Lesson 2

Glencoe Geometry Chapter 1.4 & 1.5

Segment Measurement, Midpoints, & Congruence

Last time, we looked at points, lines, and planes. Today we are going to further investigate lines, points on a line, and the segments between two points.

By the end of the show, you should be able to:

1. Find the __distance__ between two points on a number line AND between two points in a coordinate (Cartesian) plane.
2. Use the __Pythagorean__ Theorem to find the length of the hypotenuse of a right triangle.
3. Find the __midpoint__ of a segment.

We'll even get to use some good, ol' ___Algebra___ today!!

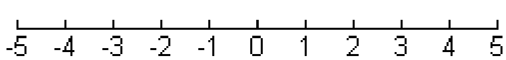
Let's review from last episode.

Remember that a line segment is a **piece** of a line that consists of two **endpoints** and all the points between them. It is denoted by the capital letters of the two endpoints with a line above them, such as $\overline{AB}$. The ___length____ of the segment matters!

There are two ways to measure the lengths of segments: with a ruler or to calculate them using number lines. We are going to focus on the second method. But first, we need look at some definitions . . .

<u>Definitions:</u>

The number that corresponds to a point on a number line is called the ___coordinate____ of the point

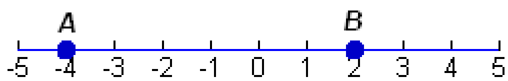


The number of units from zero to any number on the number line is called its ___absolute__ ______value______. We use the __| |________ symbol to denote this. The absolute value is ALWAYS positive!!

Example:

$|5| = 5$ $\quad |-5| = 5$ $\quad |2+5| = 7$ $\quad |2-5| = 3$

We are now ready to find the distance between two points on the number line below, namely points *A* and *B*. Finding this length is the same as finding the length of ___segment______ $\overline{AB}$, and we denote this measure by writing *AB* WITHOUT a __bar_____ above it!!!



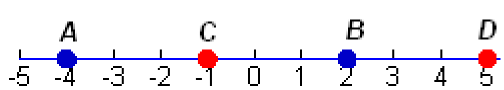
We can find AB by ___counting__ the number of units or by _subtracting__ the two coordinates.

$AB = 6$ or $AB = |-4-2| = |-6| = |2-(-4)| = |6| = 6$

Notice above that $AB = BA$ since they represent the ***same*** segment. But what if another segment that was different had the same length?! It's time for another definition.

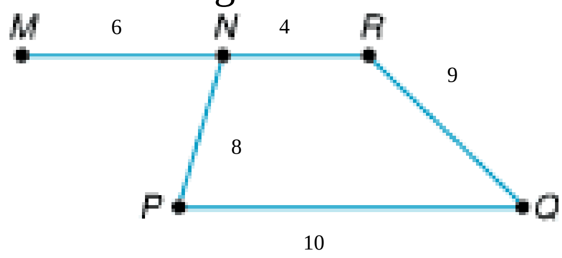
Two segments are __congruent__ if and only if they have the same length. The symbol used is $\cong$. For instance to show that segments $\overline{AB}$ and $\overline{CD}$ are congruent, we write

$$\overline{AB} \cong \overline{CD}$$



Example:

Determine which segments are congruent.



A. $\overline{PQ}$ and $\overline{MR}$ B. $\overline{MR}$ and $\overline{MP}$

C. $\overline{MN}$ and $\overline{NR}$ D. $\overline{NP}$ and $\overline{RQ}$

Example:

If $\overline{FG} \cong \overline{UV}$ and $\overline{UV} \cong \overline{PQ}$, then_____. (by transitive property)

B. $\overline{GU} \cong \overline{PV}$

C. $\overline{FP} \cong \overline{VQ}$ D. $\overline{FV} \cong \overline{UQ}$

Try this. It involves some thinking and some Algebra. It may be helpful to make a drawing of the given information.

Example:

Find *DF* if *D* is between *E* and *F*, *ED* = 6*x* - 4, *DF* = 3*x* + 5, and *EF* = 46.

$DF:$

$$ED + DF = EF$$

$$(6x+4)+(3x+5)=46$$

$$9x+1=46$$

$$9x=45$$

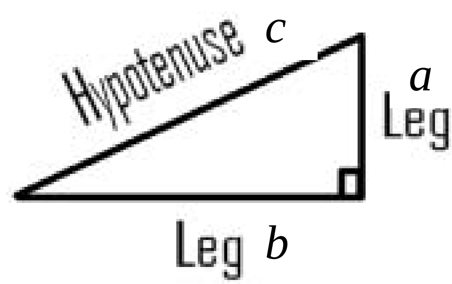
$$x=5$$

so $ED = 6(5)-4 = 26$ & $DF = 3(5)+5 = 20$

Now we will step up a dimension and find the distance between two points on the coordinate plane. To do this, we must use the **Pythagorean Theorem**. What is a theorem??? I'm glad you might have asked.

A ___Theorem___ is a statement that can be justified using logical reasoning or proven using other theorems and axioms. Basically, it's like a law or a true statement.

Here's the Pythagorean theorem:
The sum of the square of the two legs of a right triangle equals the square of the hypotenuse.
or
$a^2 + b^2 = c^2$



http://mac.ysu.edu/~jmartin/pythagorean/pythagorean1.gif

Here's how it works:
If one leg of a right triangle is 17.2 cm and the other leg is 22.5 cm, what is the length of the hypotenuse?

28.3cm **See page 30, Example 3.**

Now to get a bit more interesting.

The formula to determine the distance, *d*, between two coordinates (x_1, y_1) and (x_2, y_2) on the coordinate plane is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

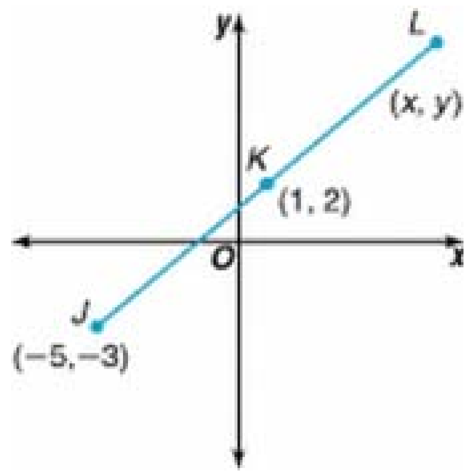
or

$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$ where Δ means "change in"

Example:

Find the length of $\overline{JK}$ in the figure at right.
For this, we can use the distance formula:

$$d = \sqrt{(-5-1)^2 + (-3-2)^2}$$
$$= \sqrt{(-6)^2 + (-5)^2} = \sqrt{36+25}$$
$$= \sqrt{61} \approx 7.810$$



note the order in which we subtract does not matter, since we are squaring each quantity.

Example:

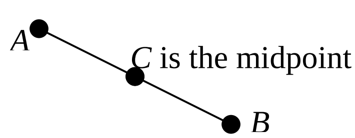
Find *RT* for *R*(-2, 5) and *T*(-3, -6). Round to the nearest tenth.

A. 9.8 B. 8.5 C. 10.6 D. 11.0

(Hint: Use the distance formula **See page 31, Example 4.**)

Now we look at finding the Midpoint of segments. Just as before, we can look at the number line or on the coordinate plane. Of course, some definitions are required.

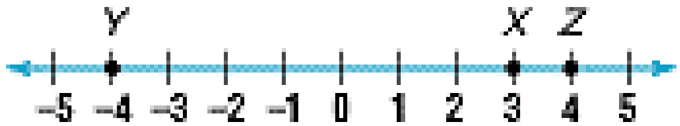
A __Midpoint___ is the point between a segment that divides the segment into two equal segments. The midpoint is said to ___bisect______ the segment. A midpoint *C* bisects $\overline{AB}$ if and only if $\overline{AC} \cong \overline{CB}$



On a number line, the coordinate of the midpoint of a segment whose endpoints have coordinates of *A* and *B* is $\frac{A+B}{2}$.

Example:

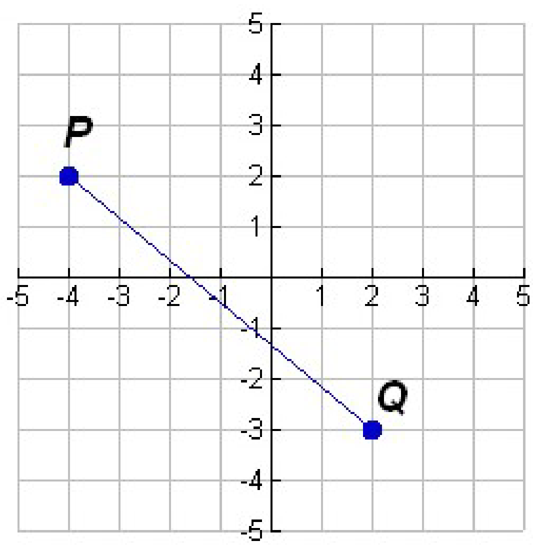
Find the midpoint of $\overline{XY}$ on the number line shown.



Again, the problems get more challenging when we are on the coordinate plane. In this case, the coordinates of the midpoint of a segment whose endpoints have coordinates (x_1, y_1) and (x_2, y_2) are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

Example:

Find the coordinates of the midpoint of $\overline{PQ}$.



$$\frac{-4+2}{2}, \frac{2+-3}{2} = \left(-1, -\frac{1}{2}\right)$$

A. $\left(-1, -\frac{1}{2}\right)$

B. $\left(-1, 2\frac{1}{2}\right)$

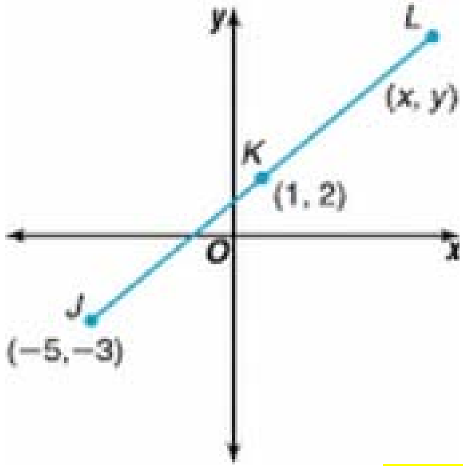
C. $(3, 2)$

D. $\left(-3, -\frac{1}{2}\right)$

Here's one a little trickier:

Example:

What are the coordinates of L if K is the midpoint of $\overline{JL}$?



$$\left(\frac{-5+x}{2}, \frac{-3+y}{2}\right) = (1,2) \text{ so}$$

$$\frac{-5+x}{2} = 1 \text{ and } \frac{-3+y}{2} = 2$$

$$-5+x = 2 \text{ and } -3+y = 4$$

$$x = 7 \text{ and } y = 7$$

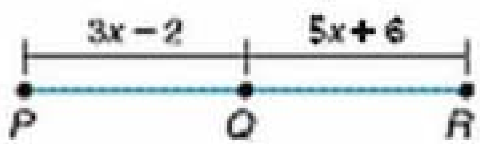
so the midpoint is $(7,7)$

A. (4, 1) B. (7, 7) C. (8, 2) D. (6, 5)

And one last interesting example:

Example:

Find the value of x if Q bisects $\overline{PR}$.



A. –4 B. 4 C. 7 D. 1

$$\overline{PQ} \cong \overline{QR} \text{ so}$$

$$PQ = QR$$

$$3x - 2 = 5x + 6$$

$$-2x = 8$$

$$x = -4$$

Say What??!!

There are a lot of different notations for lines, segments, rays, and segment measure, and they only differ by the symbol above the two-point name. Let's see if we can correctly match them up . . .

$\overrightarrow{AB}$	ray	**Segment *AB***
$\overleftrightarrow{AB}$	line	**Line *AB***
$\overline{AB}$	segment	**The measure of segment *AB***
AB	measure	**Ray *AB***

*unless otherwise noted, all images are created using TI-Interactive Software or from www.Glencoe.com