Lesson 16

Glencoe Geometry Chapter 6.3 & 6.4

Rectangles, Squares, & Rhombi

Remember:

Polygons are closed geometric shapes.

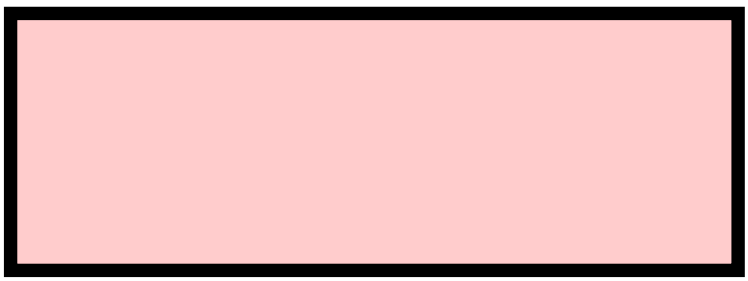
Quadrilaterals are 4-sided polygons.

Parallelograms are quadrilaterals with *both* pairs of opposite sides parallel to each other.

Today we take a closer look at three special types of Parallelograms: **Rectangles, Squares, and Rhombi**.

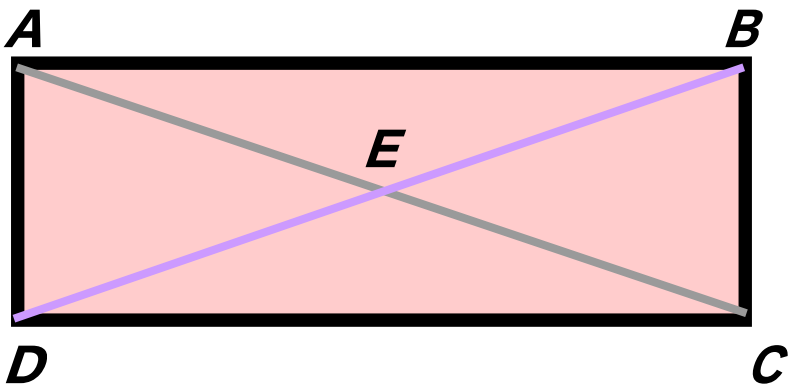
Definition:

A **rectangle** is a quadrilateral with four right angles.



A Rectangle's diagonals have a special property:

THEY ARE CONGRUENT!!



Because rectangle $ABCD$ is a parallelogram, $\overline{AE} \cong \overline{EC}$ & $\overline{DE} \cong \overline{EB}$

BUT . . .

Since it is a rectangle, we also know that $\overline{AC} \cong \overline{BD}$

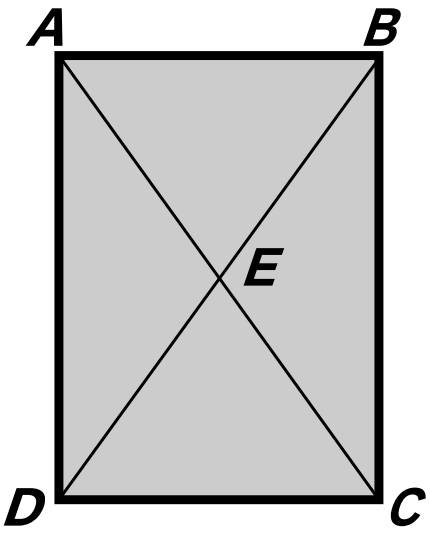
This forms two sets of congruent isosceles triangles: $\triangle AED \cong \triangle BEC$ making all the half-diagonal segments congruent to each other!!

Example:

In rectangle $ABCD$, $AC = 24$ and $DE = 2x - 8$. Find x.

A. 16 B. 13

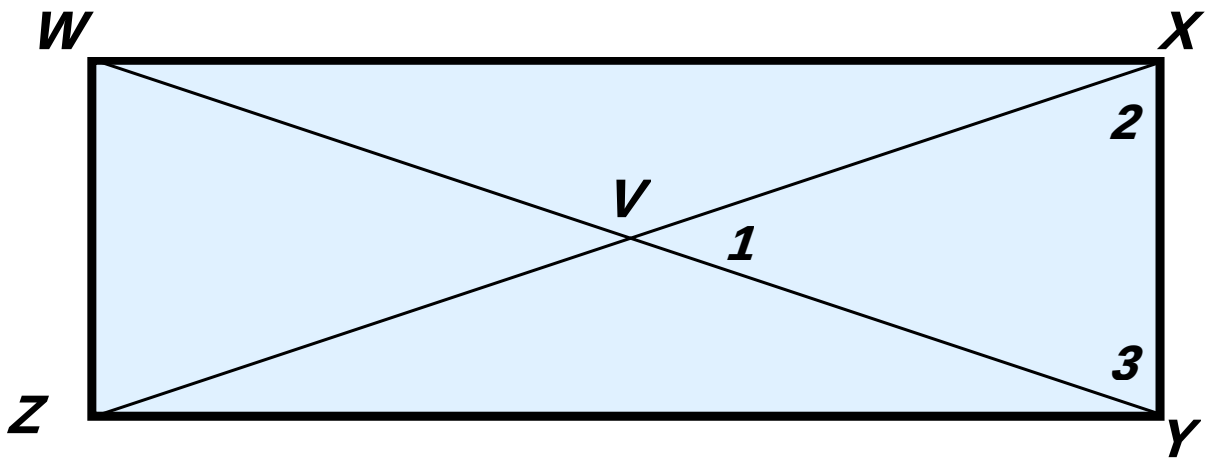
C. 7 D. 10



$AC = 24$ means $CE = AE = 24/2 = 12$.
So $DE = CE$
$2x-8 = 12$
$2x = 20$
$x = 10$

Example:

In rectangle *WXYZ*, $m\angle 1 = 40^\circ$. Find $m\angle 2$.



A. 20 B. 70 C. 140 D. 40

Triangle XVY is an isosceles angle, so the base angles 2 and 3 are congruent. Their sum is the supplement of angle 1.

So $m\angle 1 + m\angle 2 + m\angle 3 = 180$, since $m\angle 2 = m\angle 3$, we can substitute.

$40 + m\angle 2 + m\angle 2 = 180$

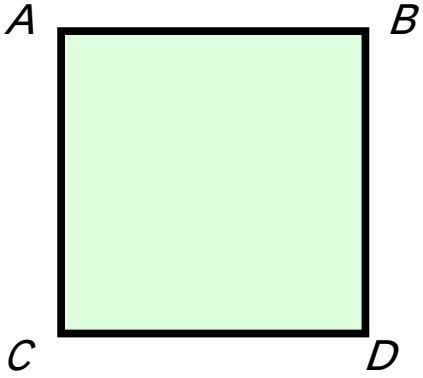
$40 + 2(m\angle 2) = 180$

$2(m\angle 2) = 140$

$m\angle 2 = 70$

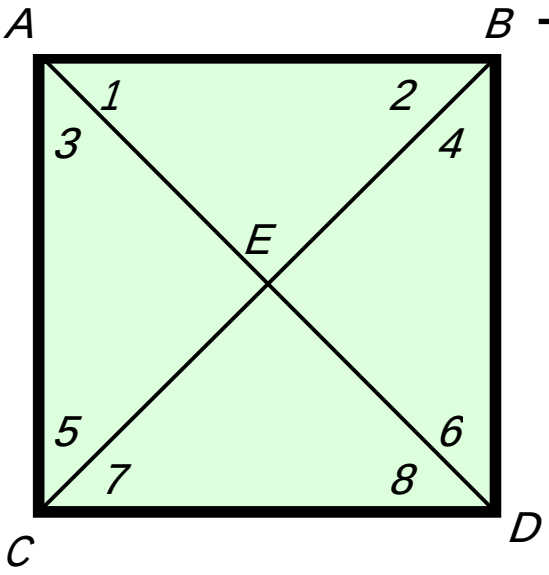
Definition:

A **square** is a quadrilateral with four right angles, and four congruent sides.



$\overline{AB} \cong \overline{BD} \cong \overline{DC} \cong \overline{CA}$

Squares have all the same properties of rectangles and some additional ones to boot!



The diagonals

1. are perpendicular

The slopes of $\overline{AD}$ and $\overline{BC}$ are negative reciprocals of each other.

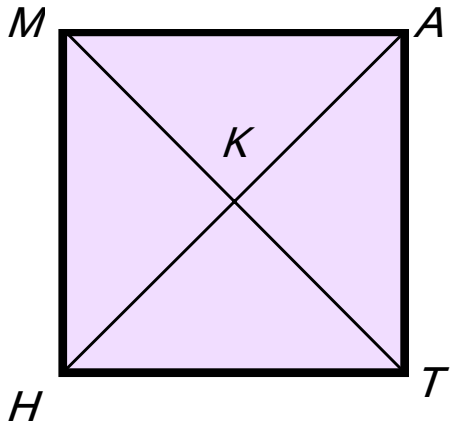
2. bisect a pair of opposite angles

$m\angle 1 = m\angle 3 = 45^\circ$. If fact, all the numbered angles are congruent and equal 45°.

The diagonals form 4 congruent isosceles triangles! What a deal!!

Example:

In square $MATH$, If $KA = x^2 + 1$ & $KT = 3x + 11$, find the value of x.



The diagonals are congruent and bisect each other, so all 4 interior segments are congruent, so

$KA = KT$

$x^2 + 1 = 3x + 11$

$x^2 - 3x - 10 = 0$

$(x - 5)(x + 2) = 0$

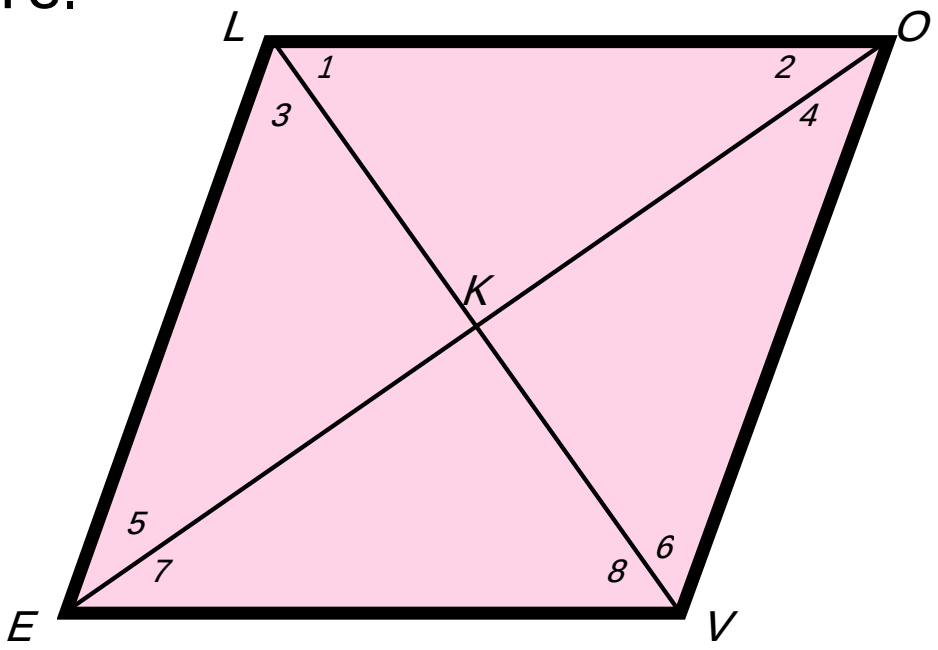
$x = 5, -2$

We are not interested in negative lengths, so $x = 5$.

Definition:

A **rhombus** is a quadrilateral with four four congruent sides.

Unlike the rectangle and the square, the angles of rhombus need not be right angles, but other than that, it's properties are similar to the square.

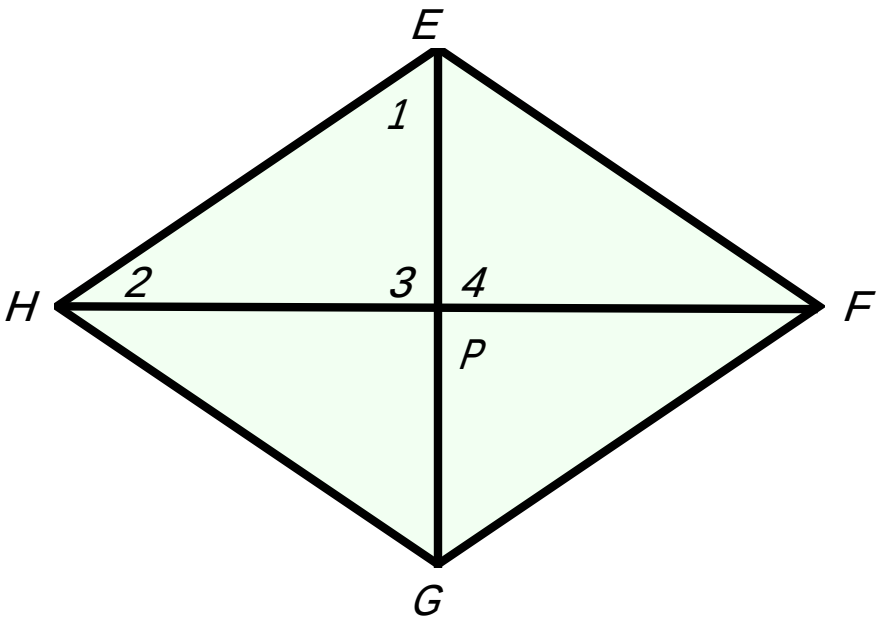


1. $\overline{LO} \cong \overline{OV} \cong \overline{VE} \cong \overline{EL}$
2. $\overline{LV} \perp \overline{OE}$
3. $\angle 1 \cong \angle 3 \cong 6 \cong 8$ and $\angle 5 \cong \angle 7 \cong \angle 2 \cong \angle 4$

(but these two sets are not necessarily congruent to each other).

Example:

In rhombus $EFGH$, $m\angle 1 = 57°$, $HE = 13$, $HP = 12$.
What is $m\angle 2$? What is PE?



Since the diagonals are perpendicular, we know $3 \quad \angle 4 = 90^\circ$.
Considering $\triangle EPH$, the sum of its interior angles equal 180.
So . . .

$$\angle 1 + \quad \angle 2 + m\angle 3 = 180$$
$$+ m\angle 2 + 90 = 180$$
$$m\angle 2 = 180 - 90 - 57$$
$$m\angle 2 = 33$$

Since the diagonals are perpendicular, we know $\triangle HEP$ is a right triangle. Using the Pythagorean Theorem, with $\overline{HE}$ being the hypotenuse, we get

$$(HP)^2 + (PE)^2 = (HE)^2$$
$$(12)^2 + (PE)^2 = (13)^2$$
$$144 + (PE)^2 = 169$$
$$PE = \sqrt{169 - 144}$$
$$PE = 5$$

Say What??!!

1. A square is a _____.

A. rhombus B. rectangle

C. parallelogram D. all of these

2. What is the best description of a quadrilateral with four right angles?

A. square B. rhombus

C. parallelogram D. rectangle

3. Which statement is false?

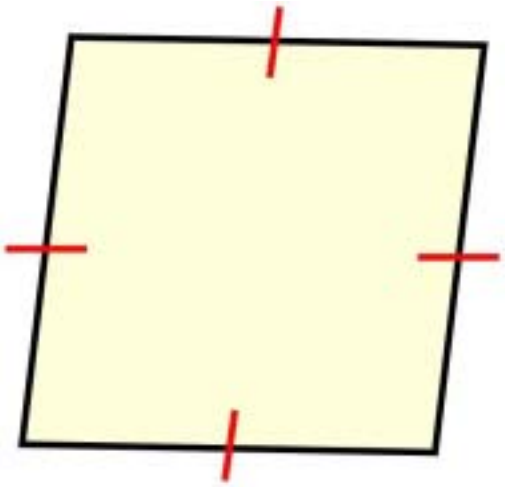
A. Every rhombus is a quadrilateral.

B. Every rhombus is a parallelogram.

C. Every rhombus is a square.

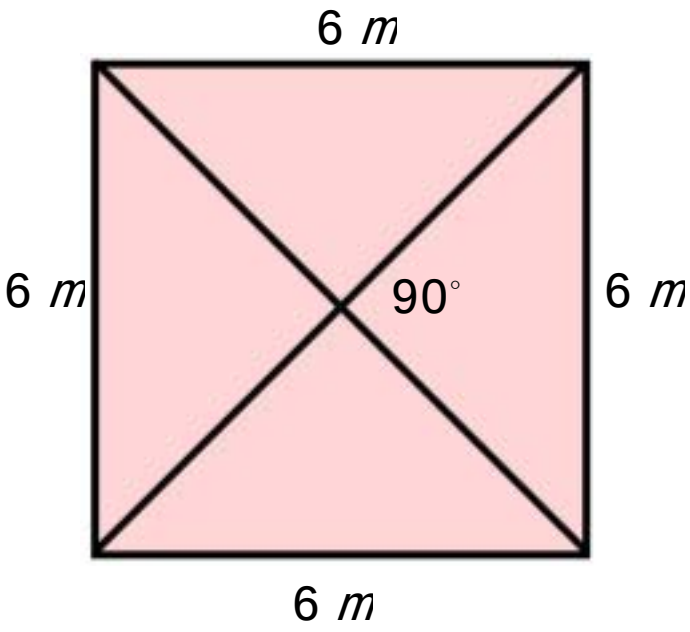
D. Every square is a rhombus.

4. The most specific description of the quadrilateral below is _____.



A. square
B. rhombus
C. rectangle
D. parallelogram

5. Choose the most precise description for the figure shown.



A. parallelogram
B. square
C. rhombus
D. quadrilateral

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