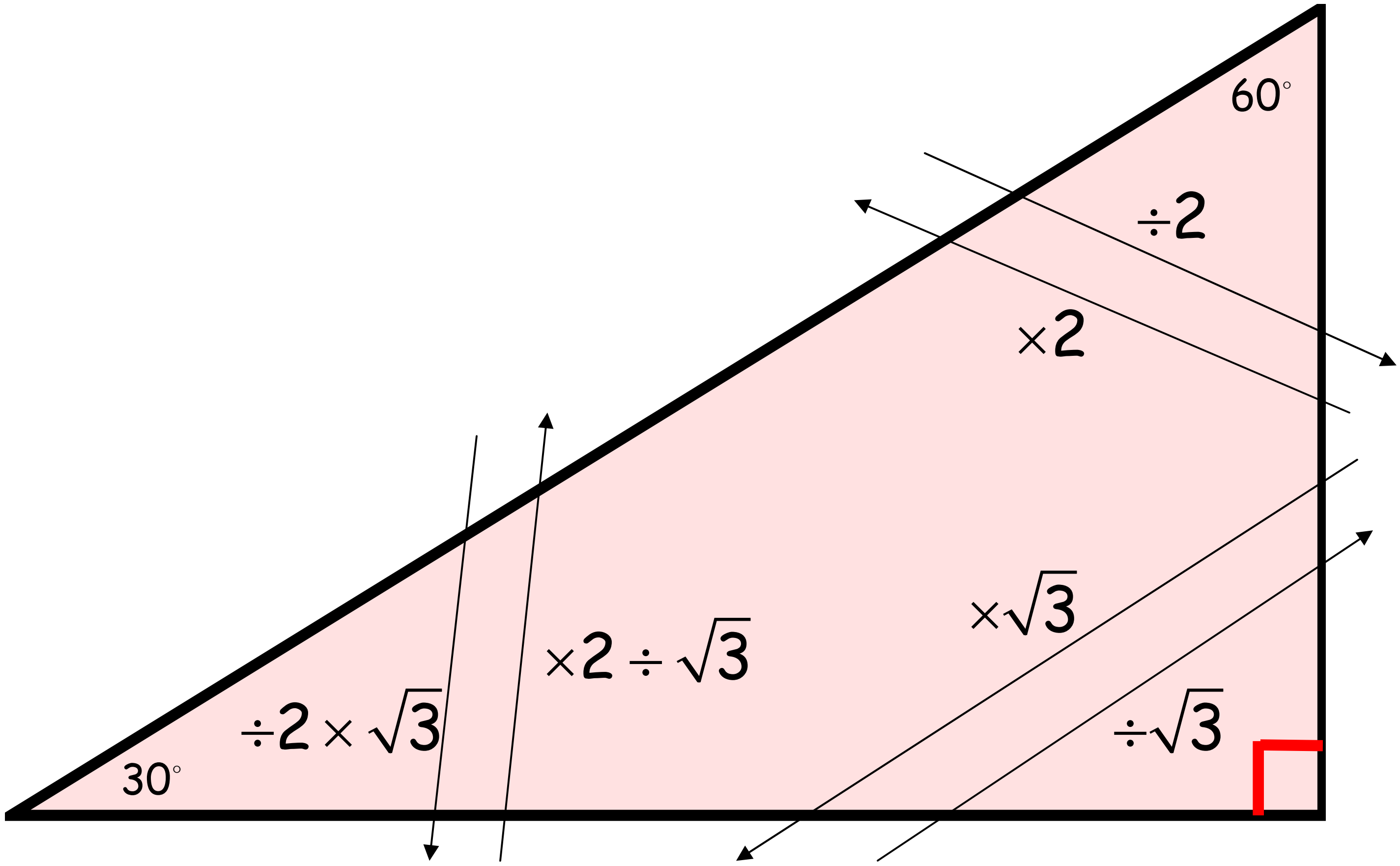
Example:

Find the value of y and z.



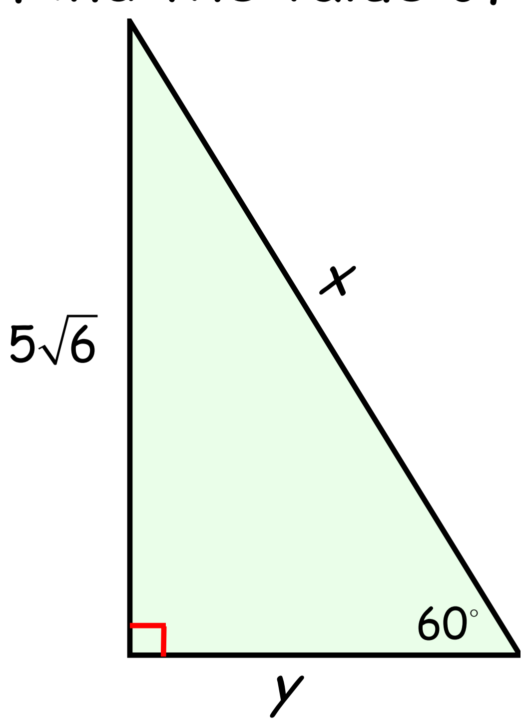
$30° - 60° - 90°$ Rule of Thumb:
Memorize the unit lengths, then appropriately multiply or divide by the correct value to get one side from another!!

Let me show you what I mean . . .



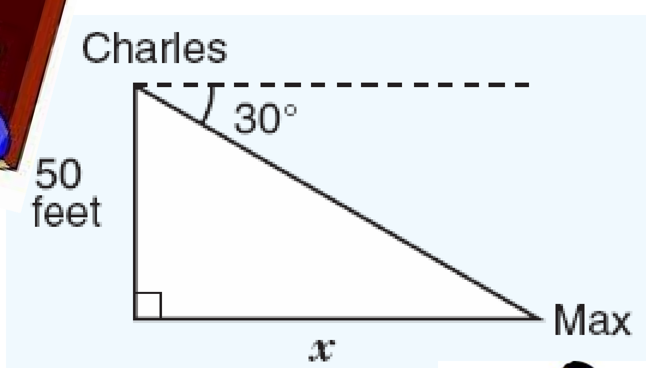
Example:

Find the value of *x*.

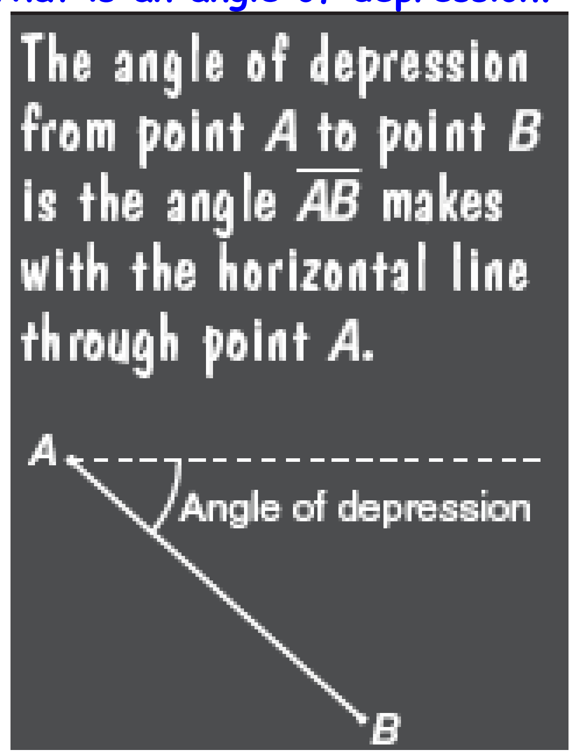


Say What?!?!

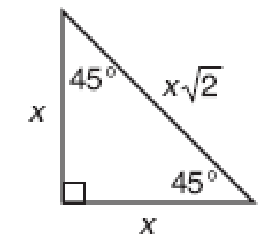
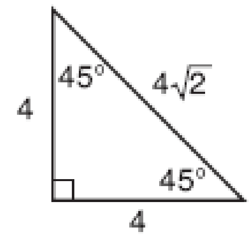
Charles is looking out a window from a point 50 feet above the ground. When Charles looks down at an angle of depression of 30°, he sees his dog Max. To the nearest foot, how far is Max from the base of the building?



What is an angle of depression?



Summary

<table>
<tr><th>Special Right Triangle</th><th>Side Relationships</th><th>Example</th></tr>
<tr><td>45°-45°-90° Triangle
(Isosceles Right)

</td><td>The lengths of the two legs are equal.
The length of the hypotenuse is the length of a leg times $\sqrt{2}$.</td><td>
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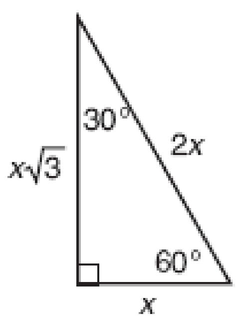
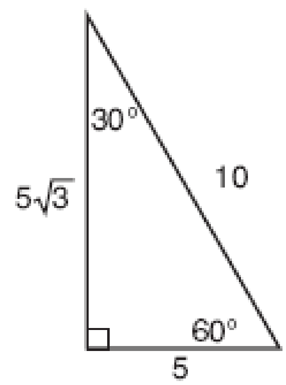
<tr><td>30°-60°-90° Triangle

</td><td>The length of the hypotenuse is twice the length of the shorter leg.
The length of the longer leg is equal to the length of the shorter leg times $\sqrt{3}$.</td><td>
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