Lesson 26

Glencoe Geometry Chapter 11.7

Surface Area and Volume of SPHERES!!!!

Today we look at the fascinating sphere.

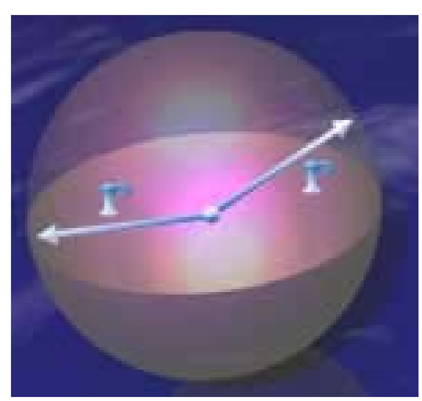
Of all the shapes, a sphere has the smallest surface area for a volume. Or put another way it can contain the greatest volume for a fixed surface area.

Example: if you blow up a balloon it naturally forms a sphere because it is trying to hold as much air as possible with as small a surface as possible.

The sphere appears in nature whenever a surface wants to be as small as possible. Bubbles and water drops, for example

Here are some other interesting things to notice about spheres:

- It is perfectly symmetrical
- It has no edges or vertices
- It is **not** a polyhedron
- All points on the surface are the same distance from the center

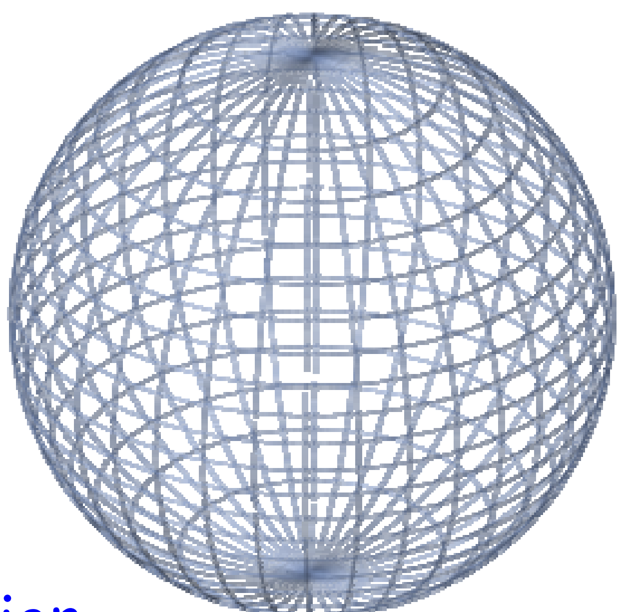


http://www.mathsisfun.com/geometry/sphere.html

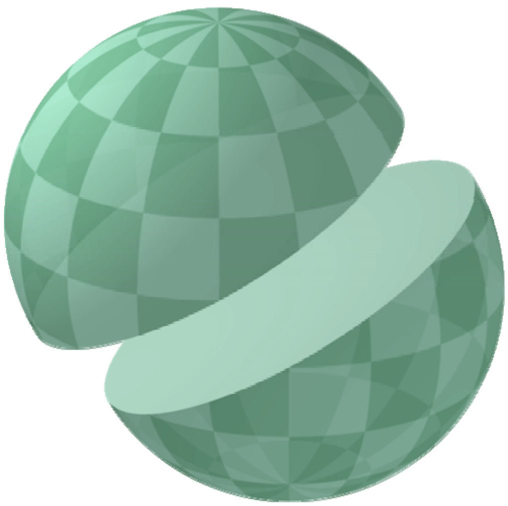
Here's some terminology:

A **great circle** is a circle on the sphere that has the same center and radius as the sphere, and consequently divides it into two equal parts. The shortest distance between two distinct non-polar points on the surface and measured along the surface, is on the unique great circle passing through the two points.

If a particular point on a sphere is designated as its **north pole**, then the corresponding antipodal point is called the **south pole** and the equator is the great circle that is equidistant to them. Great circles through the two poles are called lines (or meridians) of longitude, and the line connecting the two poles is called the axis of rotation.



Circles on the sphere that are parallel to the equator are lines of latitude. This terminology is also used for astronomical bodies such as the planet Earth, even though it is actually a geoid.



A sphere is divided into two equal **hemispheres** by any plane that passes through its center.

Now let's look at two formulas for Spheres that will help us find some important quantities.

The Surface Area of a Sphere with radius ***r*** is given by

$$SA = 4\pi r^2$$

The Volume of a Sphere is given by

$$V = \frac{4}{3}\pi r^3$$

Notice that there are only two variable in each equation. Remember $\pi \approx 3.14159\ldots$

If we know the **radius**, we can find the **surface area** or **volume**.

If we know the **surface area** or **volume**, we can find the **radius**.

You will have several types of problems. The first is when you are **given the radius**. These will be "plug-and-chug" varieties.

Example:

You and your friends are on Spring Break playing with a beach ball. Your "mathy" friend asks, ***"Hey, how much vinyl makes up this ball, and how much air is inside?"*** Your friend is simply inquiring about the sphere's surface area and volume, respectively.

Let's assume the ball's diameter is 3 feet.

http://www.cistyles.com/forms.htm

The radius is half the diameter, or 1.5 feet. So, the surface area is
$SA = 4\pi(1.5)^2 = 9\pi \approx 28.274\ ft^2$

The volume of the sphere is
$V = \frac{4}{3}\pi(1.5)^3 = 4.5\pi \approx 14.137\ ft^3$

Sometimes you may need to find the radius when given either the volume or surface area. Remember these equations are really formulas or literal equations and can be used in any equivalent form.

Example:

After a game of beach ball, **Jenna** replenishes her electrolytes by peeling and eating a delicious **orange**. When she is done, relaxing in the cool spring breeze, she calculates the amount of **orange** peeling from her snack to be roughly **80 square inches**. Now she wants to know what the radius of her **orange** was!

http://www.nutratechinc.com/advz/

We essentially know the surface area, and we want to know the radius. We can get the working equation by solving the surface area equation for the radius.

$SA = 4\pi^2$ so $r = \sqrt{\frac{SA}{4\pi}}$ plugging in our quantities, we get

$$r = \sqrt{\frac{80}{4\pi}} = \frac{\sqrt{20}}{\sqrt{\pi}} = \frac{2\sqrt{5}}{\sqrt{\pi}} \approx 2.523\ in$$

The other way to solve this type of equation is to plug in all your numeric values, THEN solve for the desired variable.

$$SA = 4\pi r^2 \rightarrow 80 = 4\pi r^2 \rightarrow r^2 = \frac{80}{4\pi} \rightarrow r = \sqrt{\frac{80}{4\pi}}$$

Now the problems get more interesting when you are given neither the radius, diameter, surface area, OR the volume, but instead some other quantity. In this case, you will have to extract the needed information, essentially squeezing your own orange juice.

Example:

Find the volume of a sphere whose great circle has a circumference of 31.4 in.

The radius of a great circle is the same as the radius of a sphere. We first need to find the radius of the circle using the circumference formula:

$$C = 2\pi r \rightarrow 31.4 = 2\pi r \rightarrow r = \frac{31.4}{2\pi} \approx 5 \ in$$

So the radius of the sphere is also 5 inches.

The volume is then

$$V = \frac{4}{3}\pi r^3$$

$$V = \frac{4}{3}(\pi)\left(5^3\right) = 523.599 \ in^3$$

Here's one that requires a bit more work (so obtaining its result will be more satisfying)!

Example:

Find the volume of a sphere with a surface area of 615.8 in^2.

We still need to extract the radius. We can do that from any formula, whether it is the circumference or the surface area:

$$SA = 4\pi r^2$$

$$615.8 = 4\pi r^2 \rightarrow r = \sqrt{\frac{615.8}{4\pi}} \approx 7 \ in$$

Now

$$V = \frac{4}{3}\pi r^3 \rightarrow V = \frac{4}{3}(\pi)\left(7^3\right) \approx 1436.755 in^3$$

Sometimes the calculations require a little ingenuity and a combination of formulas.

Example:

A pharmacist is filling medicine capsules. The capsules are cylinders with half spheres on each end. If the length of the capsule is 16 mm and the radius is 2 mm, how many cubic mm of medication can one capsule hold?

The capacity of the capsule is its volume. The top and bottom of the capsule are two equal hemispheres, so together, they make one sphere. The middle portion of the capsule is a right circular cylinder. All we need to do, then, is to add the volume of the sphere (two hemispheres) to the volume of the cylinder.

Volume of sphere (we need the radius)

$$V = \frac{4}{3}(\pi)(2)^3 = \frac{32\pi}{3} \approx 33.510 \ mm^3$$

Volume of the cylinder (we need the radius AND the height/length)

For the height, we must subtract out the radius of the hemispherical caps from each side.

$$h = 16 - 2(2mm) = 16 - 4 = 12mm$$

Now the Volume of the cylinder is

$$V = \pi r^2 h$$

$$V = \pi(2)^2(12) = 48\pi \approx 150.796 \ mm^3$$

Now we add the two together:

$$V_{capsule} = \frac{32\pi}{3} + 48\pi = \frac{176\pi}{3} \approx 184.307 \ mm^3$$

Say What?!?!

Archimedes part II

Eureka Moment

The tyrant Hiero of Syracuse once approached Archimedes seeking a solution to an unusual puzzle:

Having commissioned an artisan to produce a crown, Hiero had given the man some gold. When the crown was complete, Hiero suspected the gold had been mixed with silver (a less expensive metal), enabling the artisan to quietly pocket the difference. Was there, Hiero wondered, any way to put his suspicions to the test?

King Hiero
www.livius.org

According to the traditional tale, the answer occurred to Archimedes while he was bathing; he noticed that as he immersed himself in the tub, not only did the water level rise, but his apparent weight seemed to decrease.

http://teacherlink.ed.usu.edu

He is said to have leaped from the bath and run naked through the streets of Syracuse crying, **"Eureka! Eureka!"** *(I have found it! I have found it!)*

He discovered the principle of fluid buoyancy. He realized that two objects of equal weight will displace different volumes of water when immersed unless their densities are equal (a point now known as Archimedes' Principle). Because silver is less dense than gold, he soon discovered that Hiero's gold had in fact been adulterated.

Trivia: Running naked through the streets might not have seemed odd in Syracuse. The Greeks habitually exercised in the nude. Indeed, the word "gymnasium" derives from the Greek root "gymna," meaning naked.

Here's a related example:

Seven steel balls, each of diameter 4cm, are dropped into a tall cylinder tank of radius 5cm, which contains water. By how much does the water level rise? (Assume the balls are entirely submerged.)

- By the Archimedean principle of fluid buoyancy, the balls displace an equal volume of water, so it will rise ***x*** cm.
- Vol. of 7 balls $= 7\left[\frac{4}{3}\pi\left(2^3\right)\right] = \frac{224\pi}{3}\ cm^3$
- Increase in vol. Of water $= \pi r^2 x = \pi(5)^2 x = 25\pi x$
- Setting the two volumes equal,

$$\frac{224\pi}{3} = 25\pi x$$

$$x = \frac{224\pi}{3(25\pi)} = \frac{224}{75} \approx 2.987\ cm$$

So, The water level rises about 3 centimeters.

Eureka!!

Resources:

www.glencoe.com

http://en.wikipedia.org/wiki/Sphere