Lesson 24

Glencoe Geometry Chapter 11.3, 11.4

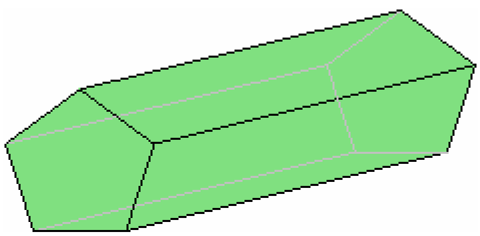
Surface Area: Prisms, Cylinders, Pyramids, and Cones

At the end of the last episode, we looked at lateral surface area of a cylinder. Today, we explore the surface areas of some other special 3-D solids. Being so special, they naturally have special area formulas we will be using.

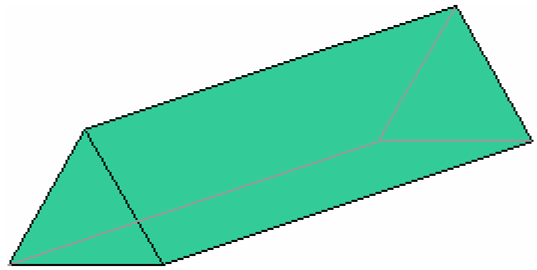
The TAKS test has them on handout.

Area	rectangle	$A = lw$ or $A = bh$
	triangle	$A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$
	trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$ or $A = \frac{(b_1 + b_2)h}{2}$
	circle	$A = \pi r^2$
Surface Area	cube	$S = 6s^2$
	cylinder (lateral)	$S = 2\pi rh$
	cylinder (total)	$S = 2\pi rh + 2\pi r^2$ or $S = 2\pi r(h + r)$
	cone (lateral)	$S = \pi rl$
	cone (total)	$S = \pi rl + \pi r^2$ or $S = \pi r(l + r)$
	sphere	$S = 4\pi r^2$

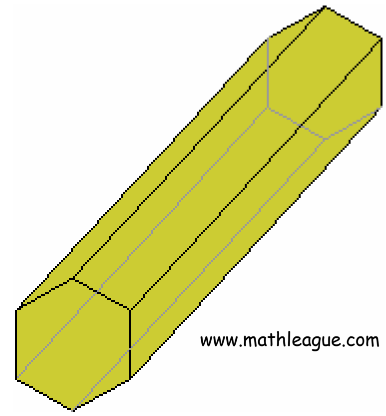
Remember a Prism is a polyhedron with two congruent bases in parallel planes. They have a uniform cross-section. Here are some examples:



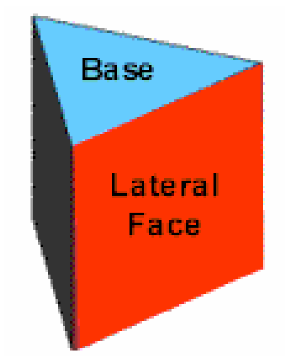
Pentagonal prism



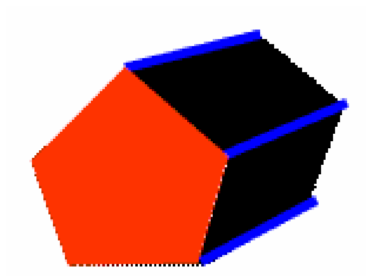
Triangular prism



Hexagonal prism

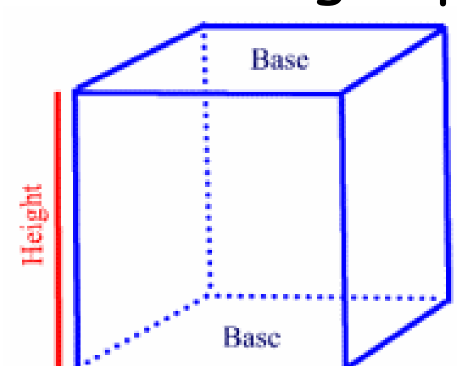


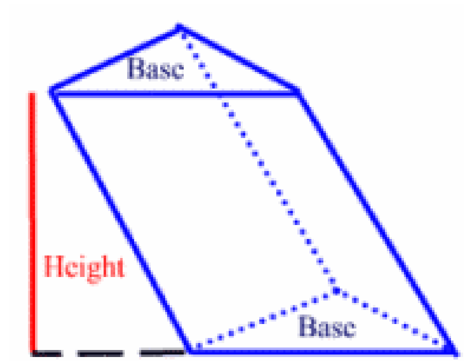
The congruent faces are called bases. The sides connecting the bases are called lateral faces and are parallelograms. A lateral edge is formed where two lateral faces meet.



Lateral edges: There are 5 congruent and parallel lateral edges in this prism.

These are called Right prisms. The height is an altitude (perpendicular to the base). If a prism is not a right prism, we say it is oblique.





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Now we can find the surface area of these prisms as we did before, but we can also derive a formula that is easier to use, and makes the calculations much faster.

The **Lateral Surface Area** of a right prism, ***L***, is the product of the perimeter, ***P***, of a base, times the altitude, or height, ***h***.

$$L = Ph$$

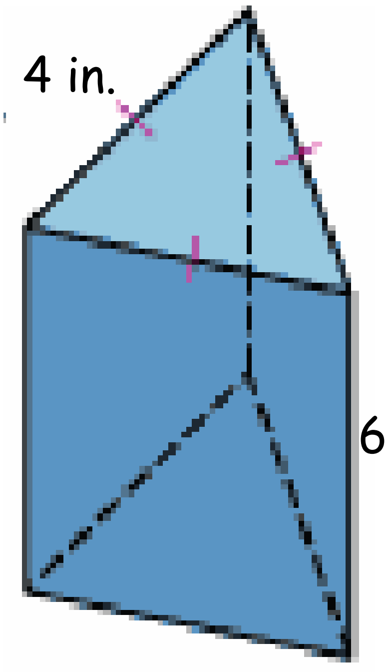
To get the entire Surface area, we simple add the two congruent areas of the bases.

The **Total Surface Area** of a right prism, ***SA***, is the lateral area plus twice the area, ***B***, of a base.

$$SA = Ph + 2B$$

Be sure to read questions very, very, very, very carefully to be sure you are finding the correct quantity!!

Example:



Find the lateral and total surface area of the triangular prism.

$L = Ph = (4+4+4)(6)in^2 = 72in^2$

$SA = L + 2B$

The area of the base is the area of a triangle, $A = \frac{1}{2}bh$.

Since it is equilateral, we can find the height by the Pythagorean theorem.

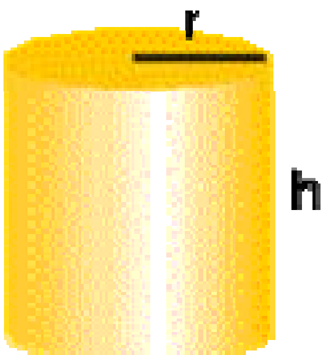
$2^2 + h^2 = 4^2 \rightarrow h = \sqrt{16-4} = \sqrt{12} = 2\sqrt{3} \approx 3.164in$

so

$SA = L + 2B = 72in^2 + 2\left[\frac{1}{2}(4)(2\sqrt{3})\right]in^2 = (72 + 8\sqrt{3})in^2$

$\approx 85.856in^2$

A cylinder is another type of solid. *It is not a polyhedra. Remember why?* You can think of a cylinder as a prism with circular bases. We can have right and oblique cylinders.

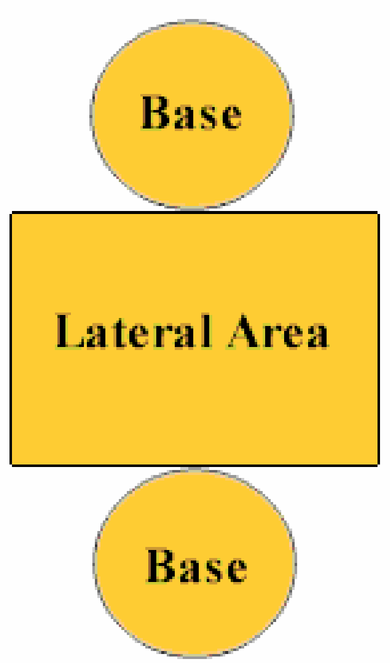


$h = height\ (altitude)$

$r = radius$

The lateral surface area, L, of a cylinder is $L = 2\pi rh$

The area, A, of a circle is $A = \pi r^2$



A net of a right circular cylinder

The **total surface area of a cylinder** is the lateral area plus twice the area of a circular base.

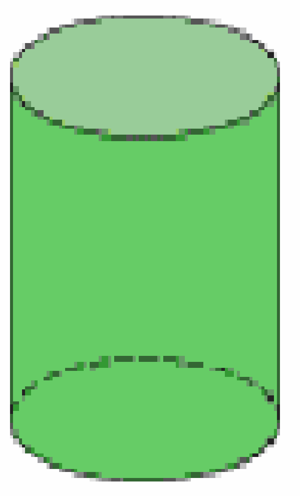
$$SA = 2\pi rh + 2\pi r^2$$

or

$$SA = 2\pi r(r + h)$$

Example:

Find the surface area of the green, right, circular cylinder.



$d = 10ft$

$h = 15ft$

The lateral area is

$L = 2\pi(5ft)(15ft) = 150\pi ft^2 \approx 471.239ft^2$

The area of a circular base is

$A = \pi(5ft)^2 = 25\pi ft^2 \approx 78.540ft^2$

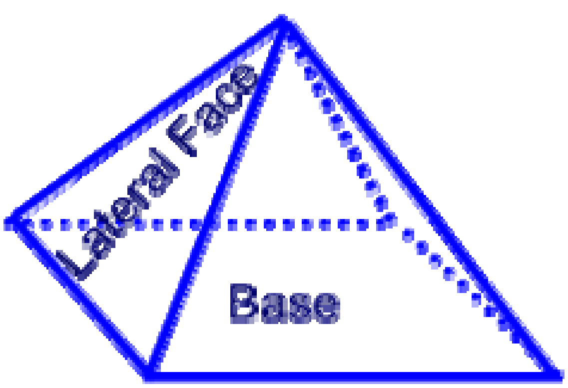
So the Surface Area of the cylinder is

$SA = 150\pi ft^2 + 2[25\pi]ft^2 = 200\pi ft^2 \approx 628.319ft^2$

Pyramids are three-dimensional closed surfaces. The **one base** of the pyramid is a polygon and the lateral faces are **always triangles** with a common vertex. The vertex of a pyramid (the point, or apex) is not in the same plane as the base.

Pyramids are also called polyhedra since their faces are polygons.

The most common pyramids are **regular pyramids**. A regular pyramid has a *regular polygon* for a base and its height meets the base at its center. The **slant height** is the height (altitude) of each lateral face.



In a **regular pyramid**, the lateral edges are congruent.
Since the base is a regular polygon, whose sides are all congruent, we know that the lateral faces of a regular pyramid are congruent isosceles triangles.

The lateral surface area, ***L***, of a regular pyramid with base perimeter ***P*** and slant height of ℓ is given by

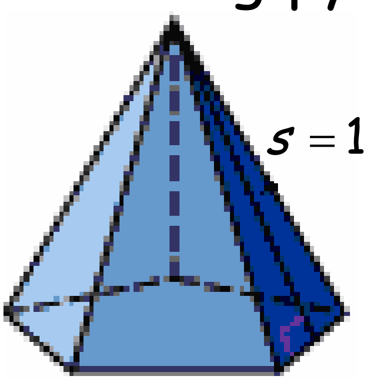
$$L = \frac{1}{2}P\ell$$

To get the total surface area, we just add the area of the base, ***B***.

$$SA = \frac{1}{2}P\ell + B$$

Example:

Find the lateral and total surface areas of the following pyramid.



$s = 15\ cm$

$8\ cm$

Apothem of base

a = 5.5 cm

The base is a regular pentagon whose perimeter is $8(5) = 40cm$

The lateral surface area is

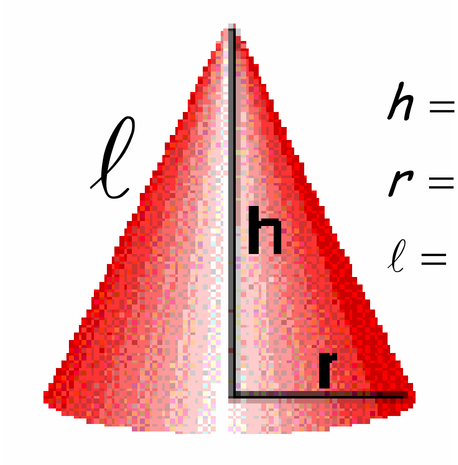
$L = \frac{1}{2}P\ell = \frac{1}{2}(40cm)(15cm) = 300cm^2$

Remember the area of a regular pentagon is $A = \frac{1}{2}Pa$, where a is the apothem. So its area is $\frac{1}{2}(40cm)(5.5cm) = 110cm^2$

The total surface area then is

$SA = 300cm^2 + 110^2 cm^2 = 410cm^2$

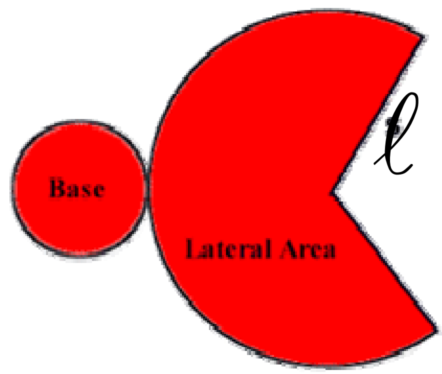
Cones are three-dimensional closed surfaces. In general use, the term *cone* refers to a right circular cone with its end closed to form a circular base surface. The vertex of the cone (the point) is not in the same plane as the base.



$h = height\ (altitude)$

$r = radius\ of\ base$

$\ell = slant\ height$



The net of a cone

Cones are **not** called polyhedra since their faces are not polygons. In many ways, however, a cone is similar to a pyramid. A cone's base is simply a circle rather than a polygon as seen in the pyramid.

The Lateral Surface Area, L, of a cone is given by

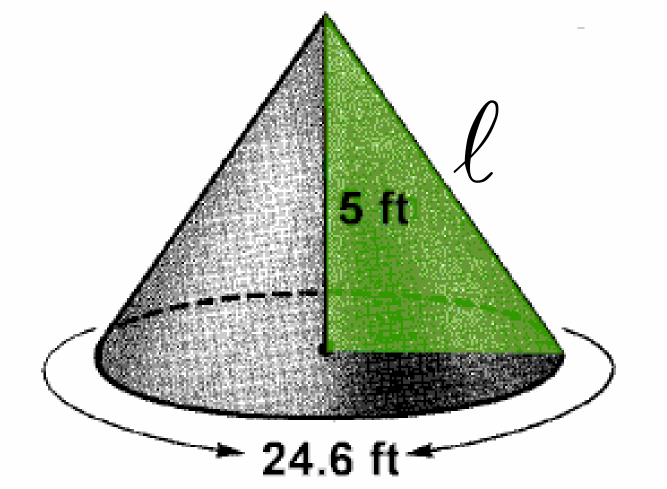
$$L = \pi \ell r$$

The Total Surface Area, SA, is given by

$$SA = \pi \ell r + \pi r^2$$

Example:

Find the surface area of the following cone.



We need to find the radius for both terms in the formula. However, we are given the circumference of the base. The formula for circumference of a circle is $C = 2\pi r$, so $24.6 = 2\pi r \rightarrow r = 24.6/(2\pi) \approx 3.915ft$

Now we need the slant height, ℓ. We get this from the Pythagorean Theorem: $\ell^2 = 3.915^2 + 5^2 \rightarrow \ell = \sqrt{40.328} \approx 6.350ft$

So the lateral surface area is $L = \pi \ell r = \pi(6.350)(3.915) \approx 78.101ft^2$

So the total surface area, SA is $SA = 78.101ft^2 + \pi(3.915ft)^2 \approx 126.253ft^2$

Say What?!?!

The Great Pyramid of Egypt

http://www.unmuseum.org/bpyramid.jpg

The Great Pyramid of Khufu, at Giza, Egypt, is 756 feet long on each side at the base, is 481 feet high, and is composed of approximately 2 million blocks of stone, each weighing more than 2 tons. The maximum error between side lengths is less than 0.1%. The base covers more than 13 acres!

The sloping angle of its sides is 51.5°. Each side is oriented with the compass points of north, south, east, and west. Each cross section of the pyramid (parallel to the base) is a square.

Until the 19th century, the Great Pyramid at Giza was the tallest building in the world. At over 4500 years in age, it is the only one of the famous Seven Wonders of the Ancient World that remains standing.
According to the Greek historian Herodotus, the Great Pyramid was built as a tomb for the Pharaoh Khufu.

In 450 B.C., Herodotus also determined that the square of its height equals the area of each triangular face.

He was remarkably close:

$h = 481ft$ so $h^2 = 481^2 \approx 231361ft^2$

To find the area of the face, we need to use the Pythagorean theorem to find the slant height, ℓ, of the face. The triangle is formed by the height of the pyramid, half the base length, with s being the hypotenuse.

$$\ell = \sqrt{\left(\frac{756}{2}\right)^2 + (481)^2} \approx 611.756ft$$

So the area of one face, using the area of a triangle formula:

$$A = \frac{1}{2}(756)(611.756) \approx 231244ft^2$$

He was only off by about 0.05% or $1/20^{th}$ of a percent!!