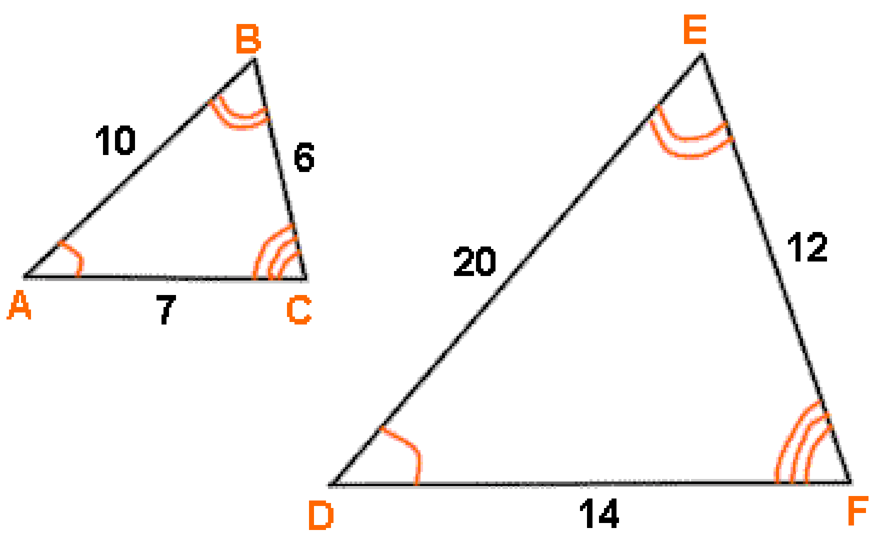
Lesson 19

Glencoe Geometry Chapter 7.3

Similar Triangles

On a previous episode, we talked about proving congruent triangles. There were certain combinations of given angles and sides that were sufficient to prove congruency.

One combination that did NOT work was the **AAA** or **Angle-Angle-Angle** combination.

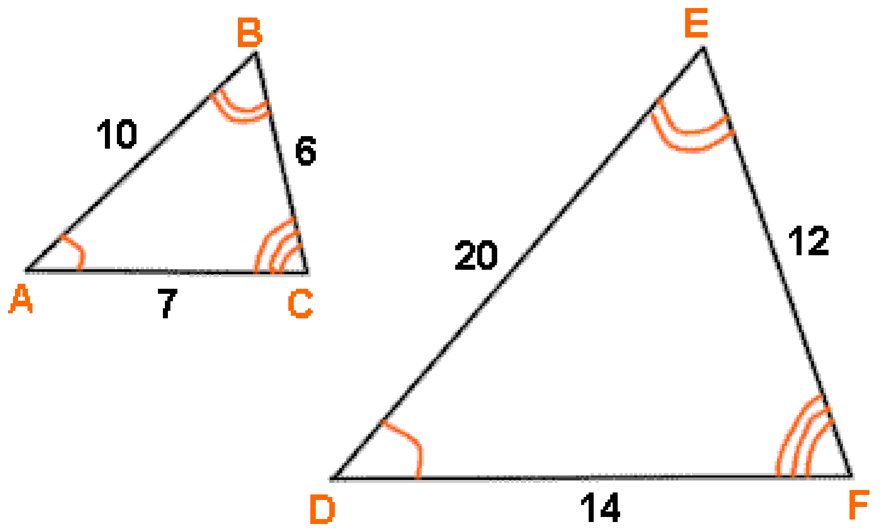


$$\triangle ABC \sim \triangle DEF$$

If two shapes are similar, one is an enlargement, or **DILATION**, of the other. The shapes will have the same **SHAPE** but not the same **SIZE**. A dilation is NOT an **ISOMETRY**, or rigid transformation.

This means that the two shapes will have the same angles and their sides will be in the same **PROPORTION**.

$\triangle ABC \sim \triangle DEF$

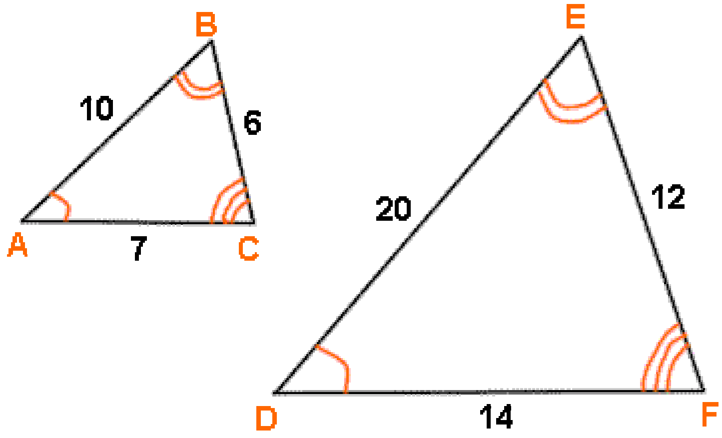


Facts about similar triangles:

$$\angle A \cong \angle D$$
$$\angle B \cong \angle E$$
$$C \quad \angle F$$

AND

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$$



$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$$

$$\frac{10}{20} = \frac{6}{12} = \frac{7}{14}$$
$$\frac{1}{2} = \frac{1}{2} = \frac{1}{2}$$

The number $\frac{1}{2}$ in this case is called the **ratio of** SIMILITUDE.

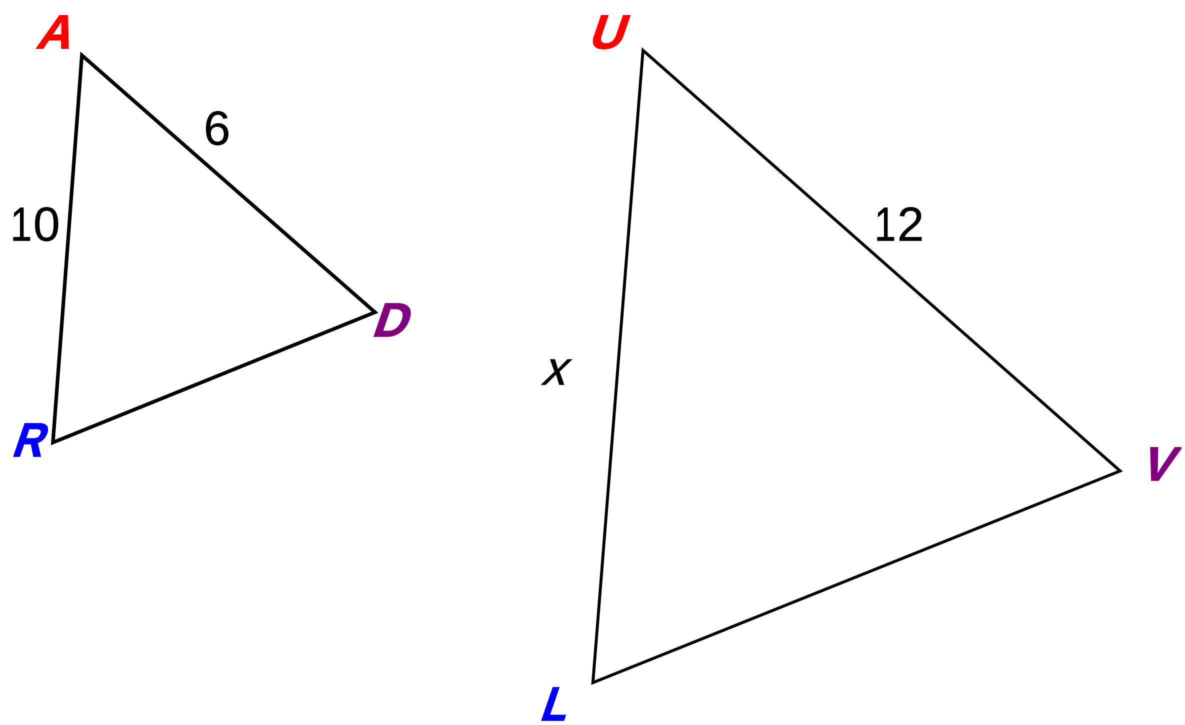
The ratios could all equivalently be reciprocated to get 2. This means that the smaller triangle is HALF the size of the larger, or the larger is TWICE the size of the smaller.

Let's look at some problems involving similar triangles. There are many different TYPES of these problems, so there are also several different STRATEGIES to deal with them. *There may be more than one correct way to arrive at the correct answer.*

The easiest problems dealing with similar triangles are those that involve two separate triangles.

Example:

For similar triangles *RAD* and *LUV*, solve for *x*.



These two triangles are sitting such that their corresponding parts are in the same position in each triangle.

If the triangles are not sitting in this manner, you can match the corresponding sides by looking across from the angles which are equal in each triangle.

Creating a proportion in one of two ways matching the corresponding sides:

a) small triangle on top: $\frac{10}{x} = \frac{6}{12} \rightarrow \frac{10}{x} = \frac{1}{2} \rightarrow 20 = x$

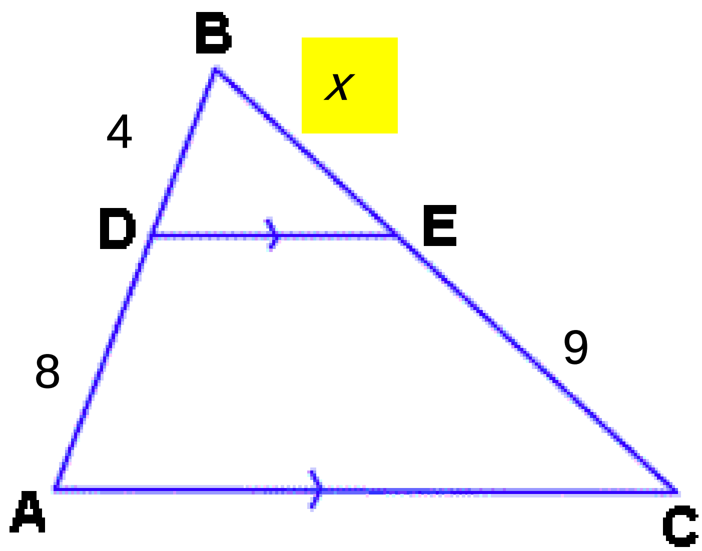
b) large triangle on top: $\frac{x}{10} = \frac{12}{6} \rightarrow \frac{x}{10} = 2 \rightarrow x = 20$

Many problems involving similar triangles have one triangle ON TOP OF another triangle.

Example:

Find *BE*. Call it x

Since *DE* is marked to be parallel to *AC*, we know that we have angle *BDE* equal to angle *DAC* (corresponding angles). Angle *B* is shared by both triangles, so the triangles are similar.



There are two ways to attack this problem:

1. Use **FULL** sides of the two triangles when dealing with the problem. Do not use *DA* or *EC* since they are not sides of triangles.

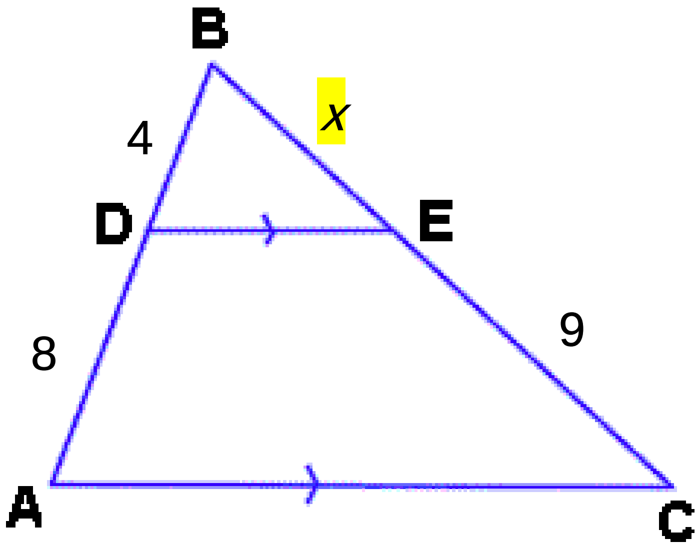
EASIER METHOD TO USE

$x \quad \rightarrow 1 = \quad x \quad \rightarrow x + 9 = 3x \rightarrow 2x = 9 \rightarrow x =$

or

$\rightarrow$

2. Another way to do this problem is to realize a special property: if a line is parallel to one side of a triangle, it divides the other sides proportionately.



$\frac{4}{8}=\frac{x}{9}\to\frac{1}{2}=\frac{x}{9}\to 9=2x\to x=4.5$

This property is not true for all polygons, and you must REMEMBER this property in order to use it, but it does simplify the math a bit.

Example:

Find *EC*. Call it x

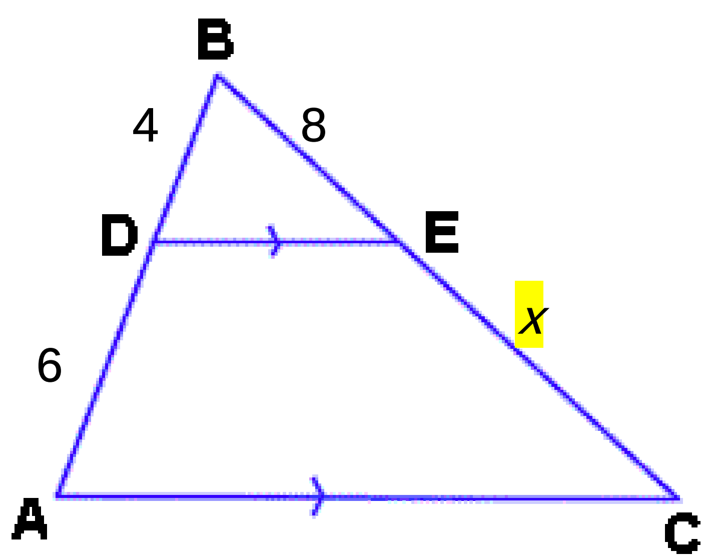
$\frac{4}{10}=\frac{8}{8+x}\to\frac{2}{5}=\frac{8}{8+x}\to$

$2(8+x)=40\to 8+x=20\to x=12$

or from the parallel property,

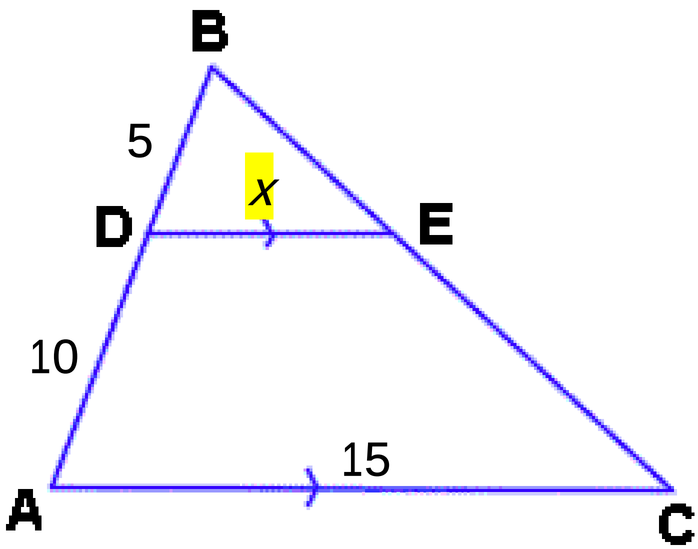
$\frac{6}{4}=\frac{x}{8}\to\frac{3}{2}=\frac{x}{8}\to$

$24=2x\to x=12$



Example:

Find *DE*. Call it x



Can't use the parallel property here!!
We MUST use the full sides of the triangles.

$$\frac{5}{15}=\frac{x}{15}\rightarrow\frac{1}{3}=\frac{x}{15}\rightarrow 15=3x\rightarrow x=5$$

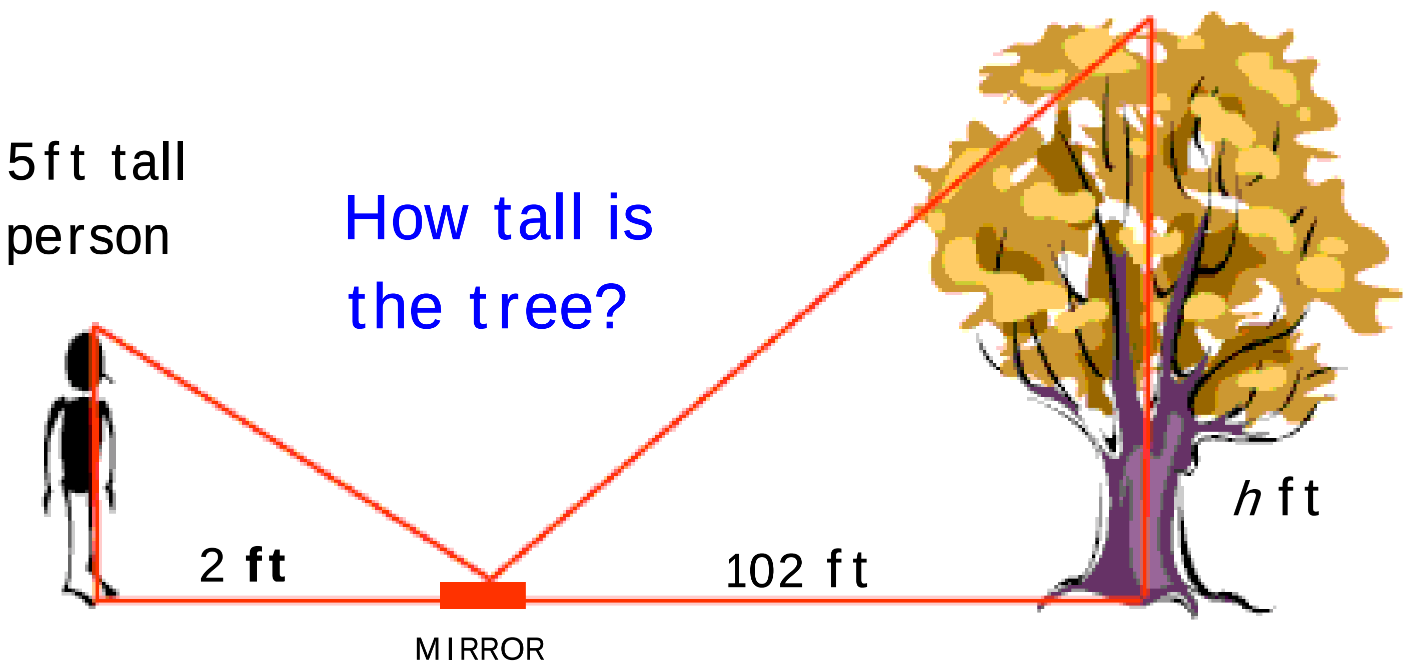
OR

$$\frac{}{5}=\frac{15}{15}\rightarrow\frac{}{5}=1\rightarrow x=5$$

HINT: If the triangles which are **on top of** one another are causing you problems, simply redraw the triangles as two separate figures.

Say What!?!?

Have you ever wanted to measure the height of a tall tree but didn't have a long enough tape measure, a ladder, or the guts? Did you know that all you really need to do this are a tape measure, a mirror, and your very own cerebral cortex?!?!



Because both you and the tree are at right angles to the ground and the angle of incidence and reflection at the mirror are congruent, the two triangles formed are congruent, so we can set up a proportion to solve of *h*.

$$\frac{h}{5} = \frac{102}{2} \rightarrow \frac{h}{5} = 51 \rightarrow h = 255ft.$$