Lesson 21

Glencoe Geometry Chapter 10.1

Polygons and Area

We have been working with special types of polygons throughout the year. Rectangles, Squares, Trapezoids, and, yes, Triangles are all classified as part of the larger family of closed geometric shapes called polygons. But . . . there are many, many others. Today we look at some.

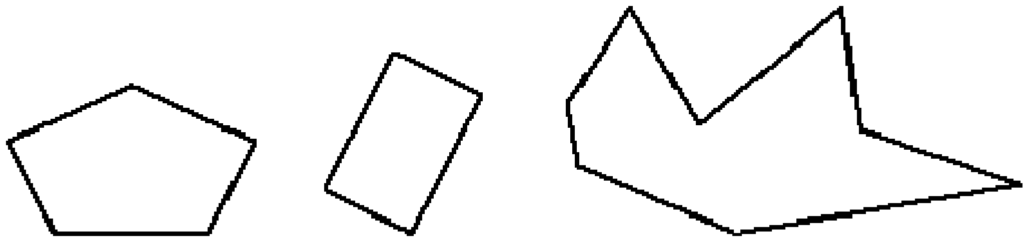
First we need a more precise, rigorous definition:

Definition of a Polygon:

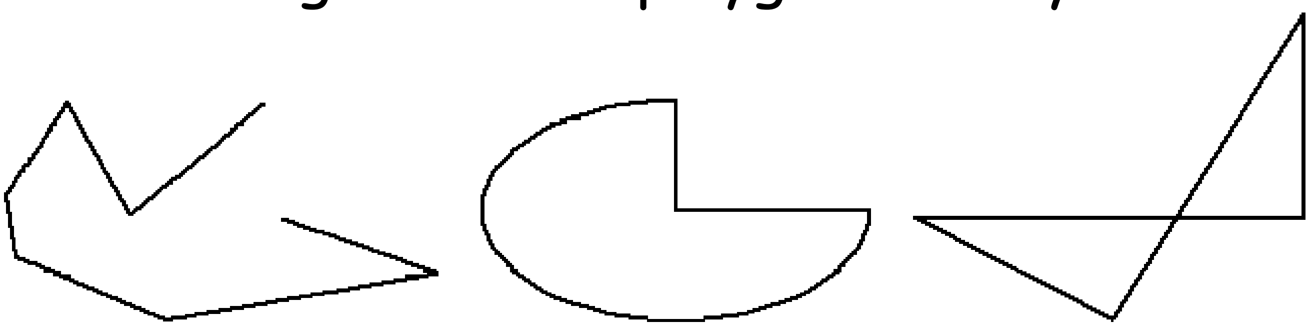
A polygon (*from Greek, literally "many-angle")* is a closed figure formed by a finite number of coplanar segments such that:

1. **the sides that have a common endpoint are non-collinear, and**
2. **each side intersects exactly two other sides, but only at their endpoints.**

The following are examples of polygons:



The following are NOT polygons. Why not?



not a closed figure

not made of line segments

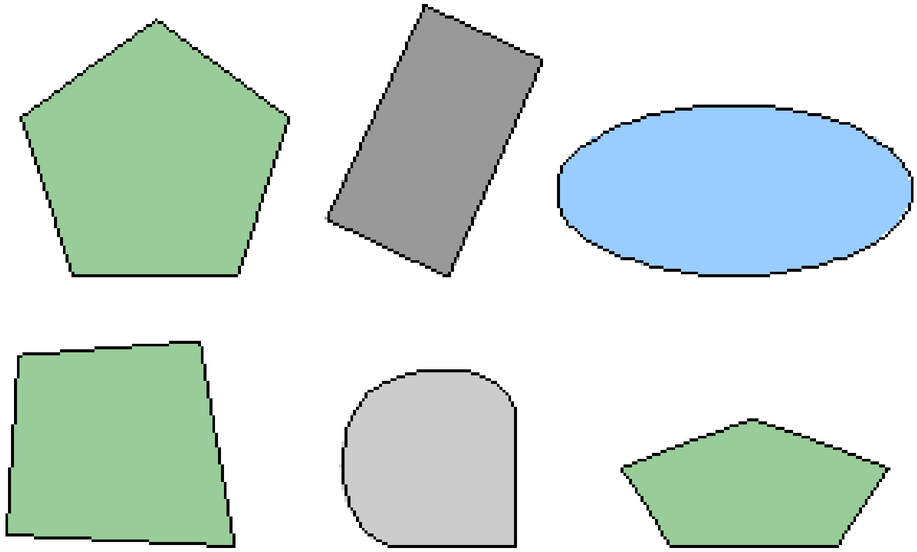
sides do not intersect in exactly two places each

Polygons can be classified based on their concavity.

- A figure is Convex if every line segment drawn between any two points inside the figure lies entirely inside the figure.

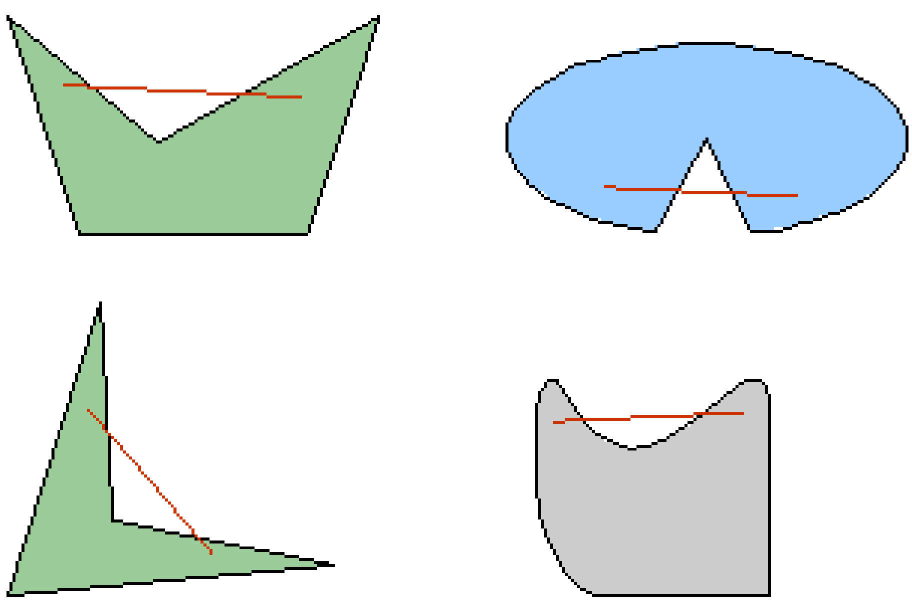
- A polygon is Concave if it is not convex.

The following figures are convex.



(which of these are NOT polygons??)

The following figures are concave. Note the **red** line segment drawn between two points inside the figure that also passes outside of the figure.



(which of these are NOT polygons??)

Polygons are named according to the number of sides, combining a Greek-derived numerical prefix with the suffix ***-gon***, e.g. ***pentagon***, ***dodecagon.*** The triangle and the quadrilateral are exceptions, however.

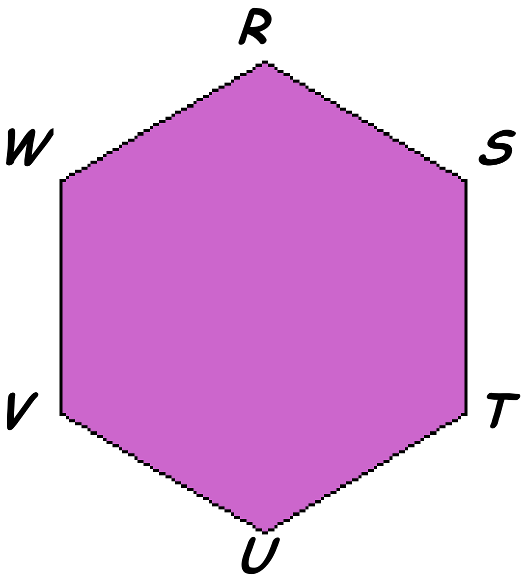
For larger numbers, we sometimes write the numeral itself, e.g. ***17-gon***. A variable can even be used, usually ***n-gon***.

POLYGON NAMES

# sides	name
1	Monogon**
2	Digon**
3	Trigon, triangle
4	Tetragon, quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon
11	Hendecagon
12	Dodecagon
13	Triskaidecagon
n	n-gon

**Degenerate polygons. Are considered impossible objects in Euclidean Geometry, but have applications in non-Euclidean geometry.

When referring to polygons, we use its name and list the vertices in consecutive order.



Two possible correct names for the polygon at the right are

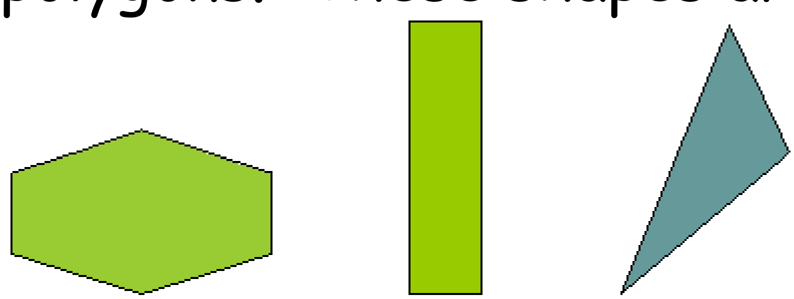
1. Hexagon *RSTUVW*

Or

2. Hexagon *TSRWVU*

Notice in this purple, **convex** hexagon that all **sides appear congruent**, and all **angles appear congruent** also. Polygons with these characteristics (purpleness aside), are called Regular polygons.

Squares and equilateral triangles are examples of regular polygons. These shapes are NOT!

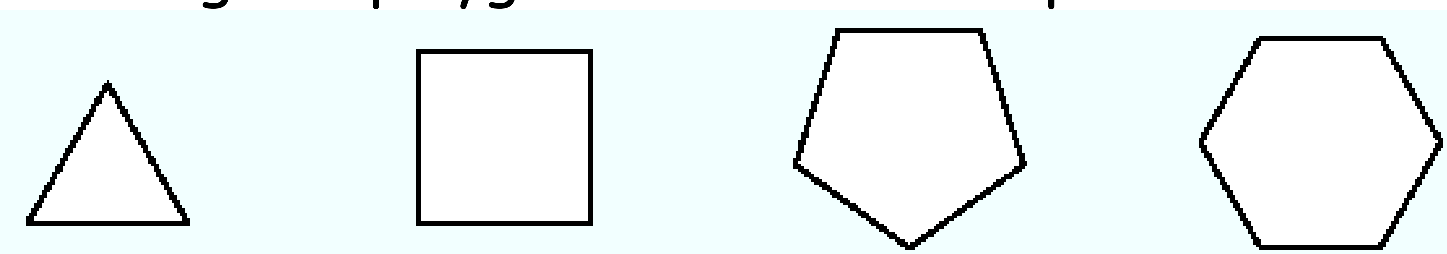


Before we look at an example, we need a definition:

The ***Perimeter*** of a polygon is the sum of all the side lengths.

Example:

The regular polygons below form a pattern:



Perimeter = 6in   Perimeter = 12in   Perimeter = 20in   Perimeter = 30in

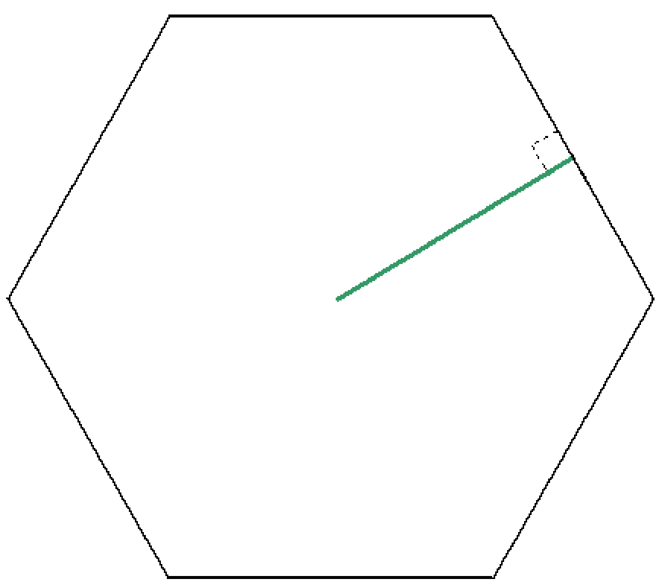
What is the perimeter of the seventh figure in the pattern?

Dividing the perimeter by the number of sides give the side lengths of 2, 3,4, and 5 respectively. Notice this is one less than the number of sides. Since the number of sides increases by one each time, the seventh figure will have 9 sides (nonagon) and will have a side length of 8. Adding all nine values of 8, we get (9)(8) = 72 inches.

Say What?!?!

We will often be asked to solve problems involving the **Area** of a regular polygon. The area is just the measure of the size of the region enclosed by a figure.

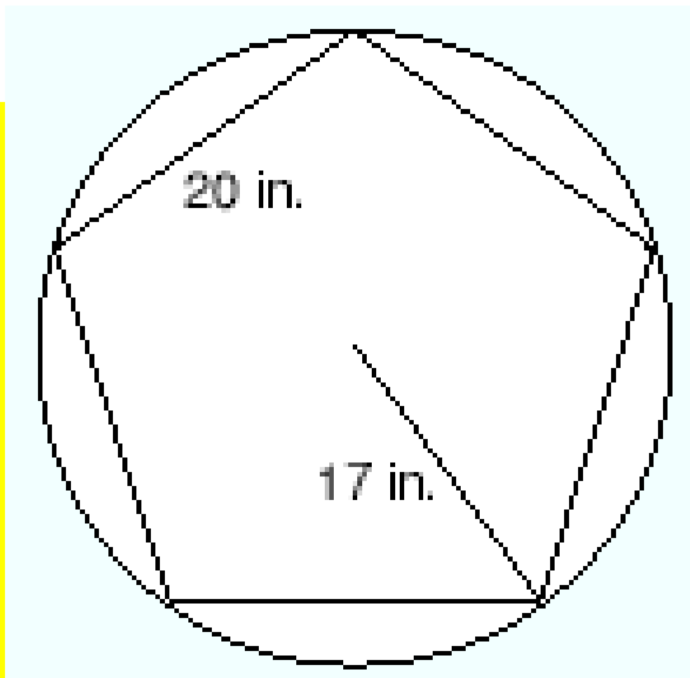
The area of the formula involves a special length called the **Apothem**. The apothem of a regular polygon is the length of the perpendicular line segment from the center of the polygon to a side of the polygon. It bisects the side to which it is drawn.



For any regular polygon with perimeter P and apothem of length *a*, the area, *A*, is given by the following formula:

$$A = \frac{1}{2}Pa$$

Find the area of a regular pentagon with a side length of 20 inches inscribed in a circle with a radius of 17 inches.



The apothem will bisect a side length, forming a right triangle with legs of 10 and the apothem length, *a*. The radius of the circle is the hypotenuse. By the Pythagorean theorem.

$10^2 + a^2 = 17^2$

$a = \sqrt{289 - 100}$

$a = \sqrt{189} \approx 13.75in$

The perimeter is 5 lengths of 20, so
$P = 5(20) = 100in$

We are now ready to apply the formula:

$A = \frac{1}{2}Pa$

$A = \frac{1}{2}(100in)\left(\sqrt{189}in\right)$

$A = (50in)\left(\sqrt{189}in\right) \approx 687.5in^2$