



Lesson 2

Glencoe Geometry Chapter 1.4 & 1.5

Segment Measurement, Midpoints, & Congruence

Last time, we looked at points, lines, and planes. Today we are going to further investigate lines, points on a line, and the segments between two points.

By the end of the show, you should be able to:

1. Find the _____ between two points on a number line AND between two points in a coordinate (Cartesian) plane.
2. Use the _____ Theorem to find the length of the hypotenuse of a right triangle.
3. Find the _____ of a segment.

We'll even get to use some good, ol' _____ today!!

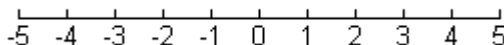
Let's review from last episode.

Remember that a line segment is a **piece** of a line that consists of two **endpoints** and all the points between them. It is denoted by the capital letters of the two endpoints with a line above them, such as $\overline{AB}$. The _____ of the segment matters!

There are two ways to measure the lengths of segments: with a ruler or to calculate them using number lines. We are going to focus on the second method. But first, we need look at some definitions . . .

Definitions:

The number that corresponds to a point on a number line is called the _____ of the point



The number of units from zero to any number on the number line is called its . We use the symbol to denote this. The absolute value is ALWAYS positive!!

Example:

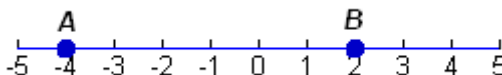
$|5| =$

$|-5| =$

$|2 + 5| =$

$|2 - 5| =$

We are now ready to find the distance between two points on the number line below, namely points A and B . Finding this length is the same as finding the length of $\overline{AB}$, and we denote this measure by writing AB WITHOUT a above it!!!



We can find AB by the number of units or by the two coordinates.

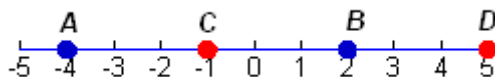
$AB =$

or $AB =$

Notice above that $AB = BA$ since they represent the **same** segment. But what if another segment that was different had the same length?! It's time for another definition.

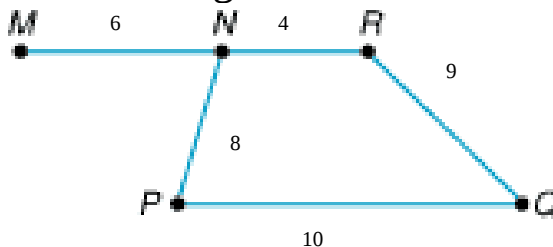
Two segments are if and only if they have the same length. The symbol used is $\cong$. For instance to show that segments $\overline{AB}$ and $\overline{CD}$ are congruent, we write

$$\overline{AB} \cong \overline{CD}$$



Example:

Determine which segments are congruent.



- A. $\overline{PQ}$ and $\overline{MR}$ B. $\overline{MR}$ and $\overline{MP}$
 C. $\overline{MN}$ and $\overline{NR}$ D. $\overline{NP}$ and $\overline{RQ}$

Example:

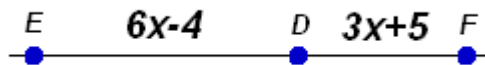
If $\overline{FG} \cong \overline{UV}$ and $\overline{UV} \cong \overline{PQ}$, then_____.

- A. $\overline{FG} \cong \overline{PQ}$ B. $\overline{GU} \cong \overline{PV}$
 C. $\overline{FP} \cong \overline{VQ}$ D. $\overline{FV} \cong \overline{UQ}$

Try this. It involves some thinking and some Algebra. It may be helpful to make a drawing of the given information.

Example:

Find DF if D is between E and F , $ED = 6x - 4$, $DF = 3x + 5$, and $EF = 46$.



DF :

Now we will step up a dimension and find the distance between two points on the coordinate plane. To do this, we must use the **Pythagorean Theorem**. What is a theorem??? I'm glad you might have asked.

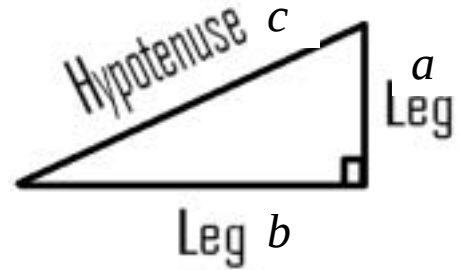
A _____ is a statement that can be justified using logical reasoning or proven using other theorems and axioms. Basically, it's like a law or a true statement.

Here's the Pythagorean theorem:

The sum of the square of the two legs of a right triangle equals the square of the hypotenuse.

or

$$a^2 + b^2 = c^2$$



<http://mac.yasu.edu/~jmartin/pythagorean/pythagorean1.gif>

Here's how it works:

If one leg of a right triangle is 17.2 cm and the other leg is 22.5 cm, what is the length of the hypotenuse?

Now to get a bit more interesting.

The formula to determine the distance, d , between two coordinates, (x_1, y_1) and (x_2, y_2) on the coordinate plane is

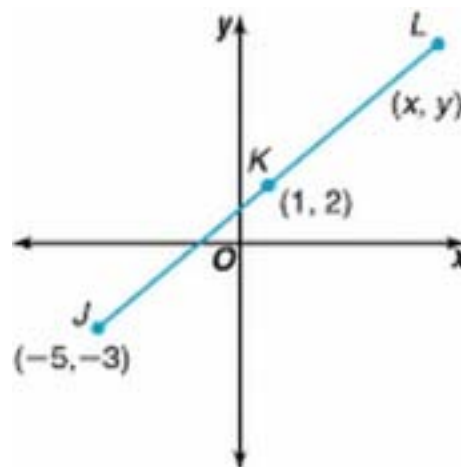
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

or

$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2} \text{ where } \Delta \text{ means "change in"}$$

Example:

Find the length of $\overline{JK}$ in the figure at right.
For this, we can use the distance formula:



note the order in which we subtract does not matter, since we are squaring each quantity.

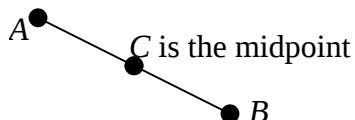
Example:

Find RT for $R(-2, 5)$ and $T(-3, -6)$. Round to the nearest tenth.

- A. 9.8 B. 8.5 C. 10.6 D. 11.0

Now we look at finding the Midpoint of segments. Just as before, we can look at the number line or on the coordinate plane. Of course, some definitions are required.

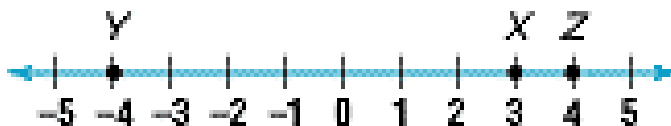
A _____ is the point between a segment that divides the segment into two equal segments. The midpoint is said to _____ the segment. A midpoint C bisects $\overline{AB}$ if and only if $\overline{AC} \cong \overline{CB}$



On a number line, the coordinate of the midpoint of a segment whose endpoints have coordinates of A and B is $\frac{A+B}{2}$.

Example:

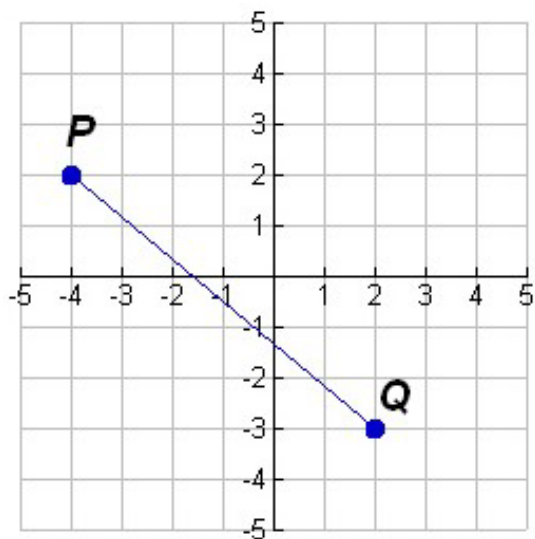
Find the midpoint of $\overline{XY}$ on the number line shown.



Again, the problems get more challenging when we are on the coordinate plane. In this case, the coordinates of the midpoint of a segment whose endpoints have coordinates (x_1, y_1) and (x_2, y_2) are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Example:

Find the coordinates of the midpoint of $\overline{PQ}$.



A. $\left(-1, -\frac{1}{2}\right)$

B. $\left(-1, 2\frac{1}{2}\right)$

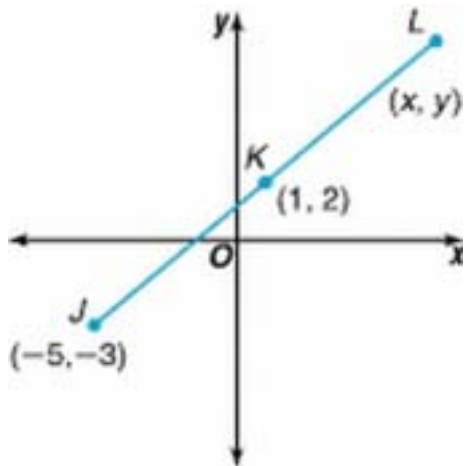
C. $(3, 2)$

D. $\left(-3, -\frac{1}{2}\right)$

Here's one a little trickier:

Example:

What are the coordinates of L if K is the midpoint of $\overline{JL}$?



A. $(4, 1)$

B. $(7, 7)$

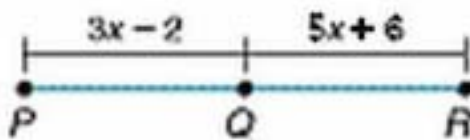
C. $(8, 2)$

D. $(6, 5)$

And one last interesting example:

Example:

Find the value of x if Q bisects $\overline{PR}$.



A. -4

B. 4

C. 7

D. 1

Say What??!!

There are a lot of different notations for lines, segments, rays, and segment measure, and they only differ by the symbol above the two-point name. Let's see if we can correctly match them up . . .

 $\overline{AB}$

Segment AB

 $\overleftrightarrow{AB}$

Line AB

 $\overline{AB}$

The measure of segment AB

 $\overrightarrow{AB}$

Ray AB