Lesson 25

Glencoe Geometry Chapter 11.5, 11.6

Volume: Prisms, Cylinders, Pyramids, and Cones

This week we look at the same solids as last time, we just add an extra dimension. Surface area is a two-dimensional quantity, while volume is a **three-dimensional** quantity.

Understanding today's lesson comes down again to **using the correct formula** and **finding the correct values** to plug into it.

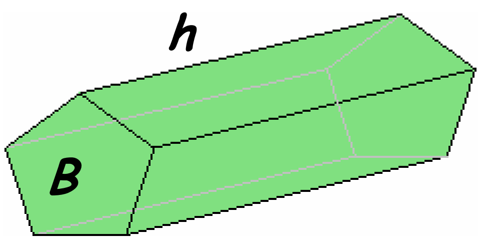
Depending on your teacher, you may have to **memorize** some of these formulas, but most of them will be found on a **formula chart**.

Volume of a Right Prism

The Volume, ***V***, of a Right Prism with a base area of ***B*** square units and a height of ***h*** units is given by

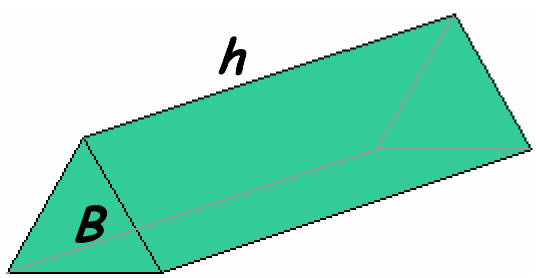
$$V = Bh$$

To use this formula, you must correctly identify and find the area of the base. These will vary from shape to shape, but the formula can be found on a chart.



Pentagonal prism

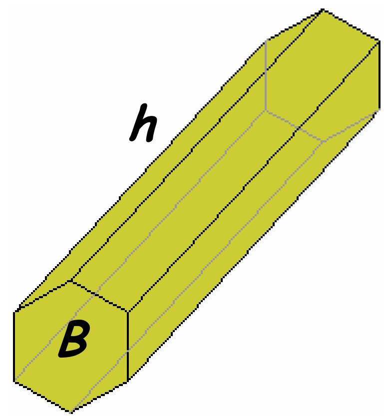
$$B = \frac{1}{2}Pa$$



Triangular prism

$$B = \frac{1}{2}bh^*$$

***CAUTION: This h is the height of the triangle, not the height of the prism. They are different quantities in different contexts represented by the same variable!!*

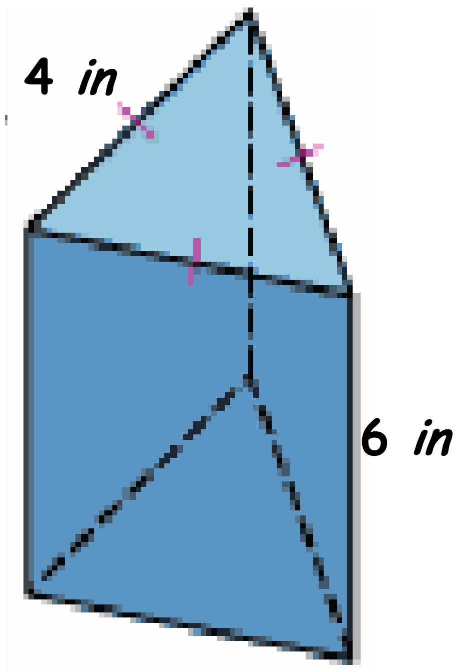


Hexagonal prism

$$B = \frac{1}{2}Pa$$

Example:

Find the Volume of the right, triangular prism.



The height of the prism, 6 in, is given. To find the area of the triangular base, we must first find the height from the Pythagorean Theorem.

$h^2 + 2^2 = 4^2 \rightarrow h = \sqrt{12} = 2\sqrt{3} in$

So

$V = Bh$

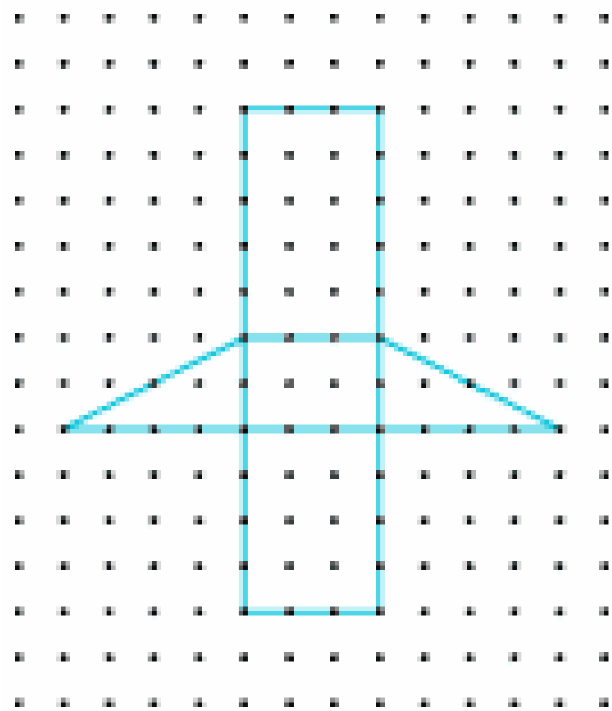
$V = \left(\left(\frac{1}{2}\right)(4)\left(2\sqrt{3}\right)\right) \bullet 6$

$V = 24\sqrt{3}\ in^3 \approx 41.569\ in^3$

Although the **Net** representation is more useful for finding **Surface Area**, we can also use it to find the **Volume**, we just must be more careful in identifying the base and height.

Example:

Find the volume of the right triangular prism given the net of the solid.



Cutting this out (or at least imagining that you are folding up) can help. The bases are the two congruent right triangles, we need the area of one of them.

Counting we get the base is 4 units and the height is 2 units. So the area is

$B = \frac{1}{2}(bh) = \left(\frac{1}{2}\right)(4)(2) = 4\ units^2$

The height of the prism is less obvious. Remember it is the length of a segment connecting the two bases, which is 3 units here.

So $V = Bh = \left(4\ units^2\right)\left(3\ units\right) = 12\ units^3$

Volume of a Right Cylinder

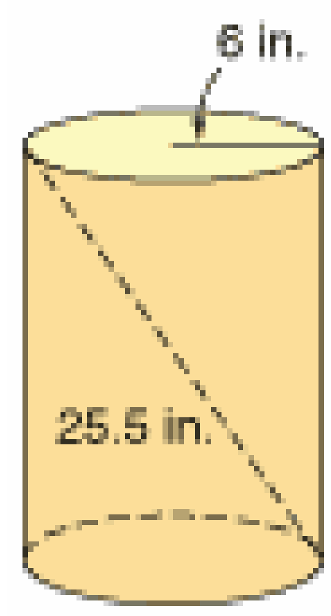
The Volume, *V*, of a Right Cylinder with a height of *h* units and a radius of *r* units is given by

$$V = \pi r^2 h$$

Notice as before that the Volume is just the area of the base times the length of a lateral side!!!

Example:

Find the volume of the cylinder.



For the volume we need only the radius (given as 6 in) and the height (not so given). To find the height in a RIGHT cylinder, we can use the Pythagorean theorem on the Right triangle created by the diameter ***d***, the height ***h***, and the given drawn diagonal.

$$h^2 + (2 \bullet 6)^2 = 25.5^2$$

$$h = \sqrt{650.25 - 144} = \sqrt{506.25} = 22.5in$$

Now to find the volume:

$$V = \pi r^2 h$$

$$V = \pi (6in)^2 (22.5in) = 810\pi \ in^3 \approx 2544.690 \ in^3$$

Formulas are also called **Literal Equations**, since they contain multiple variables *(Literal means "letters")* and constants (like π). We can use the formula to solve for ***ANY*** variable, as long as we know (or can find) all the others. This really increases the usefulness of the formulas. Here's what I mean . . .

Example:

Tate is constructing a cylinder that can hold 90.8 cubic centimeters. If the diameter of the cylinder is 3.4 centimeters, what should the height measure?

Drawing a picture will really help. The formula for the volume of a cylinder requires us to know any two of the three quantities: Volume, *V*; the radius, *r*; or the height, *h*. Remember π is a constant and is about 3.14159 . . . In this example, we are given the Volume, and we can easily find the radius since it is half the given diameter. This means we can plug these into the formula and solve for *h* !!!!

$V = \pi r^2 h$

$$90.8cm^3 = \pi\left(\frac{3.4cm}{2}\right)^2 h$$

$$90.8cm^3 = \left(2.89\pi cm^2\right)h$$

$$h = \frac{90.8cm^3}{2.89\pi cm^2} = 10.001cm$$

Notice that including the units, we can use them as an additional check for the reasonableness of our answer. For instance, if our final units had been square centimeters instead of centimeters, we know we would have made a mistake, since we don't measure length in square units!

Way to go, Tate!!

Volume of a Right Circular Cone

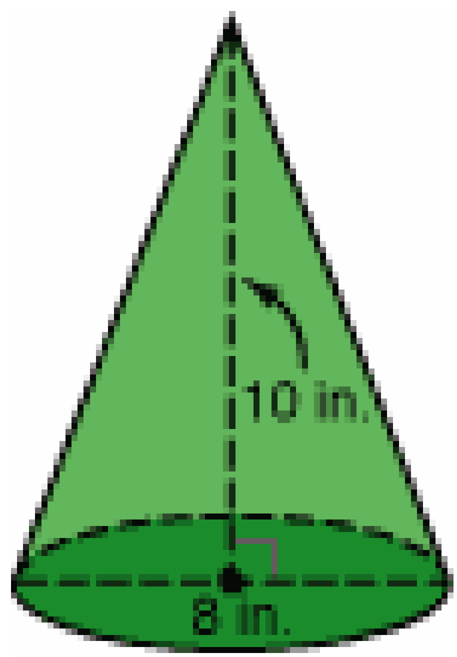
The Volume, ***V***, of a Right Circular Cone with a height of ***h*** units and a base area of ***B*** square units is given by

$$V = \frac{1}{3}Bh \quad \text{or} \quad V = \frac{\pi}{3}r^2h$$

You can think of the cone as being **one-third** of the volume of the smallest cylinder containing it.

Example:

Find the volume of the cone.



The diameter of the base is 8in. so the radius is half of that, or 4in. The height is the given altitude length of 10in.

So the Volume is

$$V = \left(\frac{1}{3}\right)(\pi)(4in)^2(10in) = \frac{160}{3}\pi in^3 \approx 167.552 in^3$$

Volume of a Right Pyramid

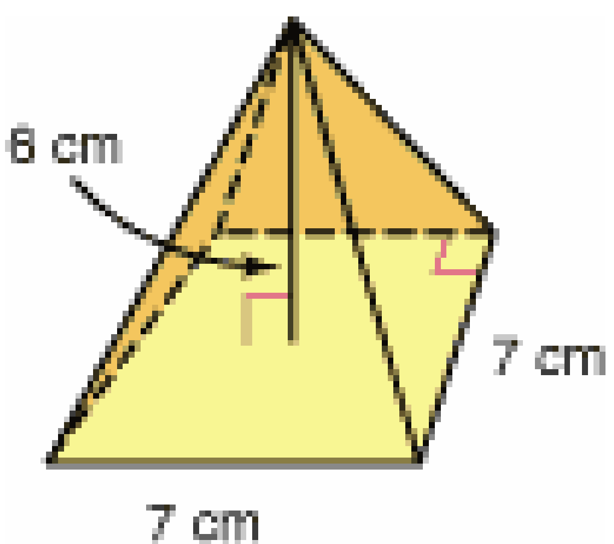
The Volume, ***V***, of a Right Pyramid with a height of ***h*** units and a base area of ***B*** square units is given by

$$V = \frac{1}{3}Bh$$

You can think of the pyramid as being **one-third** of the volume of the smallest prism containing it.

Example:

Find the volume of the pyramid.



The area of the base is the area of a square.

$B = s^2 = (7cm)^2 = 49cm^2$

The height is given to be 6cm

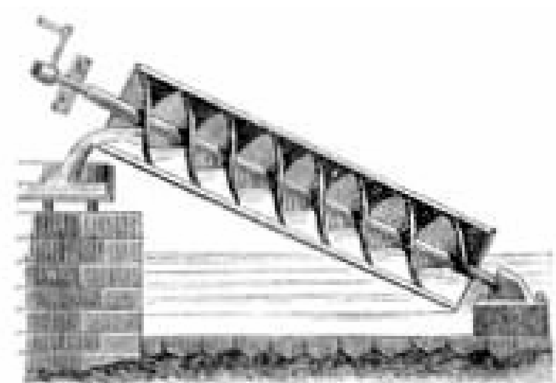
So the volume is

$V = \frac{1}{3}Bh = \left(\frac{1}{3}\right)(49cm^2)(8cm) = 130.667cm^3$

Say What?!?!


http://design-crit.com/blog/images/archimedes.gif

Regarded as one of the three greatest mathematicians and inventors of all time was **Archimedes** of Syracuse (287BC – 212BC). His inventions ranged from incredible war machines, the Archimedean screw, and the lever. He made lasting contributions in many mathematical fields including geometry (he gave the first estimate for π between $3\frac{10}{71}$ and $3\frac{10}{70}$) and calculus (he is known as the father of integral calculus even though he preceded Newton and Leibniz by almost 2000 years!!!).


Archimedean Screw
Wikipedia.org

Of all his remarkable discoveries, the one of which he was most proud was his discovery of the ratio of the volumes of a right cylinder and its inscribed sphere.

https://www.math.nyu.edu/~crorres/Archimedes/Tomb/Cicero.html

http://scidiv.bcc.ctc.edu/Math/ArchimedesTombstone.html

He found that the ratio of the volumes of the cylinder to the sphere was a tidy **3:2**. If fact, he was so proud of this discovery, it was placed on his tombstone!!

"Give me a place to stand, and I shall move the Earth."--Archimedes

Wordsworth, William (1770-1850), *The Excursion* (Book Eighth: The Parsonage, lines 220-230)

—Call Archimedes from his buried tomb
Upon the plain of vanished Syracuse,
And feelingly the Sage shall make report
How insecure, how baseless in itself,
Is the Philosophy, whose sway depends
On mere material instruments;—how weak
Those arts, and high inventions, if unpropped
By virtue.—He, sighing with pensive grief,
Amid his calm abstractions, would admit
That not the slender privilege is theirs
To save themselves from blank forgetfulness!