Lesson 22

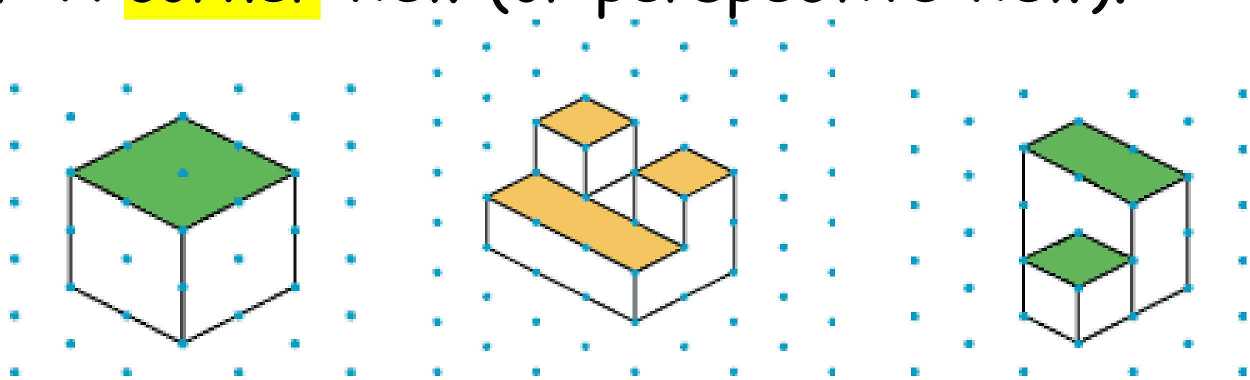
Glencoe Geometry Chapter 11.1

3-D Figures and Polyhedra

Today we will be exploring three-dimensional objects, those that possess length, width, and depth.

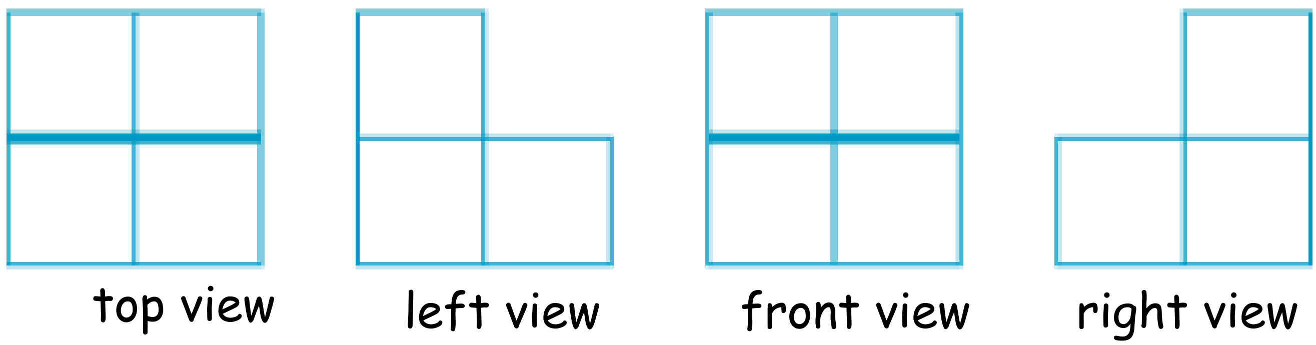
In Euclidean, or planar, geometry, we often explore 3-D objects by looking at a perspective drawing on a two-dimensional plane (sheet of paper). There are several ways to do this.

1. A corner view (or perspective view).



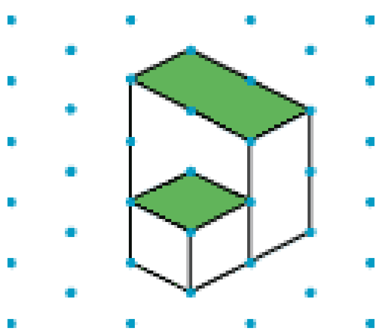
Shading is often used to give it that 3-D look!

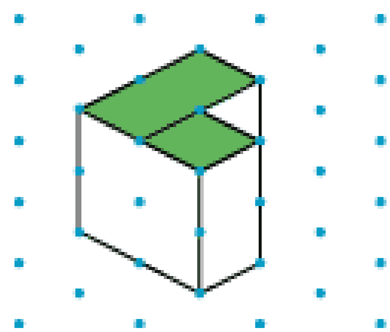
2. Multiple views of a solid: a set of 2-D depictions usually of the front, top, and side views of a 3-D object, from which a 3-D model, or perspective drawing can be made.

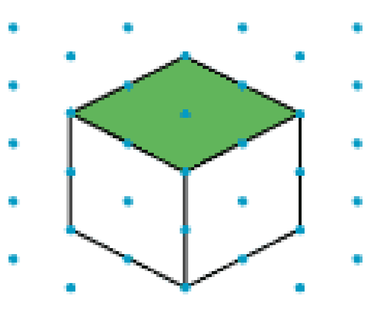


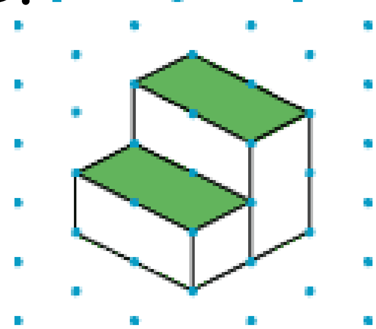
Example:

Which of the following corner views represents the above solid?

A. 

B. 

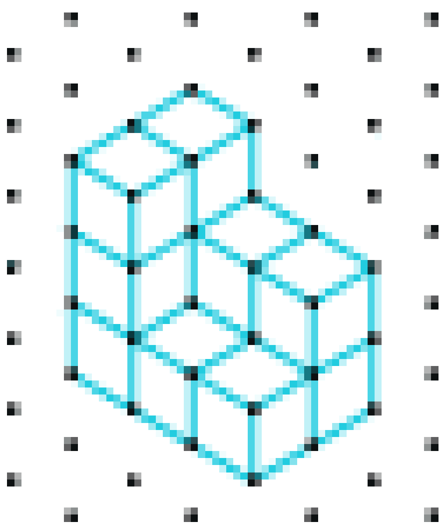
C. 

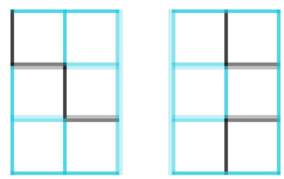
D. 

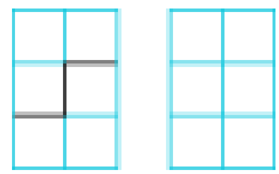
In the previous example, we went from a solid to a corner view. We can also go the opposite direction.

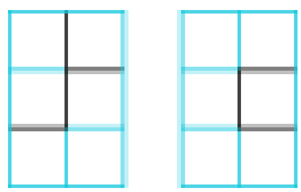
Example:

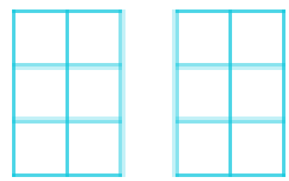
The corner view of a solid is shown. Find the right and left sides.



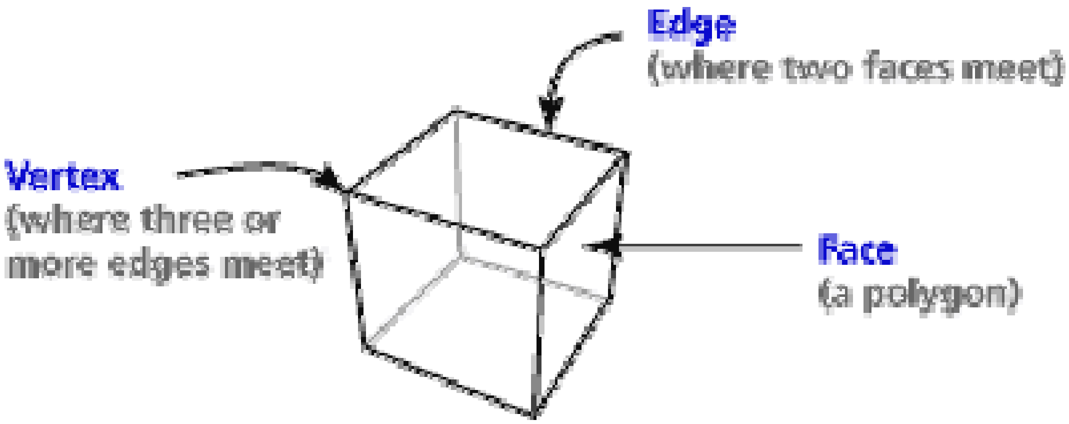
A. 

B. 

C. 

D. 

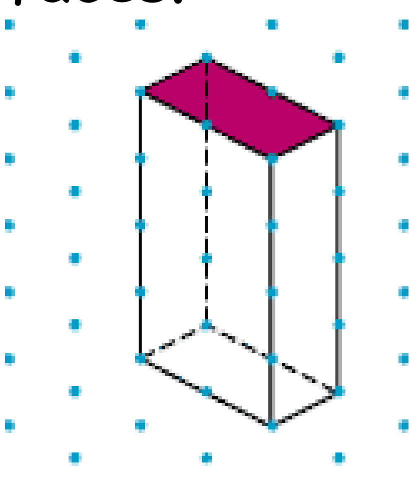
All of the surfaces in these solids are ***flat surfaces*** called faces. Solids with all flat surfaces that enclose a single region of space are called polyhedrons or polyhedra. All of the faces are polygons, and the line segments, called edges, intersect at a vertex.



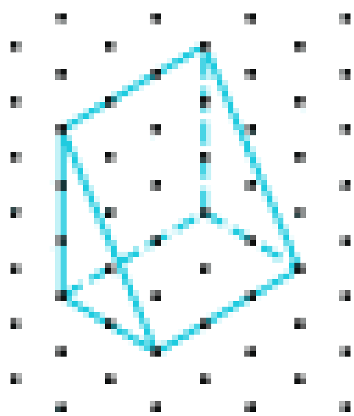
http://www.learner.org

Of course, there are special types of polyhedra:

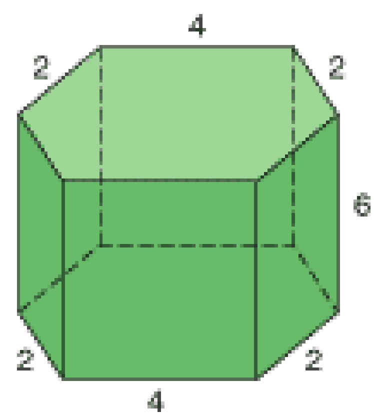
Prism: two congruent faces, called bases, that are polygons contained in parallel planes. The other faces are called lateral faces.



rectangular prism

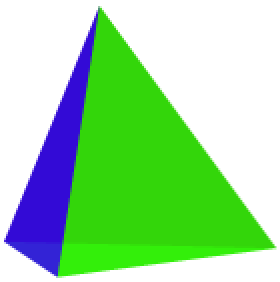


triangular prism

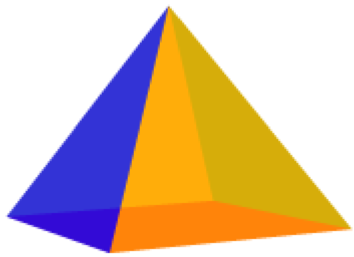


hexagonal prism

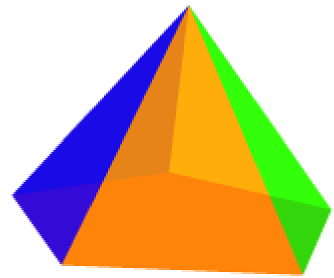
Pyramid: a polyhedron that has all faces except one intersecting at one point.



Triangular pyramid

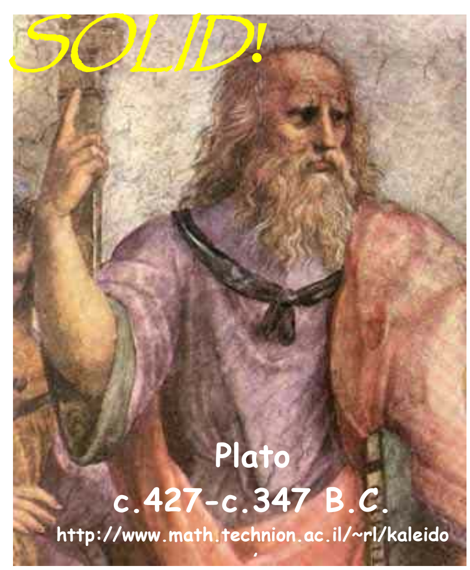


square pyramid

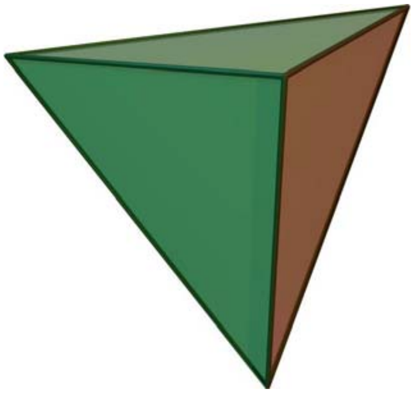


pentagonal pyramid

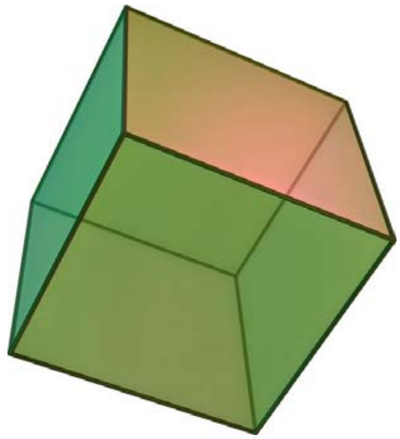
Polyhedra are said to be Regular if all of the faces are composed of congruent polygons.
This implies that all edges are also congruent.



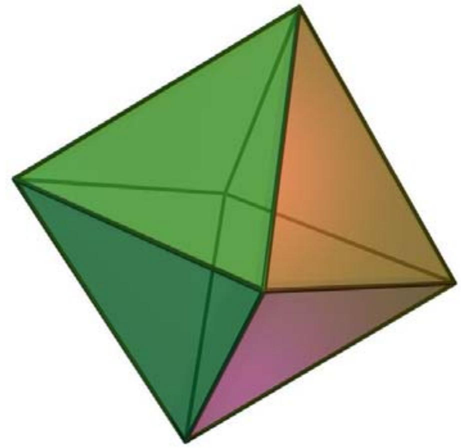
It has been proven that there are only 5 such regular (convex) polyhedra, *also known by the Ancient Greeks*. Because **Plato** described them so fully in his writings, regular polyhedra are also called Platonic Solids. Here they are!!



4 faces
tetrahedron

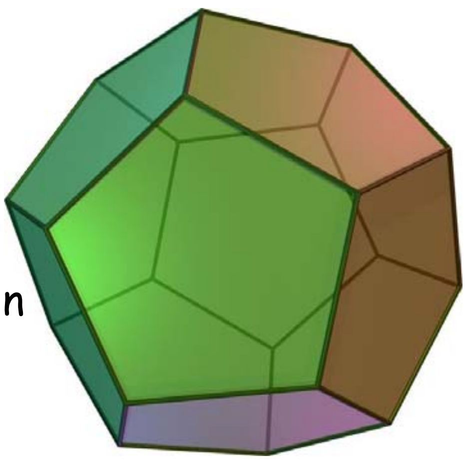


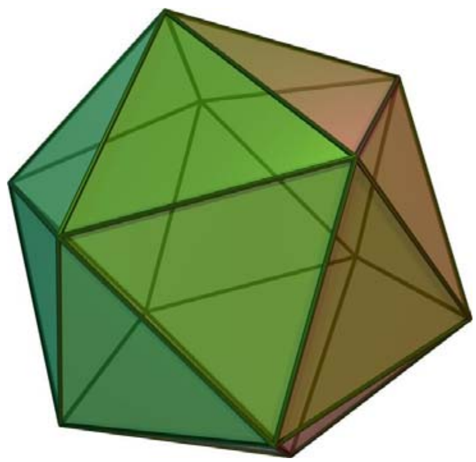
6 faces
hexahedron



8 faces
octahedron

12 faces
dodecahedron





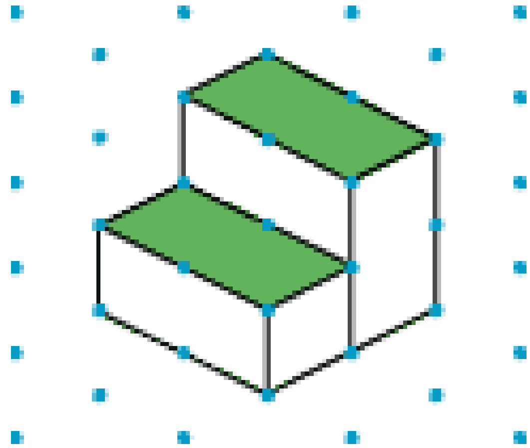
20 faces
icosahedron

Now it really gets interesting when we begin to slice a figure. When a plane intersects a solid figure parallel to the base or bases, a cross-section is obtained.

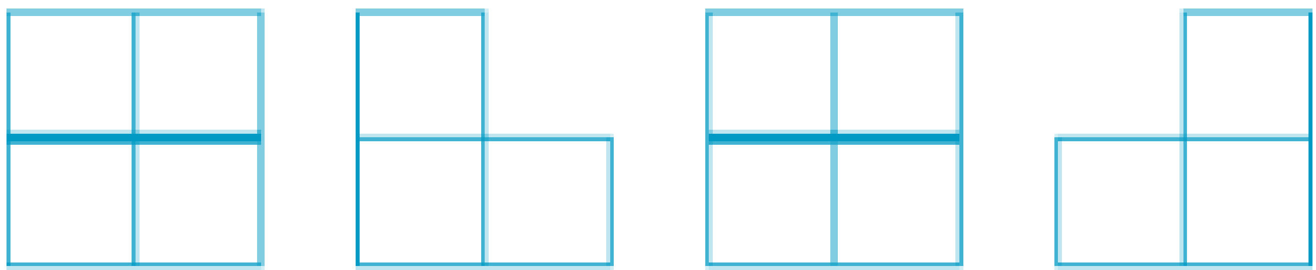
Cross-sections allow us to analyze a sample of the 3-D figure in a 2-D environment, much like our multiple 2-D depictions earlier.

Example:

Which of the following is a cross-section of the figure?



The slice must be perpendicular to the base.

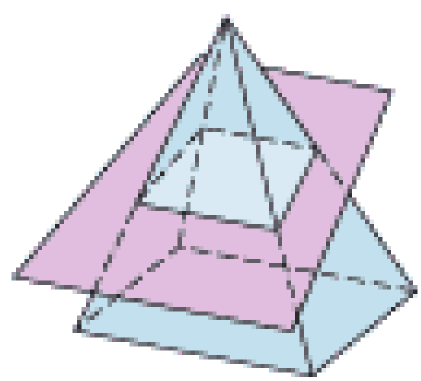


top view | left view | front view | right view

Slicing along different planes can achieve different, often very useful, shapes.

Example:

Describe the slice resulting from a plane cutting a square pyramid at an angle perpendicular to one of the triangular faces.



A. trapezoid   B. square   C. triangle

Example:

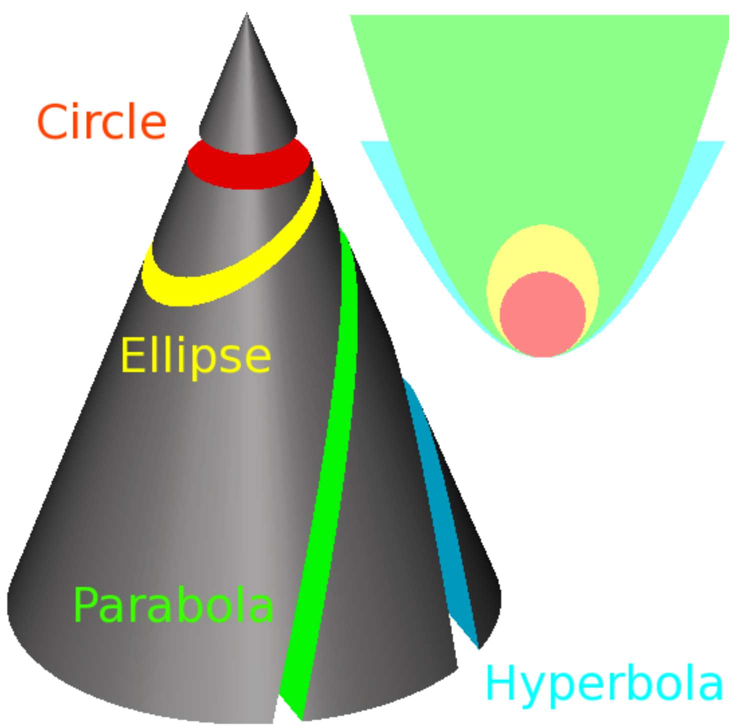
What is the shape of the intersection of a sphere and a horizontal plane?

A. trapezoid B. square

C. circle D. triangle

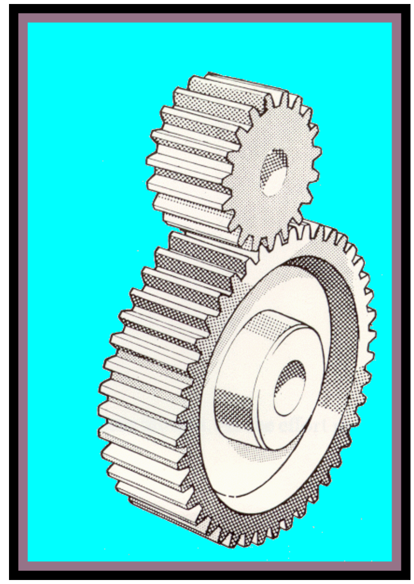
Say What?!?!

When a cone is intersected by a plane at different angles, different shapes, or **Conic Sections**, are obtained. Their applications are quite practical and numerous.



http://en.wikipedia.org/wiki/Conic_section

Circle

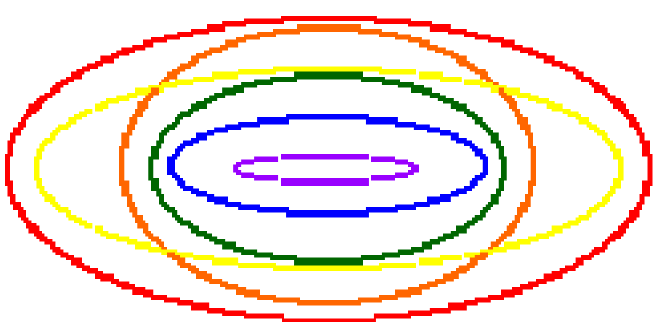


http://www.ul.ie/~nolk/Ken16.gif

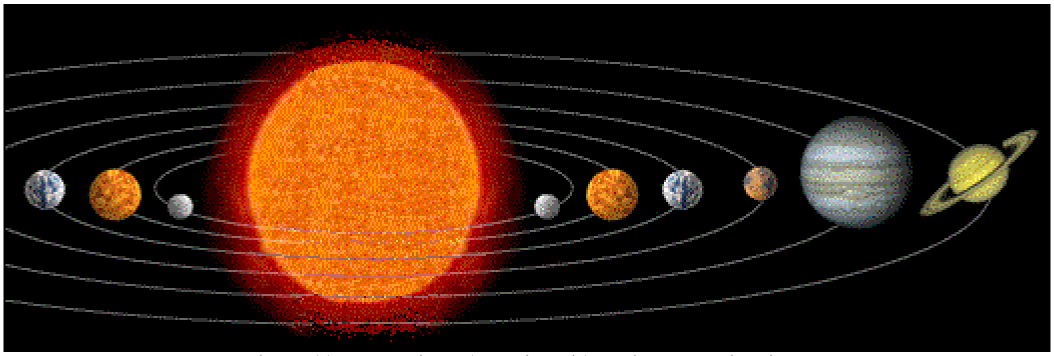
The circle, as a wheel, is one of the greatest inventions of all time and the basis of much of our transportation system. Circular gears are important elements in many of the machines we use every day, from CD players to electric saws.

Ellipse

The ellipse looks like a stretched circle. But unlike the circle, which is always the same shape, ellipses come in many shapes, from long and narrow to shorter and "fatter":

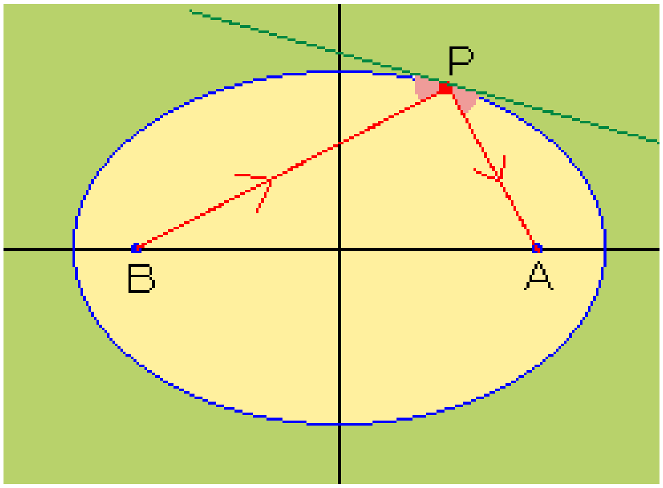


The orbits of the planets are ellipses, as well as the orbits of moons, satellites, and comets.



http://www.k12.hi.us/~mathappl/MAch3Curves.html

They also have a unique reflective property.

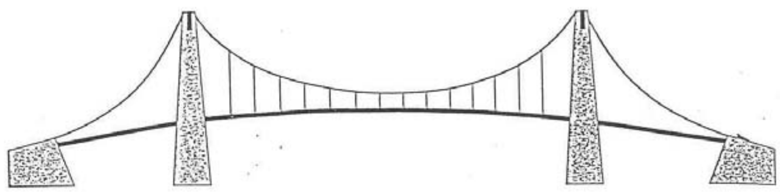


http://www.ies.co.jp/math/java/conics/focus_ellipse/focus_ellipse.html

Parabola

The Parabola is an interesting mathematical curve, one of the conics, and a curve with many applications from space travel to the sport of baseball.

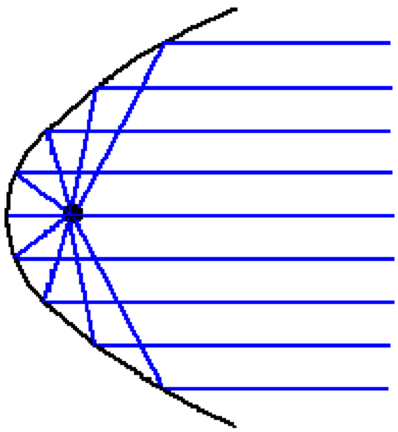
The parabola is the form taken by the path of any object thrown in the air, and is the mathematical curve used by engineers in designing some suspension bridges.



http://www.cuyctyengineers.org

The properties of the parabola make it the ideal shape for the reflector of an automobile headlight.

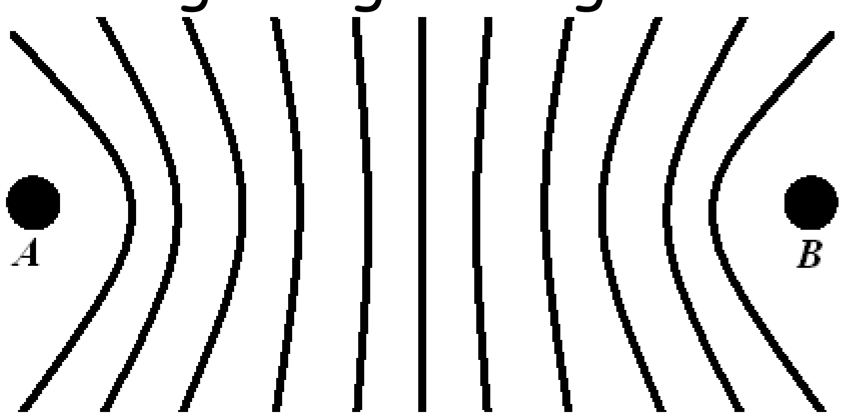
In an automobile headlight, the light from a bulb at the focus point of the metal back of the headlight reflects and is sent outward to light our way in the dark.



Hyperbola

The Hyperbola has many applications: the path of a comet often takes the shape of a hyperbola, and many telescopes use hyperbolic (hyperbola-shaped) lenses. Hyperbolic gears are used in many machines, and in industry. Sound waves travel in hyperbolic paths, and so there are applications of the hyperbola in navigation.

LORAN or Long RAnge Navigation



http://en.wikipedia.org/wiki/Image:Crude_loran_diagram.PNG

Based on time-difference principle. The difference between the time of receipt of synchronized signals from radio stations A and B is constant along each line of curve.

The following websites referenced for this program:
http://www.glencoe.com

http://www.learner.org

http://www.mathsisfun.com/geometry/pyramids.html

http://en.wikipedia.org/wiki/Regular_polyhedron

http://www.k12.hi.us/~mathappl/MAch3Curves.html