Lesson 23

Glencoe Geometry Chapter 11.2

Nets and Surface Area

When discussing 3-D solids, it is natural to talk about that solid's Surface Area, which is the sum of the areas of all its outer surfaces or faces.

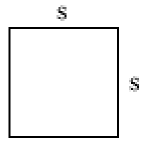
Perhaps you are painting the **exterior of your house**, or you are **wrapping a giant gift** for someone, *the amount of material used* (be it paint or paper) *depends on the surface area* of the solid object.

The ***Area of a surface***, or polygon, is the **measure of how much space it takes up in a plane**. Area is measured in "square" units. Think of the area of a figure as the number of squares required to cover it completely, like tiles on a floor. Many common shapes have formulas for finding this quantity. Here are some common ones.

Area Formulas

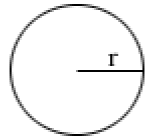
$\pi \approx 3.14159\cdots$

Square:



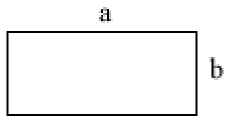
$A = s^2$

Circle:



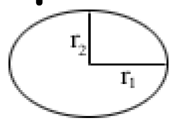
$A = \pi r^2$

Rectangle:



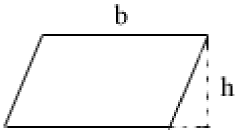
$A = ab$

Ellipse:



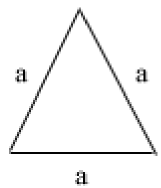
$A = \pi r_1 r_2$

Parallelogram:



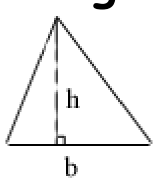
$A = bh$

Equilateral Triangle:



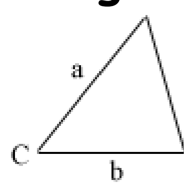
$A = \frac{\sqrt{3}}{4}a^2$

Triangle:



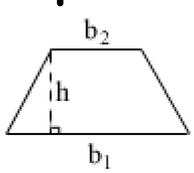
$A = \frac{1}{2}bh$

Triangle given SAS:



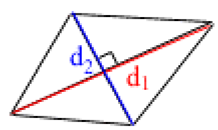
$A = \frac{1}{2}ab\sin C$

Trapezoid:



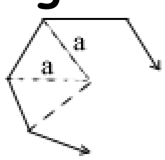
$A = \frac{1}{2}(b_1 + b_2)h$

Rhombus:



$A = \frac{1}{2}d_1 d_2$

Regular Polygon:



$A = \frac{1}{2}Pa$

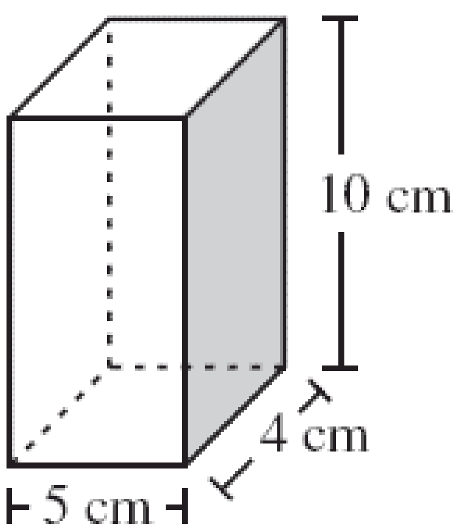
So finding surface area just involves

1. Determining which of the above shapes make up the sides
2. Applying the formula to the respective side to find its area
3. Adding them all up

NOTE: since area involves side lengths, there will be units. ***Be sure to use the same units for all measurements.*** *You cannot multiply feet times inches, it doesn't make a square measurement.*

Example:

Gift Wrapping: I am wrapping a box for my wife that is 4 centimeters long, 5 centimeters wide, and 10 centimeters high. What is the surface area of the box?



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The faces are rectangles. The area of each rectangles is length times width. Notice that the sides come in pairs. Find all areas, then add them up.

$SA = 2(5cm)(4cm) + 2(4cm)(10cm) + 2(5cm)(1$

$= 2(20cm^2) + 2(40cm^2) + 2(50cm^2)$

$= 40cm^2 + 80cm^2 + 100cm^2$

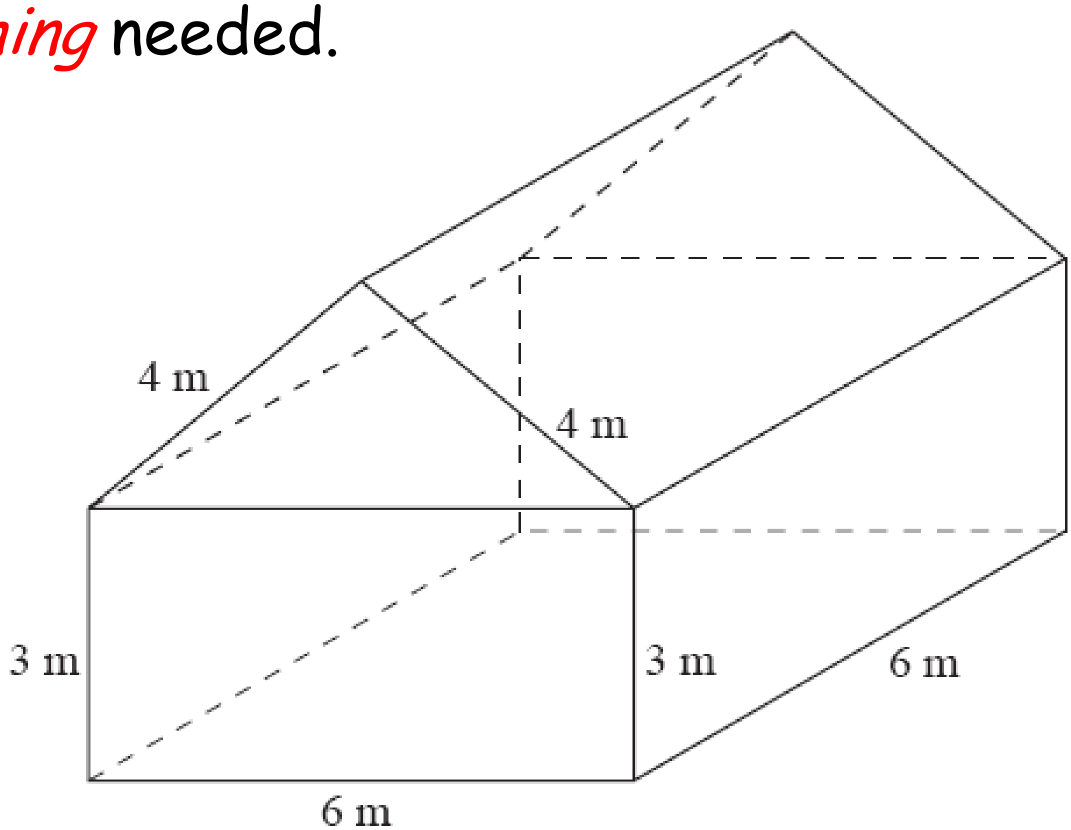
$= 220cm^2$

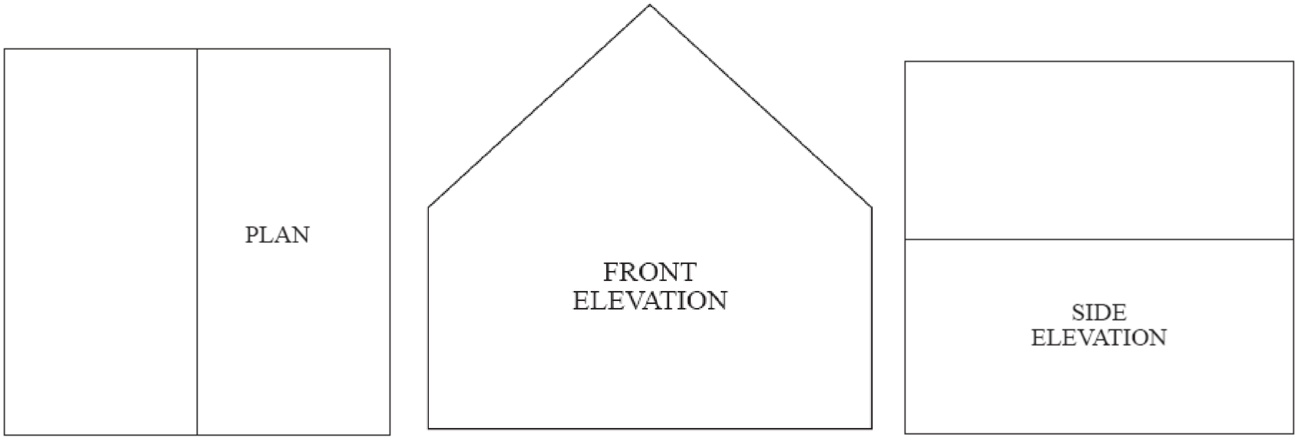
There are two helpful ways to visualize the surfaces of the solids that will help you in calculating the surface area.

1. Plan and Elevations: *These are the multiple 2-D representations we looked at last time. The plan is the view from above; the elevations are the side views.*

Example:

Draw the elevations for the following shed, and then determine the amount of exterior *sheathing* needed.





The surface area of the roof is shown in the plan:

$$SA = 2(4m)(6m) = 2(24m^2) = 48m^2$$

This means we will need about 517 square feet of roofing material!!

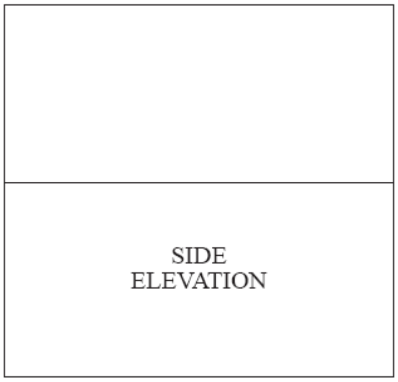
The surface area of the front and back are the same. We can find the area of the lower rectangle and triangular gable above. The triangle formula requires the height:

Since the triangle gable is isosceles, we can find the height (perpendicular bisector) by the Pythagorean theorem:
$3^2 + h^2 = 4^2 \rightarrow h = \sqrt{16-9} = \sqrt{7} \approx 2.646m$

So the surface area of the front and back combined is . . .

$$SA = 2\left[\underbrace{(3m)(6m)}_{\text{base rectangle}} + \underbrace{\left(\frac{1}{2}\right)(6m)\left(\sqrt{7}m\right)}_{\text{triangle gable}}\right]$$

$$= 2\left[18m^2 + 3\sqrt{7}m^2\right] = \left(36 + 6\sqrt{7}\right)m^2 \approx 51.874m^2$$



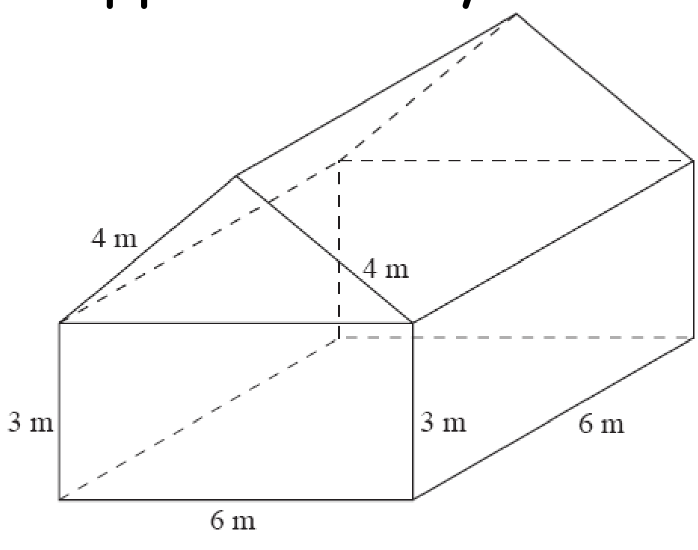
Now for the side elevations, it is important to realize that we have already found the surface area of the top rectangles (the roof). We only need calculate the surface areas of the two rectangular bases of the sides.

$$SA = 2(3m)(6m) = 2\left(18m^2\right) = 36m^2$$

The total exterior surface area is the sum of all our quantities:

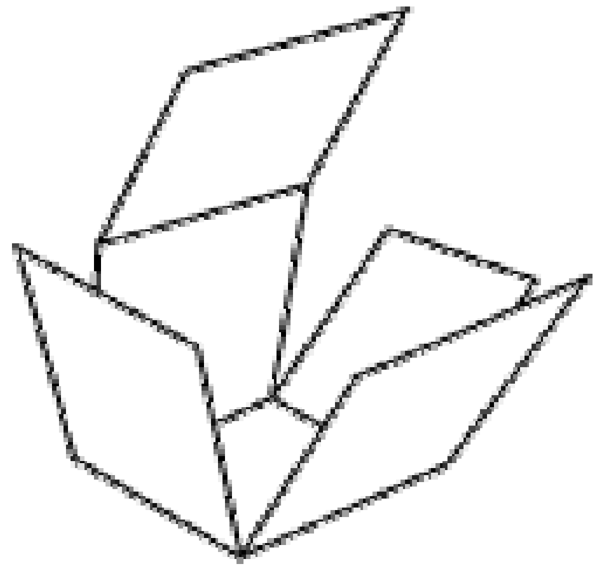
$$\boxed{SA = 48m^2 + 51.874m^2 + 36m^2 = 135.875m^2}$$

This is approximately 1462.541 ft^2



The second way to represent a 3-D object's surface area on a 2-D plane is by a Net.

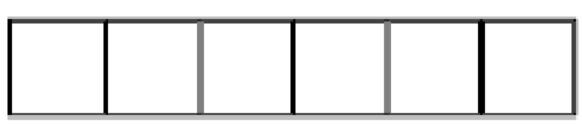
Imagine you cut a cardboard box along its edges and laid it out flat. The result would be the 2-D net. Depending on how you cut it, you might obtain a different, yet equivalent net.



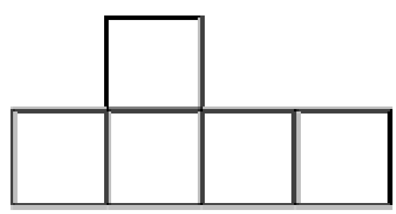
Example:

Which of the following are nets for the cube above? *Hint: Try to imagine folding each one, or print them out to explore.*

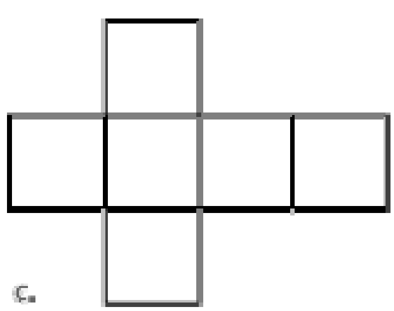
C & D



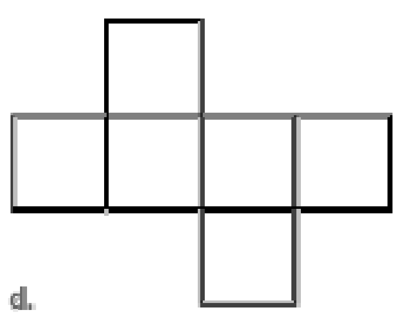
a.



b.



c.



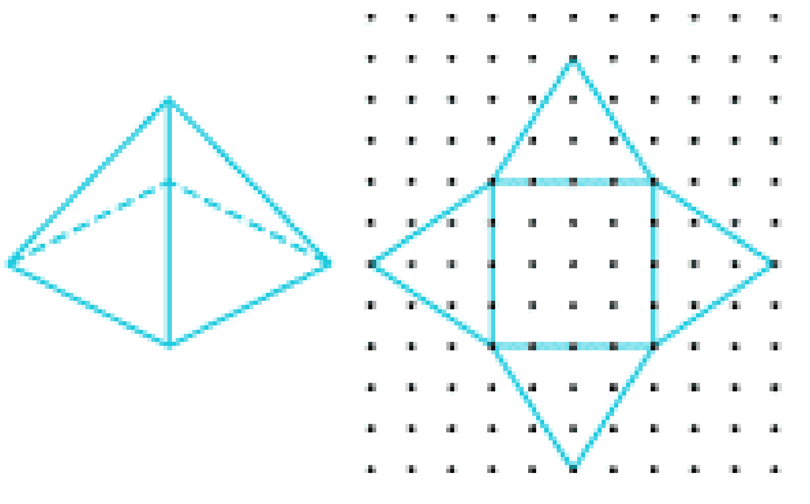
d.

The net is useful to determine surface area, because we reduce the problem to finding the area of the 2-Dimensional solid. We can subdivide the figure in squares, rectangles, triangles, or any other figure for which we know the area formula!!!

The use of isometric **dot paper** is useful for depicting nets.

Example:

To the nearest whole number, find the surface area of this square pyramid, using the net.



The surface area is the sum of the areas of the 4 congruent triangles and the square base. We can count the dots, each representing one unit, to determine the required side lengths:

$$SA = 4\left(A_{triangle}\right) + A_{square}$$

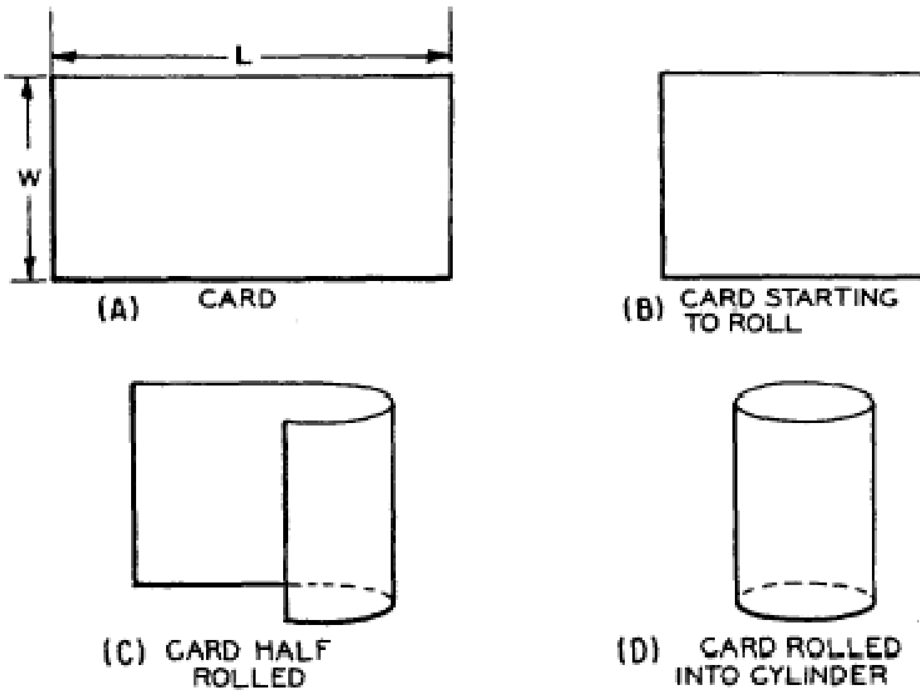
$$= 4\left[\left(\frac{1}{2}\right)bh\right] + \left(s^2\right)$$

$$= 2(4)(3) + \left(4^2\right)$$

$$= 24 + 16 = 40 \text{ square units}$$

Say What?!?!

Sometimes we are only interested in a portion of the surface area of a figure. For instance, in a right circular cylinder, like a can of food, we might want to know the surface area of the label only. This is called the Lateral **Surface Area** (remember lateral means "side").



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So the lateral surface area of a cylinder is the same as the area of the rectangle whose height is the same as the cylinder's, but whose length is the Circumference of the circle: $C_{circle} = 2\pi r$. We get the equation:

$$\text{Lateral surface area of a cone} = L = 2\pi rh$$

Example:

Find the lateral surface area of this can.

Since the diameter is 5.5, the radius is 2.75.

$$L = 2\pi rh$$
$$= 2\pi(2.75in)(7.75in)$$
$$= 42.625\pi\ in^2$$
$$\approx 133.910 \text{ square inches}$$