Lesson 18

Glencoe Geometry Chapter 7.1, 7.2

Similarity & Proportionality

Recall from algebra the idea of a ratio:

A **ratio** is a comparison of two numbers or expressions *a* and *b*. It can be expressed as a quotient of the two numbers.

$$\frac{a}{b} \text{ or } a:b,$$

Ratios are useful to compare relative lengths, to establish a scale factor, or, when given units (such as $\frac{a \text{ miles}}{b \text{ gallons}}$ or $\frac{a \text{ dollars}}{b \text{ pounds}}$), to **express a rate**.

Example:

What is the ratio of 15 to 5?

Answer: 15/5, 15:5 or 3/1, 3:1
So If Jill has $15, and Jack has $5, then Jill has **three times** more than Jack. *Although this fraction can be simplified, by not doing so, we preserve the original measurements or values.*

Sometimes it is convenient to write a ratio in simplified format, or with the denominator as a one.

Example:

The ratio of 5 to 2 can be written as . . .

$\frac{5}{2}$ or as $5:2$ or as $\frac{2.5}{1}$ or as $2.5:1$. Suppose units of dollars and pounds: We can say the ratio of price to pounds is \$5 per 2 pounds, or \$2.50 per pound.

When two ratios are set equal to each other, a proportion is obtained.

$$\frac{a}{b} = \frac{c}{d}$$

A proportion is useful in finding missing values when a scale factor is involved. The equation can be solved by cross multiplying.

$$\frac{a}{b} = \frac{c}{d}$$

$$ad = bc$$

Example:

What is 38% of 178?

38% means the ratio $\frac{38}{100}$ this represents the "part" 38 to the "whole" 100. We want to know the equivalent "part" (call is x) of the "whole" 178, so we set up the proportion accordingly:

$$\frac{38}{100} = \frac{x}{178}$$
$$(38)(178) = 100x$$
$$6764 = 100x$$
$$x = \frac{6764}{100} = 67.64\%$$

Example:

Solve the proportion $\frac{2x+1}{6} = \frac{7}{9}$.

$$\frac{2x+1}{6} = \frac{7}{9}$$
$$(2x+1)(9) = (6)(7)$$
$$18x + 9 = 42$$
$$18x = 33$$
$$x = \frac{33}{18} = \frac{11}{6} \approx 1.83\ldots$$

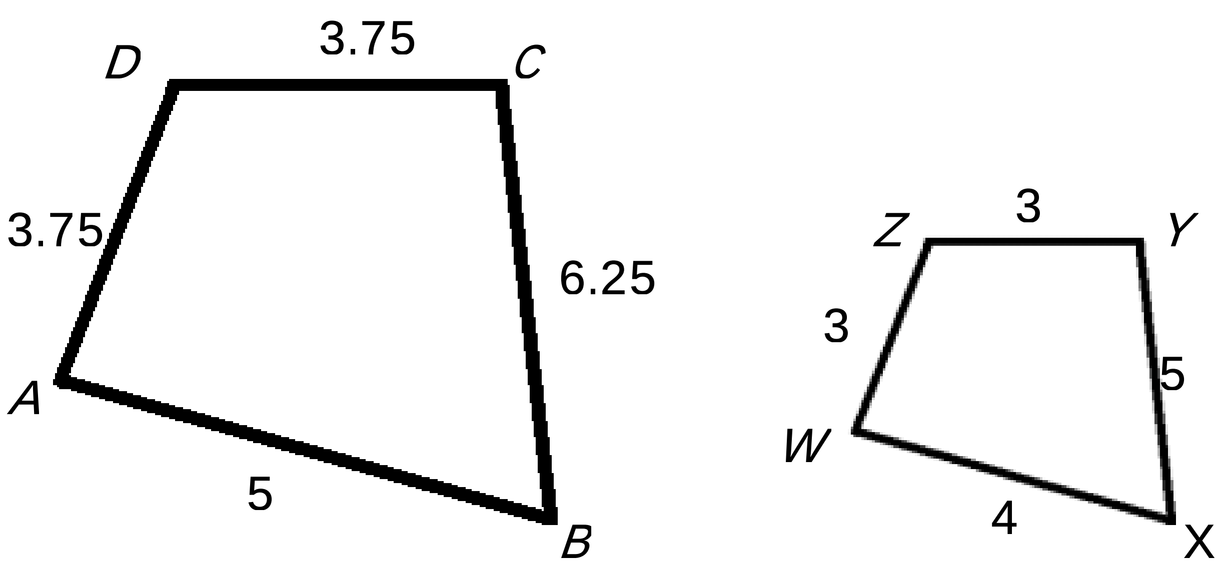
So where does the geometry come in?

Two polygons are similar ***if and only if*** their corresponding angles are congruent and the measures of their corresponding sides are proportional. **They have the same *SHAPE* but different *SIZES*.**

The ratio of the lengths of the two corresponding sides of two similar polygons is called the scale factor.

Example:

Quadrilateral *ABCD* is similar to quadrilateral *WXYZ* (***ABCD ~ WXYZ***). Find the scale factor of quadrilateral *ABCD* **to** quadrilateral *WXYZ*.

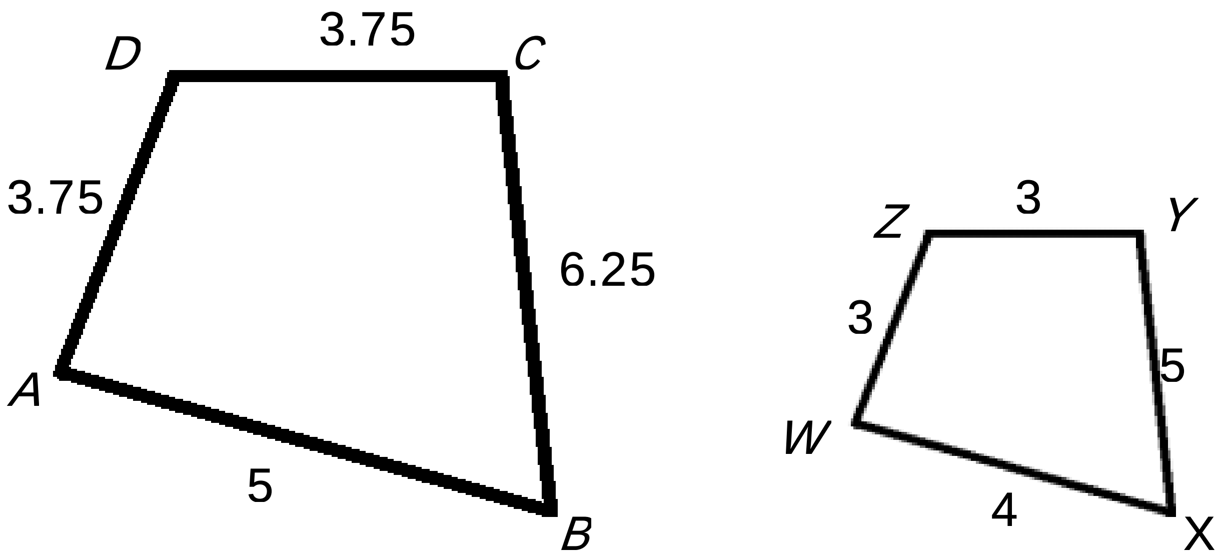


Observe the following relationships:

$$\angle A \cong \angle W,\ \angle B \cong \angle X,\ \angle C \cong Y,\ \angle D \cong \angle Z$$

AND

$$\overline{AB} \sim \overline{WX},\ \overline{BC} \sim \overline{XY},\ \overline{CD} \sim \overline{YZ},\ \overline{DA} \sim \overline{ZW}$$



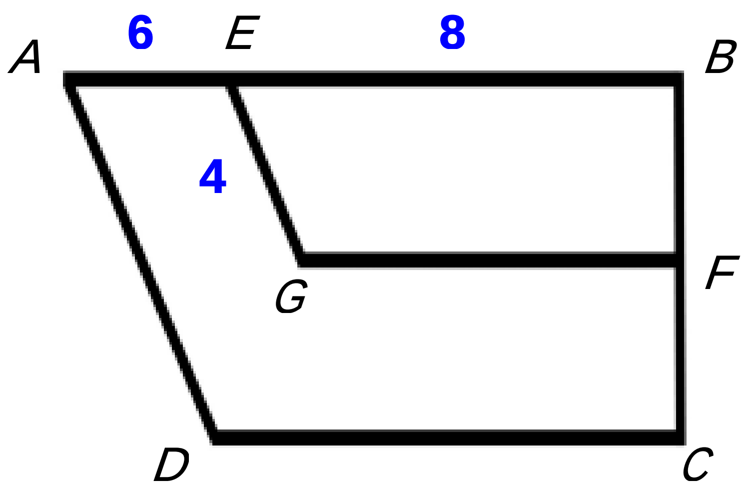
To find the scale factor, we must find the ratio of the measure of a side of *ABCD* **to** the measure of the corresponding side of *WXYZ*.
(In this example, because we have numeric measures on all sides, any pair of corresponding sides will work.)

$\frac{AB}{WX} = \frac{5}{4}$ or $\frac{1.25}{1} = 1.25$. This means *ABCD* is 1.25 times larger than *WXYZ*
or
$\frac{AD}{WZ} = \frac{3.75}{3} = \frac{1.25}{1} = 1.25$ or $\frac{BC}{XY} = \frac{6.25}{5} = \frac{1.25}{1} = 1.25$

Notice that the scale factor of *ABCD* to *WXYZ* is $1.25 = \frac{5}{4}$, so the scale factor of *WXYZ* to *ABCD* is $\frac{4}{5} = 0.8$. This means that *WXYZ* is eight-tenths the size of the larger *ABCD*.

Example:

Trapezoid *ABCD* ~ trapezoid *EBFG*. Find *AD*.



We must create a proportion relating corresponding sides in the small and large trapezoid. We must also involve the side lengths given and the side length we want. It is helpful to start with the length you are trying to find in the numerator.

$$\frac{AD}{EG} = \frac{AB}{EB}$$

$$\frac{AD}{4} = \frac{8+6}{8} = \frac{14}{8} = \frac{7}{4}$$

$$\frac{AD}{4} = \frac{7}{4}$$

$$4AD = (7)(4)$$

$$4AD = 28$$

$$AD = 28 / 4 = 7$$

Say What??!!

Map Link

On the map of France shown, 2.5 centimeters represents 200 miles.

Find the actual distance from Cherbourg to Lyon if the map distance is 4.8 centimeters.

Use a proportion to find the actual distance.

map distance → $\frac{2.5 \text{ cm}}{200 \text{ mi}} = \frac{4.8 \text{ cm}}{x \text{ mi}}$ ← *map distance*

actual distance → ← *actual distance*

$2.5(x) = 200(4.8)$ ***Cross Products***

$x = 384$

The actual distance from Cherbourg to Lyon is 384 miles.