Lesson 20

Glencoe Geometry Chapter 7.4 & 7.5

Parallel Lines & Proportional Parts

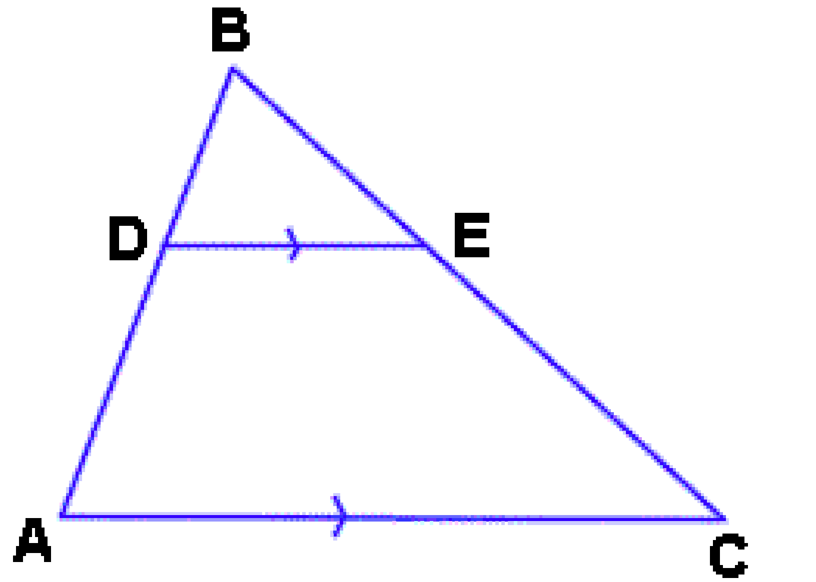
On previous episodes, we've look at both parallel lines and proportionality. Today we look at both of them together.

A very important theorem in geometry states this relationship.

The Triangle Proportionality Theorem:

If a line is parallel to one side of a triangle and intersects the other two sides in two distinct points, then it separates these sides into segments of proportional length, and *vice-versa (The Converse)*!!

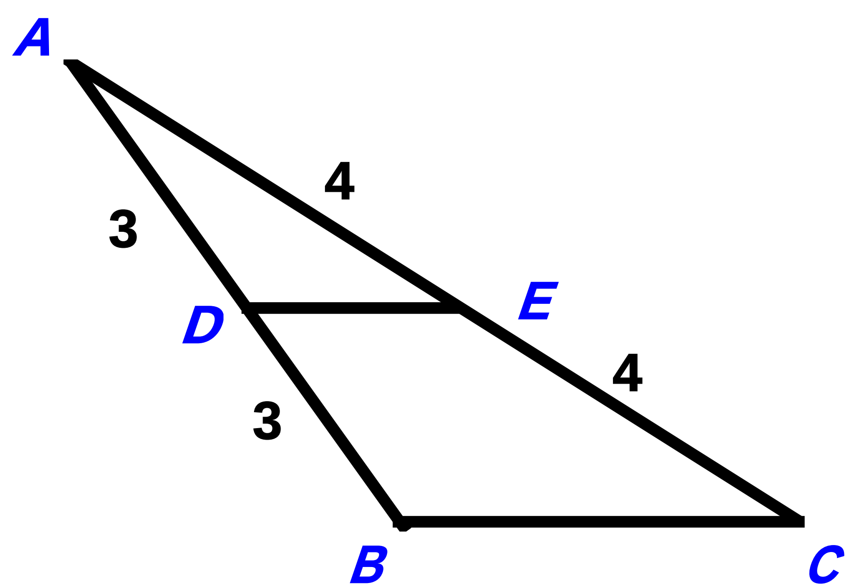
In triangle ABC, $\overline{DE} \parallel \overline{AC}$. The theorem tells us that $\frac{DA}{BD} = \frac{EC}{BE}$



We actually looked at this property informally last week, but today we will explore it in more detail.

Example:

Based on the figure at right, which statement is false?



A. $\overline{DE} \parallel \overline{BC}$ B. $\triangle ABC \sim \triangle ADE$

C. $\triangle ABC \cong \triangle ADE$ D. D is the midpoint of $\overline{AB}$.

Here's another VERY important theorem:

A segment whose endpoints are the midpoints of two sides of a triangle is parallel to the third side of the triangle, **AND** its length is one-half the length of the third side.

Example:

Find the value of x.

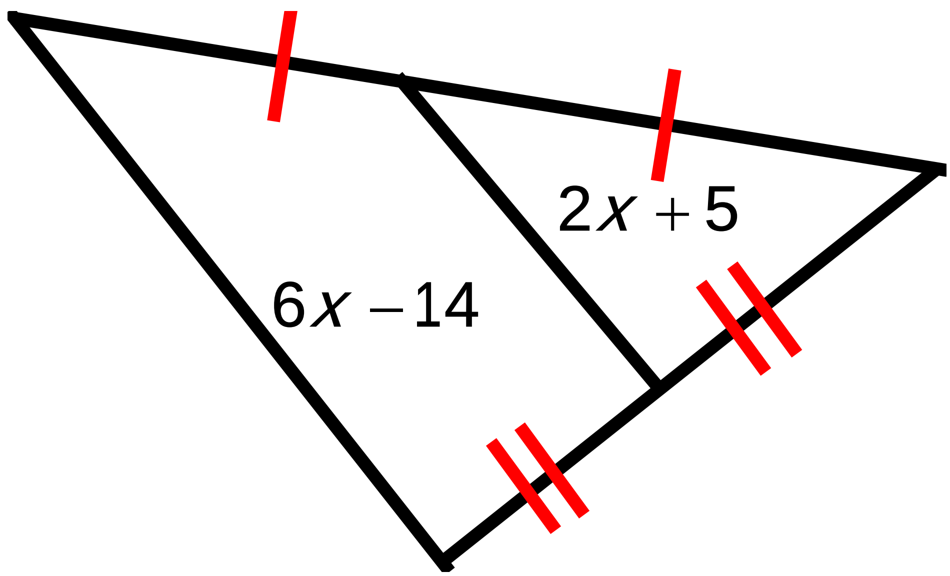
By the theorem,

$x + \quad = \frac{1}{2}(6x - 14)$

$x + \quad = 3x - 7$

$-x = -12$

$x = 12$



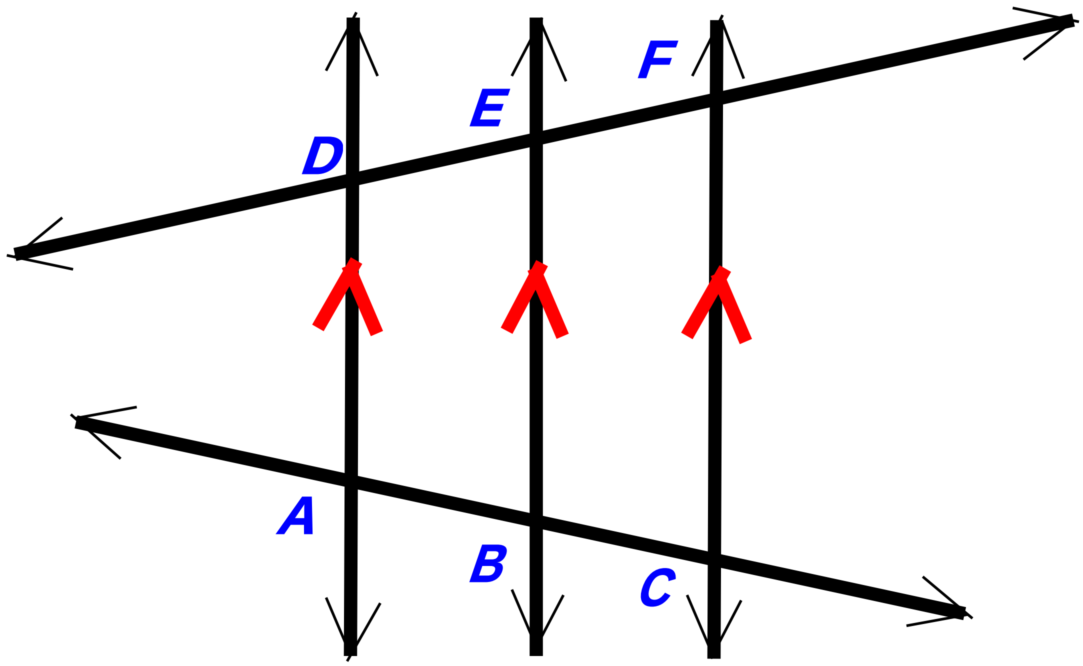
Are you ready for another theorem??? What else would you be watching for??

Theorem:

If **three** or more parallel lines intersect **two transversals**, then they cut off the transversals **proportionally**.

Example:

In the figure, $\overleftrightarrow{AD} \parallel \overleftrightarrow{BE} \parallel \overleftrightarrow{CF}$, if $AB = 4$, $DE = x + 1$, $BC = 6$, and $EF = 3x - 9$, what is the value of x?



$\frac{AB}{DE} = \frac{BC}{EF} \rightarrow \frac{4}{x+1} = \frac{6}{3x-9} \rightarrow 4(3x-9) = 6(x+1) \rightarrow 12x - 36 = 6x + 6 \rightarrow 6x = 42 \rightarrow x = 7$

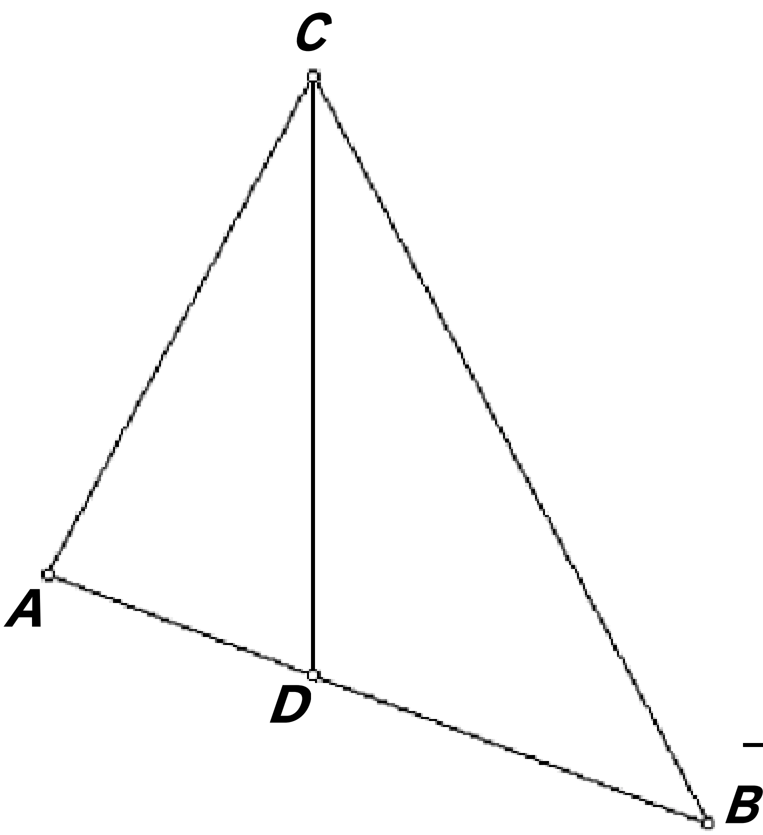
so

$DE = 7 + 1 = 8$ and $EF = 3(7) - 9 = 21 - 9 = 12$

Now, before we get on to some more important theorems, lets look at . . . well . . . another THEOREM. This one has its own special name (those are the ***really*** important ones)!

The Angle Bisector Theorem:

An angle bisector in a triangle divides the opposite side into segments that have the same ratio as the other two adjacent sides. REREAD.

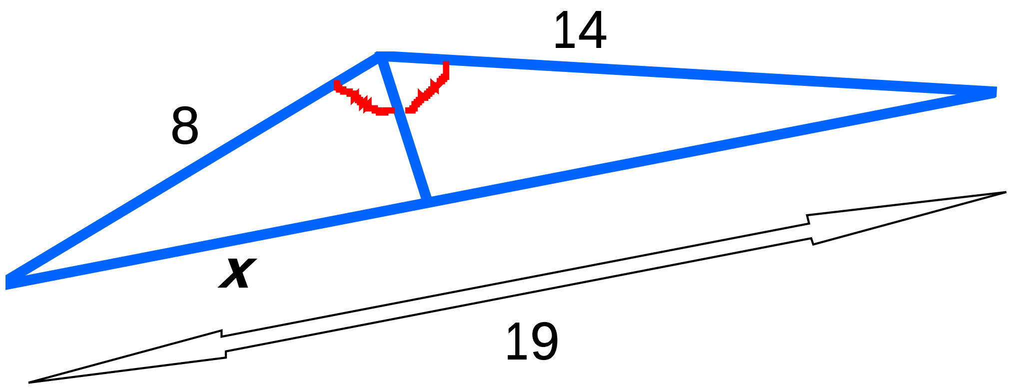


In $\triangle ABC$, $\angle ACD \cong \angle BCD$, so by the theorem,

$$\frac{AD}{BD} = \frac{AC}{BC}$$

Example:

Find the value of x.



The side opposite the angle bisector can be labeled as two lengths, x and $19 - x$.

By the theorem

$$\frac{x}{19-x} = \frac{8}{14}$$

$$\frac{x}{19-x} = \frac{4}{7}$$

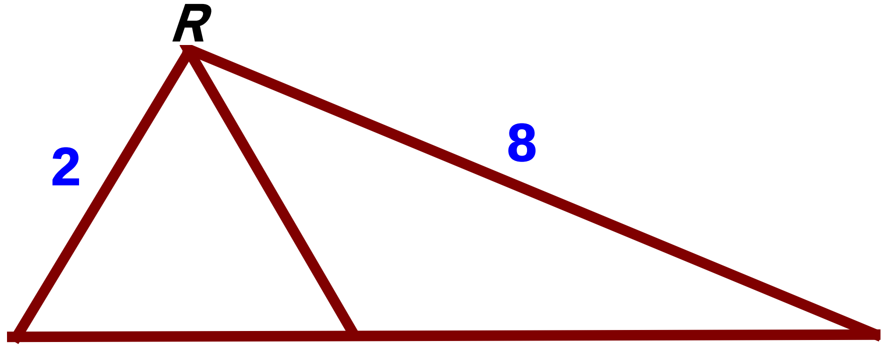
$$7x = 4(19 - x)$$

$$7x = 76 - 4x$$

$$11x = 76$$

Example:

In the figure, $\overline{RU}$ bisects $\angle TRS$. Find the value of y.

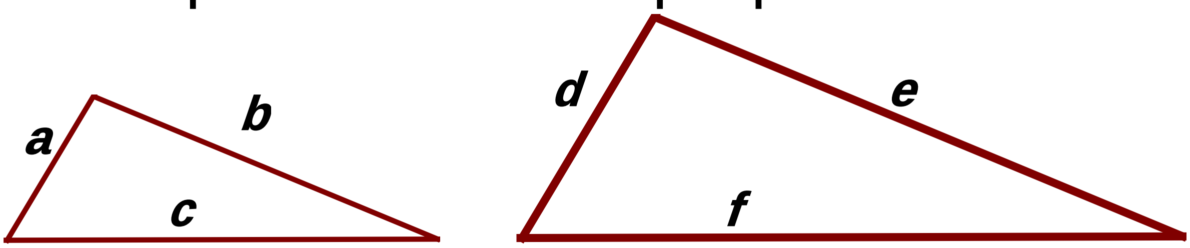


T y U 7.2 S

Say What?!?!

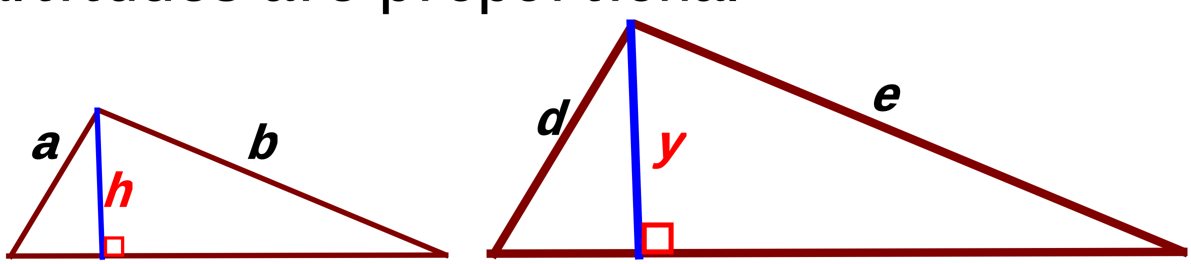
Did you know that if two triangles are similar, there are not only special parts, but also special quantities that are also proportional to the corresponding sides??? Say WHAT?!?! Here's a summary of what I mean . . .

- The perimeters are proportional



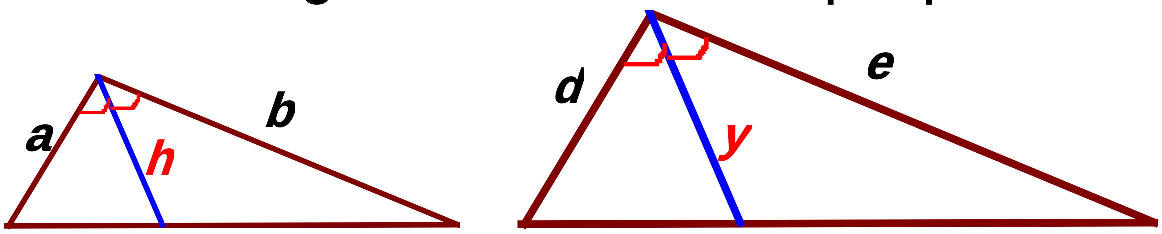
$$\frac{a+b+c}{d+e+f}=\frac{a}{d}=\frac{b}{e}=\frac{c}{f}$$

- The altitudes are proportional



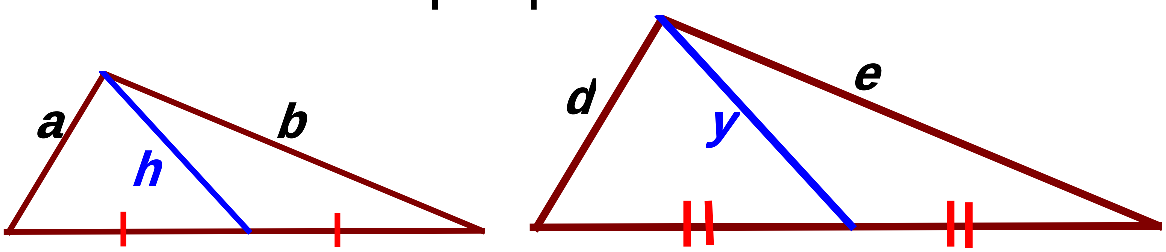
$$\frac{h}{y}=\frac{a}{d}=\frac{b}{e}$$

- The angle bisectors are proportional



$$\frac{h}{y}=\frac{a}{d}=\frac{b}{e}$$

- The medians are proportional



$$\frac{h}{y}=\frac{a}{d}=\frac{b}{e}$$

Now you know.

Go out and use your new power wisely!!

Some examples from www.glencoe.com