Lesson 17

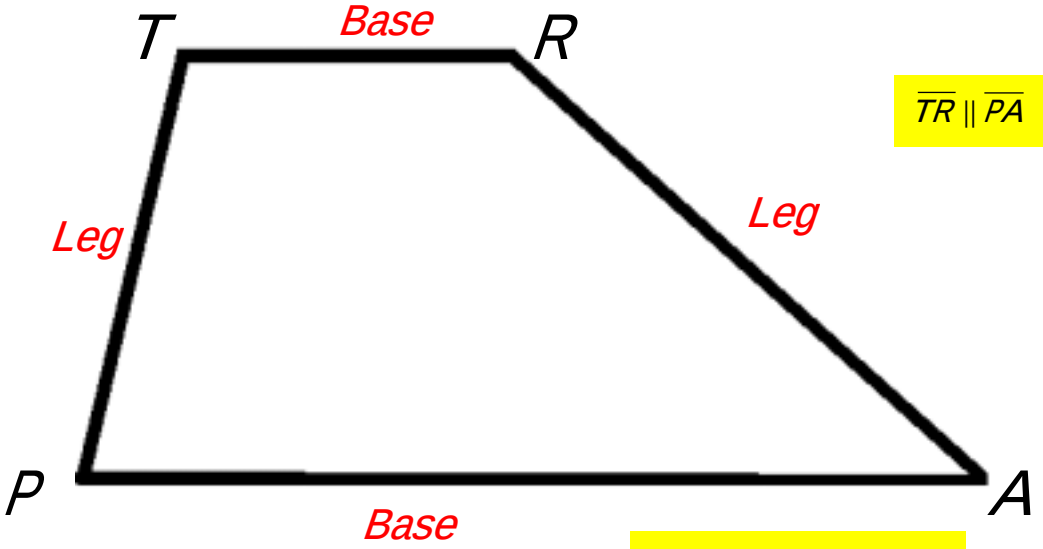
Glencoe Geometry Chapter 6.5

Trapezoids

Today we look at another special quadrilateral—**Trapezoids**!

Definition:

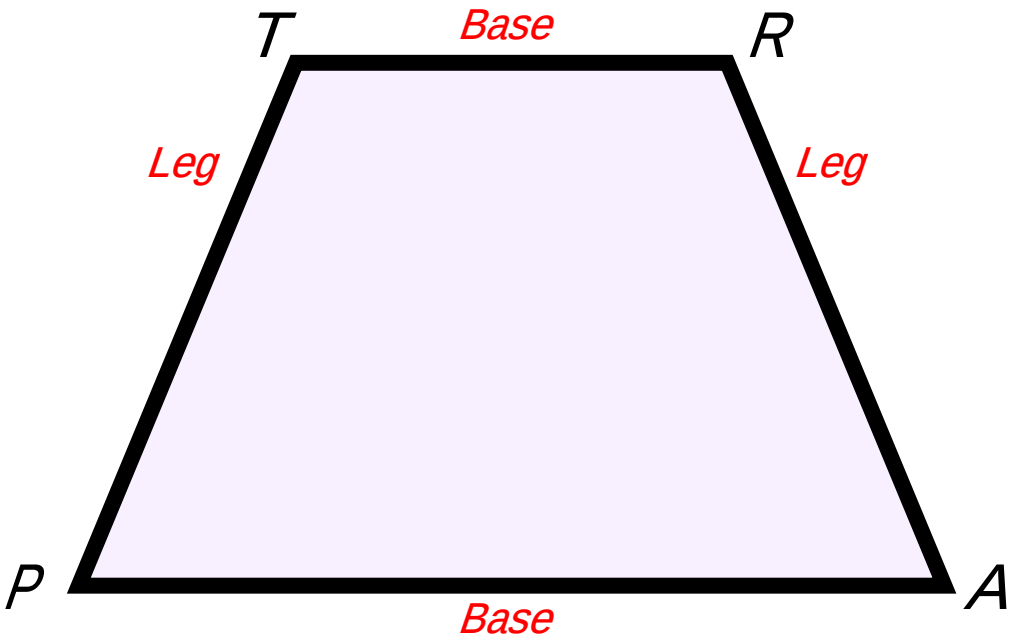
A **Trapezoid** is a quadrilateral with exactly one pair of parallel sides.



$\overline{TR} \parallel \overline{PA}$

The two parallel sides are called bases. The nonparallel sides are called legs. Angles T,R,A,& P are called Base Angles.

If the two legs of the trapezoid are congruent, then the trapezoid is an isosceles trapezoid.



Isosceles Trapezoids have special properties:

1. Both pairs of base angles are congruent.

$\angle T \cong \angle R$ AND $\angle A \cong \angle P$

2. The diagonals are congruent.

$\overline{TA} \cong \overline{RP}$

Example:

The measures of a pair of base angles of an isosceles trapezoid are $7x - 12$ and $5x + 6$. Find the value of x.

A pairs of base angles in an isosceles trapezoid are congruent, so they have equal measures:
$7x - 12 = 5x + 6$
$7x - 5x = 6 + 12$
$2x = 18$
$x = 9$

Example:

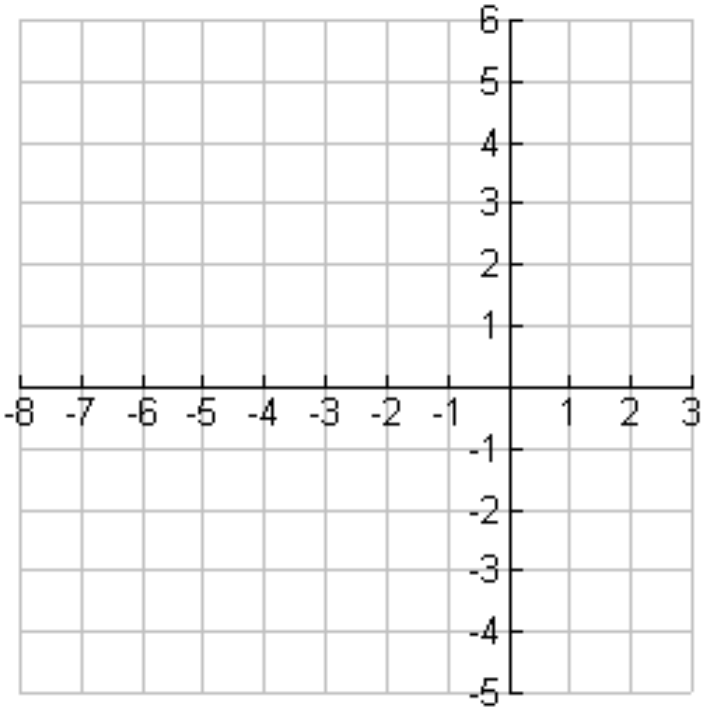
The measures of two base angles on the same side of a trapezoid are $10x + 7$ and $6x - 3$, respectively. What is the measure of the larger base angle? (HINT: The base angles in a trapezoid are formed by two parallel lines cut by a transversal.)

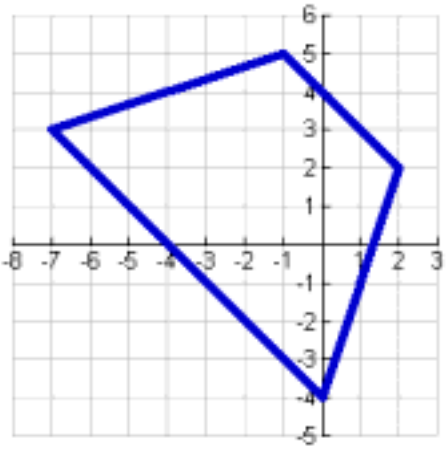
These angles form "consecutive angles." Remember that consecutive angles are supplementary, or add to 180 degrees:
$(10x + 7) + (6x - 3) = 180$
$16x + 4 = 180$
$16x = 176$
$x = 11$

so the two measures are
$10(11) + 7 = 110 + 7 = 117$ degrees
and
$6(11) - 3 = 66 - 3 = 63$ degrees
so
the larger angle is 117 degrees.

Example:

An isosceles trapezoid has vertices at (-1, 5), (2, 2), (0, -4), and (-7, 3). Find the measure of each diagonal.





The diagonals are formed by the set of ordered pairs (-7,3) and (2,2) and by (-1,5) and (0,-4). Since the lengths of diagonals are equal in an isosceles trapezoid, we only need to find the length of one of them:

Using the distance formula:

$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

$$d = \sqrt{(-7-2)^2 + (3-2)^2} = \sqrt{81+1}$$

$$= \sqrt{82} \approx 9.055\ldots$$

There is another special segment in a trapezoid that can be drawn—the median.

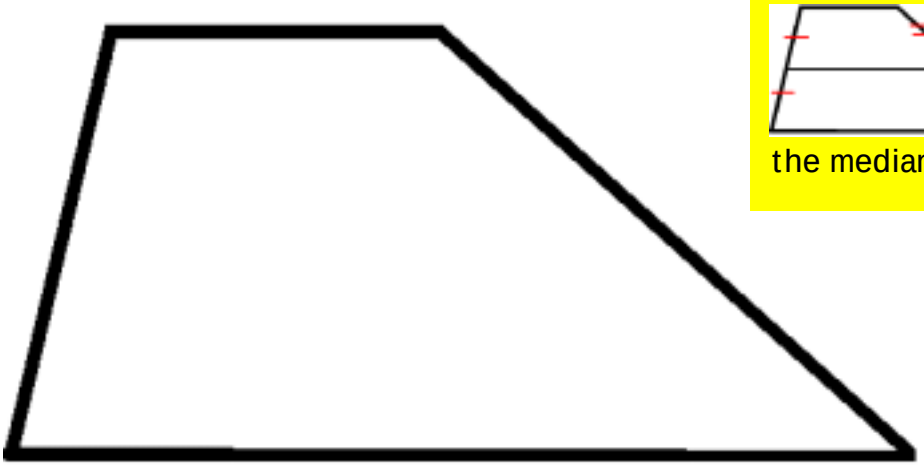
Definition:

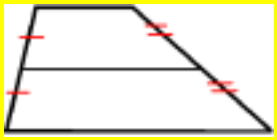
> A **Median** of a trapezoid is the segment that joins the midpoint of its legs.

The median has a special relationship to the bases—*it is always parallel to them* (even for non-isosceles trapezoids).

AND. . .

The measure of the median is always *one-half the sum of the measures of the bases!!!*

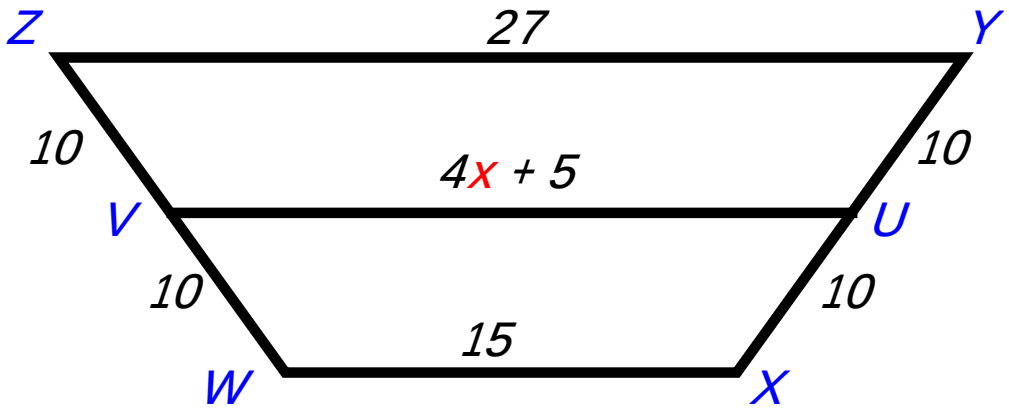




the median is show above

Example:

Given that $\overline{VU}$ is the median of trapezoid $ZYXW$, find the value of x.



The length of the median is half the sum of the bases:

$$4x + 5 = \frac{27 + 15}{2}$$

$$4x + 5 = \frac{42}{2}$$

$$4x = 21 - 5$$

$$4x = 16$$

$$x = 4$$

and the length of the median is $4(4) + 5 = 16 + 5 = 21$

Say What??!!

Food

Some cafeteria trays are shaped like isosceles trapezoids so they will save space and fit around tables.

http://www.centralrestaurant.com/

If the measures of one pair of base angles of a tray are 120 degrees, what are the measures of the other pair of base angles?

Remember that consecutive angles from two different bases are supplementary. So, if one set of base angles measure 120 degrees each, the other two angles must be the supplements of these.

That means that the other two angles each measure 180 – 120 = 60 degrees each.