Lesson 27

Glencoe Geometry Chapter 9.1 and 9.2

Circles, Angles, and Arcs

What would our world be without circles? From car tires, to CDs and DVDs, to gears, and crop circles, the circle is one of the most useful and important of all geometric shapes.

Science, and particularly *geometry* and *astronomy/astrology*, was linked directly to the divine for most medieval scholars. The compass in this 13th century manuscript is a symbol of God's act of *Creation*. Many believed that there was something intrinsically "divine" or "perfect" that could be found in circles. God has created the universe after geometric and harmonic principles, to seek these principles was therefore to seek and worship God.

God the Geometer

But what is a circle, really?

Well, we have already seen that it is

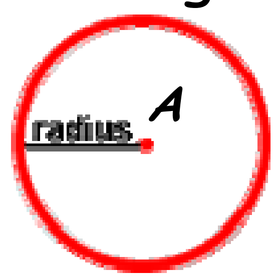
- the **shape of a cross-section of a cone** (or a cylinder)
- round
- Circularlyish
- Very round

Here's its precise ***Locus*** definition.

A ***circle*** is the set of all points in a plane that are a given distance, ***r***, from a given point, ***A***, in that plane.

r is called the radius

A is called the center



Circles are usually named by their center. This circle is called $\odot C$.

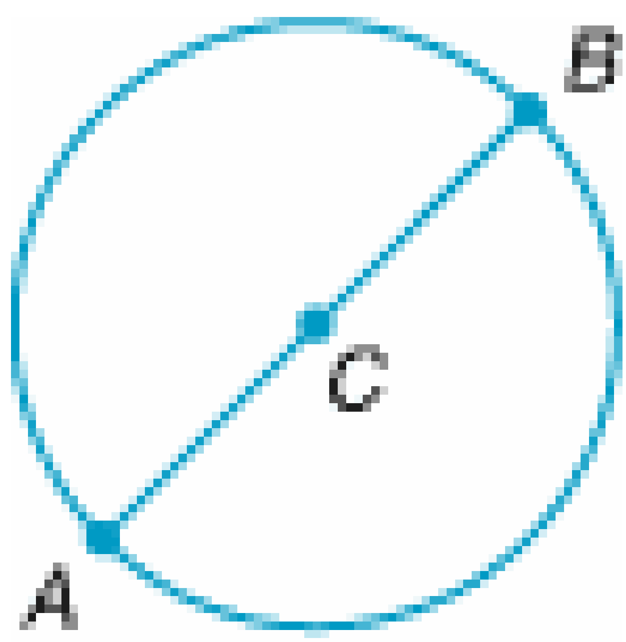
Center: C

Radius: $\overline{CA}$ or $\overline{CB}$

Diameter: $\overline{AB}$ or $\overline{BA}$

Note: The diameter, d, is twice the radius:

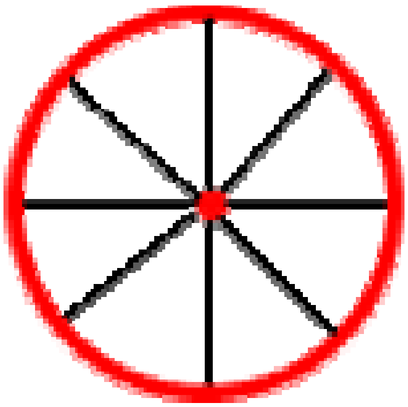
$$d = 2r \text{ or } r = \frac{d}{2}$$



Circles are simple closed curves which divide a plane into an interior and exterior.

- The ***circumference***, ***C***, of a circle means the length of the [perimeter of the] circle (the distance all the way around it). The formula is given by $C = 2\pi r$ or πd

Circumference, diameter, and radii are measured in linear units, such as inches and centimeters. A circle has **many different** radii and many different diameters, each passing through the center.



A real-life example of a radius is the spoke of a bicycle wheel.

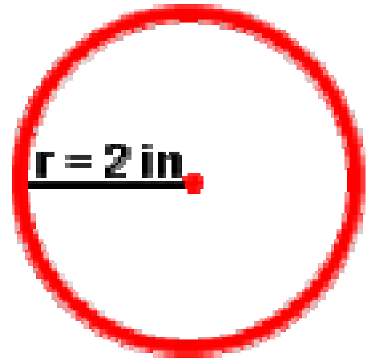
http://patentpending.blogs.com

A 9-inch pizza is an example of a diameter: when one makes the first cut to slice a round pizza pie in half, this cut is the diameter of the pizza. So a 9-inch pizza has a 9-inch diameter.

http://www.superbrandsindia.com

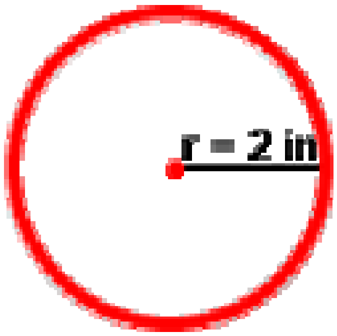
Examples:

1. The radius of a circle is 2 inches. What is the diameter?



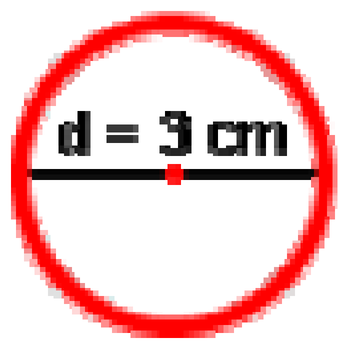
$$d = 2r$$
$$d = 2(2in)$$
$$d = 4in$$

2. The radius of a circle is 2 inches. What is the circumference?



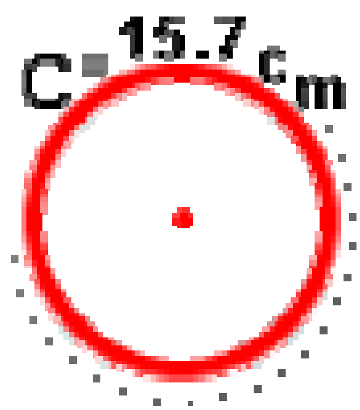
$$C = 2\pi r$$
$$C = 2(\pi)(2in)$$
$$C \approx (3.14159\ldots)(4cm) \approx 12.566in$$

3. The diameter of a circle is 3 centimeters. What is the circumference?



$$C = \pi d$$
$$C = \pi(3cm)$$
$$C \approx (3.14159\ldots)(3cm) \approx 9.425cm$$

4. The circumference of a circle is 15.7 centimeters. What is the diameter?



$$C = \pi d$$
$$15.7cm = \pi d$$
$$d \approx \frac{15.7cm}{3.14.59\ldots} \approx 4.997cm$$

Application:

The <u>diameter</u> of your bicycle wheel is 25 inches. How far will you move in one turn of your wheel?

http://patentpending.blogs.com

The key here is to realize that the translated forward distance on the bike for one revolution of the wheel is nothing more than the wheel's circumference. This is what we are looking for.

$C = \pi d$

$C = 25\pi \approx 78.540 in$

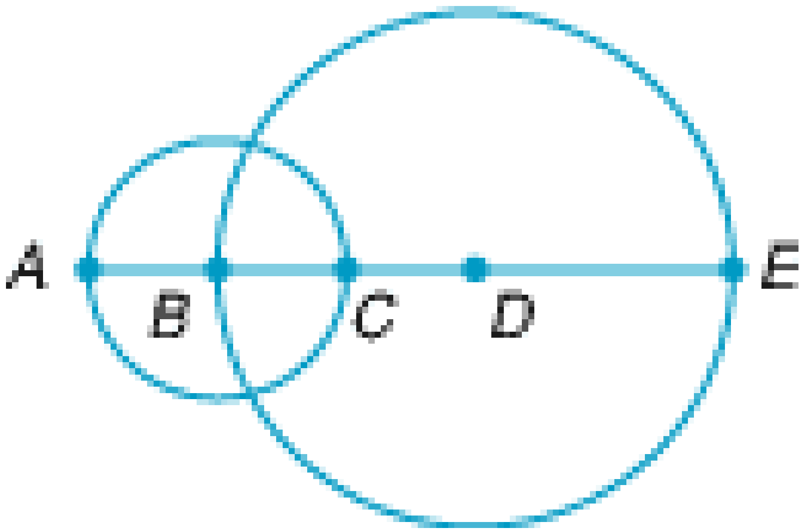
Extension:

Refer to circles *B* and *D* in the figure below. If *BC* = 5 and *CD* = 5, find *AE*.

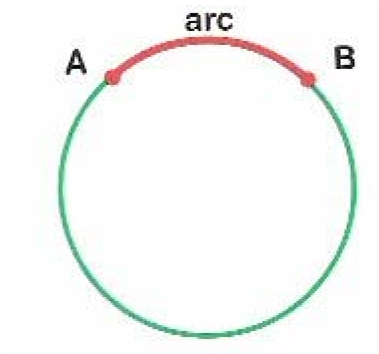
AE is the sum of the radius of $\odot B$ and the diameter of $\odot D$. If we find the radius of each circle, we can find *AE*.

The radius of $\odot B$ is $BC = 5$.
The radius of $\odot D$ is
$BC + CD = 5 + 5 = 10$
The diameter of $\odot D$ is 20.

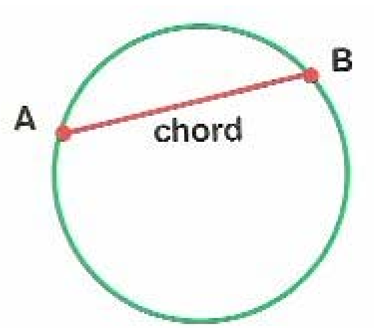
So,
$AE = 5 + 20 = 25$



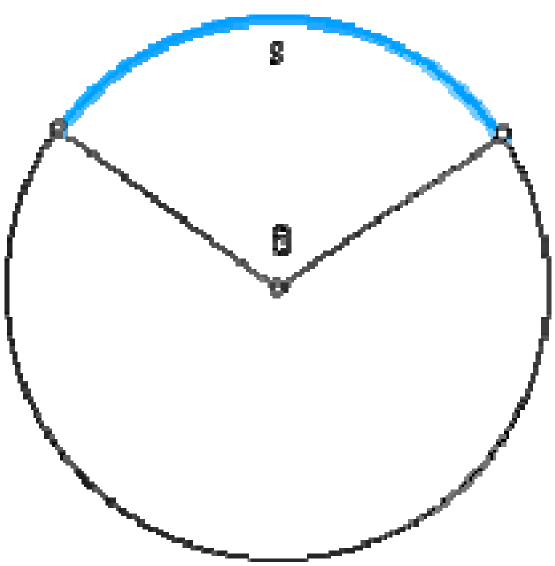
Circles have other parts as well:



- An ***arc*** is any continuous portion of a circle.
- A ***chord*** is any segment that has its endpoints on the circle. A diameter is the longest chord in a circle. Is a radius a chord? NO-one of its endpoints is at the center (not on the circle).



Here's another very important property of circles:



A central angle, θ, is an angle whose vertex is at the center of a circle. Every arc, ***s***, subtends a central angle, θ, of a circle.

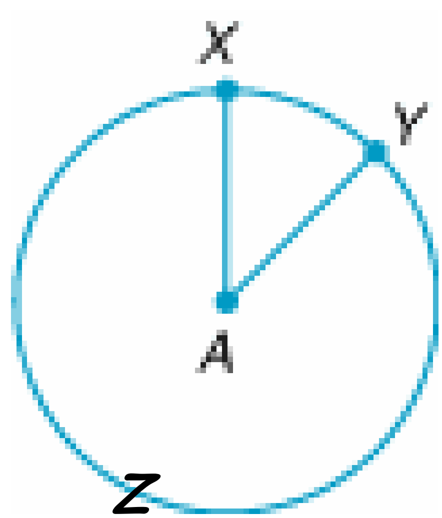
s can refer to the arc itself, or to the ***measure*** of the arc.

The sum of the measures of the central angles of a circle is 360°

This means that if you rotate the radius all the way around a circle, you will sweep out an arc that ***IS*** the circumference, and you will have rotated 360°.

Arcs are measured by their corresponding central angle, and are denoted by the letters of the points defining them with an arc over the letters. For example, in the circle below, $\overset{\frown}{XY}$

A central angle separates a circle into two adjacent arcs, by which we can classify them.



a) ***minor arc***: central angle is less than 180°. $\overset{\frown}{XY}$

b) ***major arc***: central angle is greater than 180°. $\overset{\frown}{XZY}$

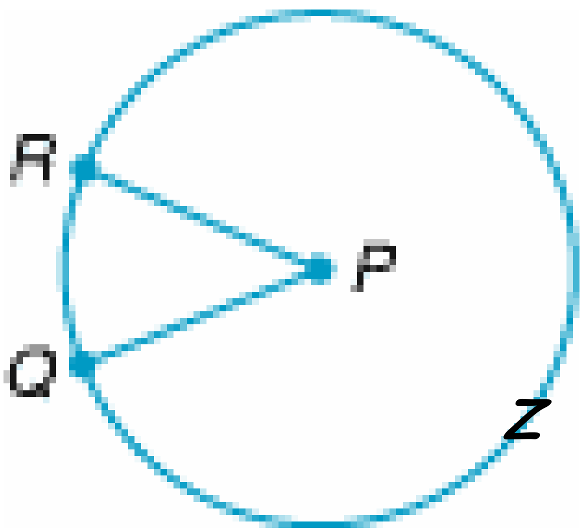
***major arcs use 3 letters to denote.**

Realize that the measure of the major arc is 360 minus the measure of the minor arc. So, if the major and minor arcs are the same measure (180° each), the arcs are called **semicircles**.

This also means the sum of the major and minor arcs equals 360, one full rotation.

Example:

In $\odot P$, $m\angle QPR = 40$. Find $m\overset{\frown}{QR}$.

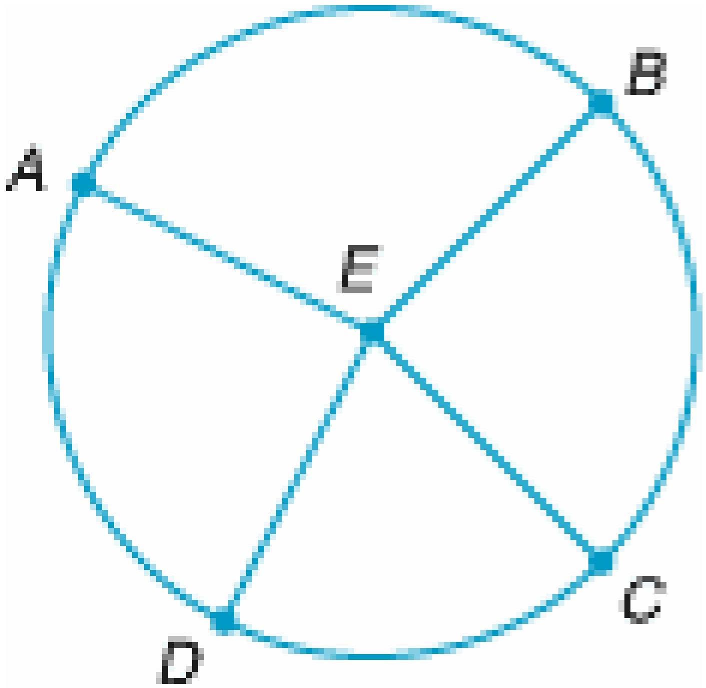


Since the center of the circle is point P, angle QPR is a central angle. The measure of arc QR is, therefore, the same as the measure of central angle QPR, which is 40. Consequently, arc QR is a minor arc.

We also can find $m\overset{\frown}{QZR}$, the measure of the major arc: $m\overset{\frown}{QZR} = 360 - m\overset{\frown}{QR} = 360 - 40 = 320$

Example: Algebra

In $\odot E$, $m\angle AEB = 4x + 18$, $m\angle BEC = 5x + 4$, $m\angle CED = 3x + 4$, and $m\angle AED = 5x - 6$. Of course, find x. Then find $m\overset{\frown}{DAB}$ and classify it as major or minor.



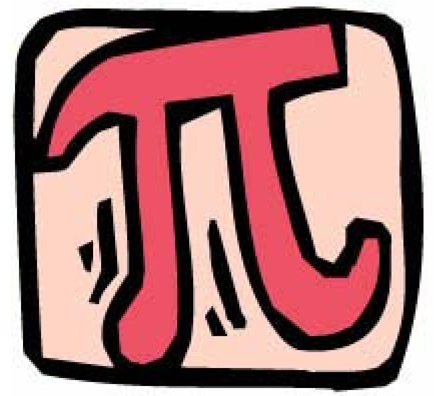
These consecutive arcs make up the entire circumference of the circle. We know that the sum of the interior angles add to 360, so all we need to do is add up the variable expressions for the angle measures, set them equal to 360, and isolate the variable:

$$(4x+18)+(5x+4)+(3x+4)+(5x-6)=360$$
$$(4x+5x+3x+5x)+(18+4+4-6)=360$$
$$17x+20=360$$
$$17x=340$$
$$x=\frac{340}{17}=20$$

$$m\overset{\frown}{DAB}=m\overset{\frown}{DA}+m\overset{\frown}{AB}$$
$$m\overset{\frown}{DAB}=(5x-6)+(4x+18)$$
$$m\overset{\frown}{DAB}=5(20)-6+4(20)+18$$
$$m\overset{\frown}{DAB}=180+12=182>180$$

So $\overset{\frown}{DAB}$ is a major arc

Say What?!?!



Archimedes: Part III

Archimedes loved circles
(remember: "please do not disturb my circles!")
He was one of the first known people to use *gears* to build elaborate machines (clocks/calendars) that rival today's mechanisms.

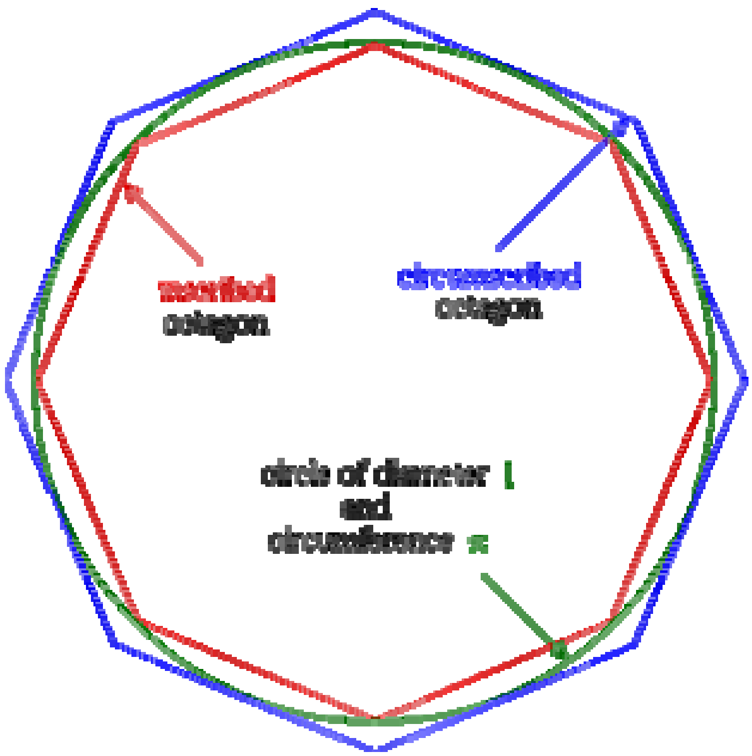
He was also the first to *give* us a theoretical, calculated (rather than measured) upper and lower bound for π. He found $3\frac{10}{70} < \pi < 3\frac{10}{71}$, which in decimals is $3.142857143 < \pi < 3.14084507$, correct to two decimals.

What is π exactly? From the formula for the circumference, $C = \pi d$, and solving for π, we get:

$$\pi = \frac{C}{d}$$

That is, π is the constant ratio between ***ANY*** circle's **circumference to its diameter**.

How did Archimedes do it? Good question!



He did this by modifying one of Euclid's theorems, developing a formula for the circumscribed perimeters of small regular polygons, and applied it eventually to a 96-sided inscribed and circumscribed polygon.

The method of Exhaustion!!

What was additionally remarkable is that he reduced a geometric calculation to a purely algebraic procedure, something that is probably still unsettling for Plato.

For more detailed information on this topic, check out the following websites:

http://itech.fgcu.edu/faculty/clindsey/mhf4404/archimedes/archimedes.html

or

http://www.pbs.org/wgbh/nova/archimedes/pi.html

Resources

- www.glencoe.com
- http://en.wikipedia.org/wiki/Circle
- http://mathworld.wolfram.com/Circle.html
- http://www.mathgoodies.com/lessons/vol2/circumference.html
- http://www.math-worksheets.info/Circle_2.html