Lesson 15

TAKS Review

Objective 6

The student will demonstrate an understanding of geometric relationships and spatial reasoning.

For this objective you should be able to

- use a variety of representations to describe geometric relationships and solve problems;
- identify, analyze, and describe patterns that emerge from two and three-dimensional geometric figures; and
- apply the concept of congruence to justify properties of figures and solve problems.

How Do You Solve Geometric Problems?

You can use diagrams, geometric concepts, properties, definitions, and theorems to solve geometric problems. When solving a geometric problem, consider the following.

- Determine which geometric properties, definitions, or theorems apply to the problem.
- Draw a diagram to represent the problem if you are not given one.
- Identify or algebraically represent any quantities in the problem.
- Use a formula if necessary. Some formulas you need are in the Mathematics Chart.
- Use an equation to represent the relationship between the quantities in the problem.

Grades 9, 10, and 11 Exit Level
Mathematics Chart

LENGTH

Metric	Customary
1 kilometer = 1000 meters	1 mile = 1760 yards
1 meter = 100 centimeters	1 mile = 5280 feet
1 centimeter = 10 millimeters	1 yard = 3 feet
	1 foot = 12 inches

CAPACITY AND VOLUME

Metric	Customary
1 liter = 1000 milliliters	1 gallon = 4 quarts
	1 gallon = 128 ounces
	1 quart = 2 pints
	1 pint = 2 cups
	1 cup = 8 ounces

MASS AND WEIGHT

Metric	Customary
1 kilogram = 1000 grams	1 ton = 2000 pounds
1 gram = 1000 milligrams	1 pound = 16 ounces

TIME

1 year = 365 days
1 year = 12 months
1 year = 52 weeks
1 week = 7 days
1 day = 24 hours
1 hour = 60 minutes
1 minute = 60 seconds

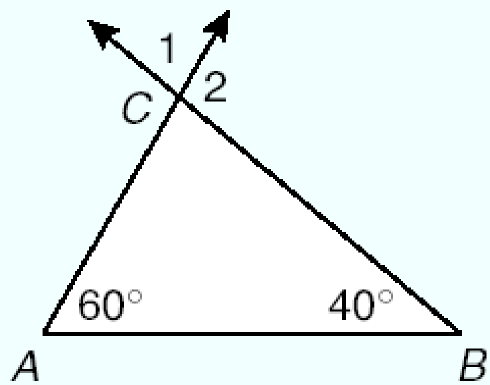
Metric and customary rulers can be found on the tear-out Mathematics Chart in the back of this book.

Grades 9, 10, and 11 Exit Level Mathematics Chart

Perimeter	rectangle	$P = 2l + 2w$ or $P = 2(l + w)$
Circumference	circle	$C = 2\pi r$ or $C = \pi d$
Area	rectangle	$A = lw$ or $A = bh$
	triangle	$A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$
	trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$ or $A = \frac{(b_1 + b_2)h}{2}$
	circle	$A = \pi r^2$
Surface Area	cube	$S = 6s^2$
	cylinder (lateral)	$S = 2\pi rh$
	cylinder (total)	$S = 2\pi rh + 2\pi r^2$ or $S = 2\pi r(h + r)$
	cone (lateral)	$S = \pi rl$
	cone (total)	$S = \pi rl + \pi r^2$ or $S = \pi r(l + r)$
	sphere	$S = 4\pi r^2$
Volume	prism or cylinder	$V = Bh$*
	pyramid or cone	$V = \frac{1}{3}Bh$*
	sphere	$V = \frac{4}{3}\pi r^3$
**B represents the area of the Base of a solid figure.*		
Pi	π	$\pi \approx 3.14$ or $\pi \approx \frac{22}{7}$
Pythagorean Theorem		$a^2 + b^2 = c^2$
Distance Formula		$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Slope of a Line		$m = \frac{y_2 - y_1}{x_2 - x_1}$
Midpoint Formula		$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
Quadratic Formula		$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Slope-Intercept Form of an Equation		$y = mx + b$
Point-Slope Form of an Equation		$y - y_1 = m(x - x_1)$
Standard Form of an Equation		$Ax + By = C$
Simple Interest Formula		$I = prt$

Example:

In $\triangle ABC$ below, find the measures of $\angle 1$ and $\angle 2$.



The sum of the measures of the angles of a triangle is 180°.

$$60^\circ + 40^\circ + m\angle ACB = 180^\circ$$

Subtract the sum $60^\circ + 40^\circ$ from 180° to find $m\angle ACB$.

$$m\angle ACB = 180^\circ - (60^\circ + 40^\circ)$$

$$m\angle ACB = 180^\circ - 100^\circ$$

$$m\angle ACB = 80^\circ$$

Since $\angle 1$ and $\angle ACB$ are vertical angles, their measures are equal.

Since $m\angle ACB = 80^\circ$, $m\angle 1 = 80^\circ$.

$\angle 1$ and $\angle 2$ are supplementary; the sum of their measures is 180°.

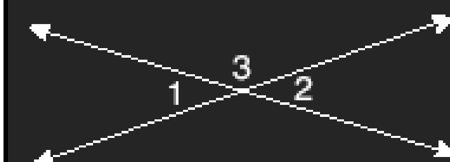
$$m\angle 1 + m\angle 2 = 180^\circ$$

$$80^\circ + m\angle 2 = 180^\circ$$

$$m\angle 2 = 180^\circ - 80^\circ$$

$$m\angle 2 = 100^\circ$$

Vertical angles are formed by the intersection of two lines. Vertical angles are always congruent. The adjacent angles formed by the intersection of two lines are always supplementary angles. The sum of the measures of supplementary angles is 180°.



$$m\angle 1 = m\angle 2$$

$$m\angle 1 + m\angle 3 = 180^\circ$$

$$m\angle 2 + m\angle 3 = 180^\circ$$

Example:

If $\angle A$ is the supplement of $\angle B$ and $\angle B$ is the supplement of $\angle C$, what is the relationship between the measures of $\angle A$ and $\angle C$?

- If $\angle A$ is the supplement of $\angle B$, then the sum of their measures is 180°.

$$m\angle A + m\angle B = 180°$$

- If $\angle B$ is the supplement of $\angle C$, then the sum of their measures is 180°.

$$m\angle B + m\angle C = 180°$$

- Since both sums equal 180°, the two quantities are equal to each other.

$$m\angle A + m\angle B = m\angle B + m\angle C$$

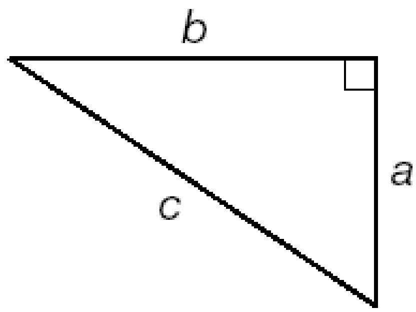
- Subtract the measure of $\angle B$ from both sides.

$$\begin{array}{rcl} m\angle A + m\angle B & = & m\angle B + m\angle C \\ - m\angle B & = & - m\angle B \\ \hline m\angle A & = & m\angle C \end{array}$$

If $\angle A$ is the supplement of $\angle B$ and $\angle B$ is the supplement of $\angle C$, then $m\angle A = m\angle C$.

What Patterns in Right Triangles Can You Use to Solve Problems?

The **Pythagorean Theorem**, $a^2 + b^2 = c^2$, can be used to solve problems involving the lengths of sides of right triangles.

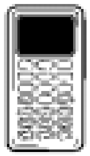


$$a^2 + b^2 = c^2$$

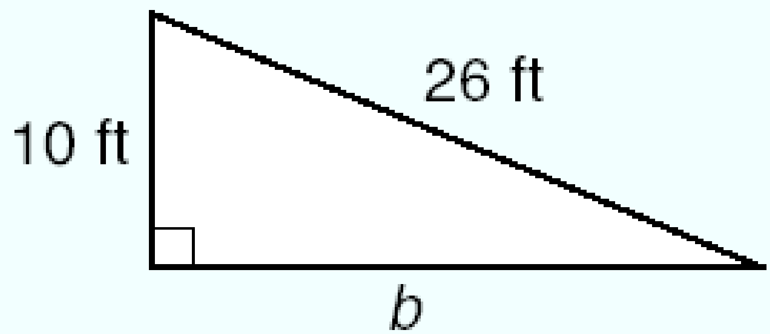
A set of three whole numbers that satisfy the Pythagorean Theorem is called a **Pythagorean triple**. Pythagorean triples can be used to find a missing side in a right triangle.

Multiply each of the numbers in a Pythagorean triple by any whole number to find another Pythagorean triple.

Example:



Find the length of side b in the right triangle below.



One way to find the length of side b is to use a Pythagorean triple.

- Multiply the Pythagorean triple {5, 12, 13} by 2 to obtain the set {10, 24, 26}.
- Since {5, 12, 13} is a Pythagorean triple, the set {10, 24, 26} is also a Pythagorean triple.
- The side lengths of the right triangle in the diagram are equal to two of the values in the set {10, 24, 26}.
- The third side length of the right triangle is equal to the remaining value in the set {10, 24, 26}.

Side b has a length of 24 feet.

Or

$$10^2 + b^2 = 26^2$$
$$100 + b^2 = 676$$
$$b^2 = 676 - 100 = 576$$
$$b = \sqrt{576} = 24$$

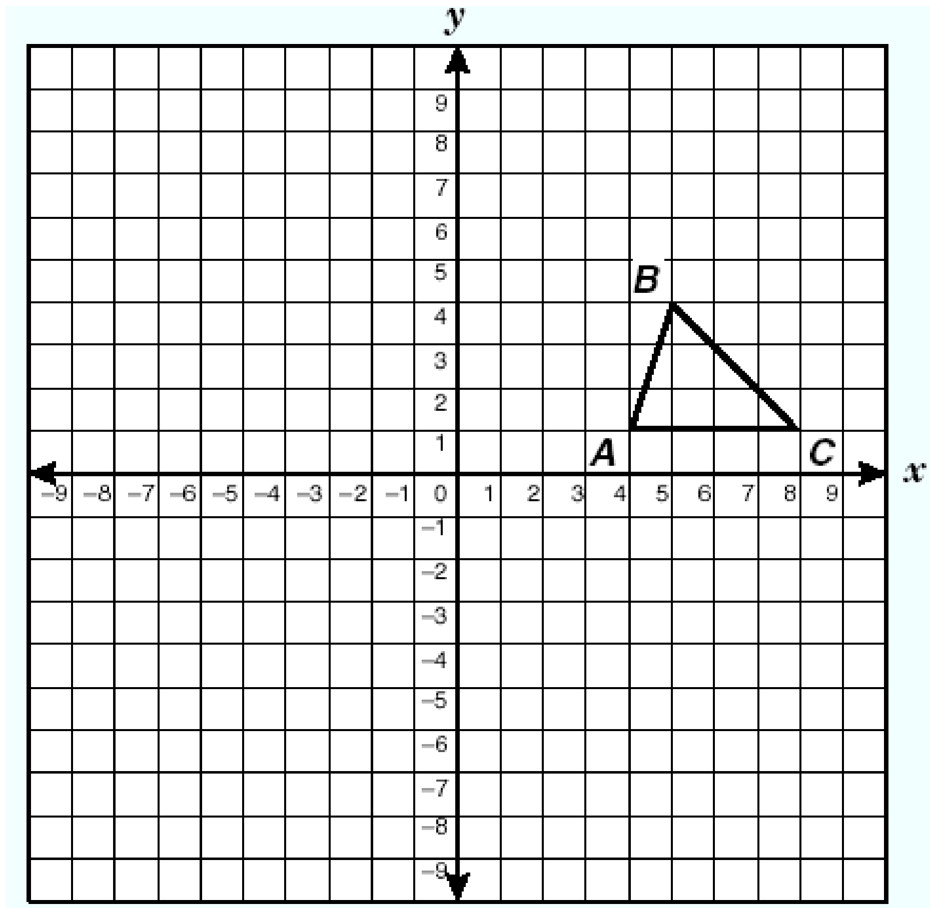
How Do You Use Transformations to Solve Problems?

Translations, reflections, and rotations are transformations of geometric figures that do not change the lengths of the segments of the figures. The original figure and its transformed image are congruent.

When figures are transformed, prime notation is often used to name the image. The point P' is the image of point P.

Example:

The graph of $\triangle ABC$ is shown below. Find the coordinates of $\triangle A'B'C'$, the image of $\triangle ABC$ reflected across the y-axis and translated 2 units up.



- Reflect A (4, 1) across the y-axis. The x-coordinate of A' is the negative of the x-coordinate of A. The x-coordinate of A' is -4.

  If the point is then translated 2 units up, the y-coordinate of A' will be 2 more than the y-coordinate of A. $1 + 2 = 3$.

  A' has coordinates $(-4, 3)$.

- Reflect B (5, 4) across the y-axis. The x-coordinate of B' is the negative of the x-coordinate of B. The x-coordinate of B' is -5.

  If the point is then translated 2 units up, the y-coordinate of B' will be 2 more than the y-coordinate of B. $4 + 2 = 6$.

  B' has coordinates $(-5, 6)$.

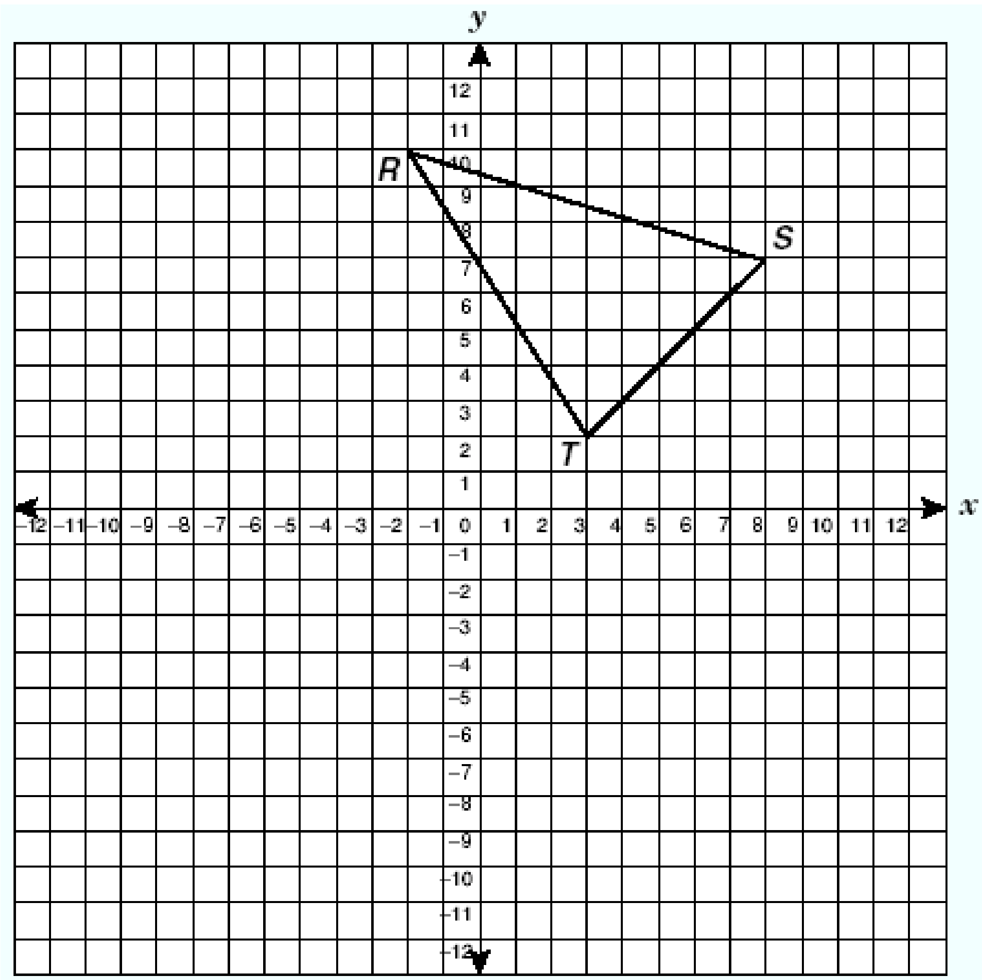
- Reflect C (8, 1) across the y-axis. The x-coordinate of C' is the negative of the x-coordinate of C. The x-coordinate of C' is -8.

  If the point is then translated 2 units up, the y-coordinate of C' will be 2 more than the y-coordinate of C. $1 + 2 = 3$.

  C' has coordinates $(-8, 3)$.

Example:

The graph of $\triangle RST$ is shown below. Find the coordinates of S' if $\triangle RST$ is rotated 90° clockwise around point T.

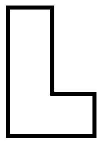


- If $\triangle RST$ is rotated 90° clockwise about point T, $\overline{ST}$ will be perpendicular to $\overline{S'T}$.
- The slopes of perpendicular lines are negative reciprocals. If the slope of $\overline{ST}$ is $\frac{5}{5}$, then the slope of $\overline{S'T}$ should be $-\frac{5}{5}$.
- Use the slope of $\overline{S'T}$, $-\frac{5}{5}$, to find point S'. Starting at point T, count 5 units down and 5 units to the right.

The point S' will have coordinates $(8, -3)$.

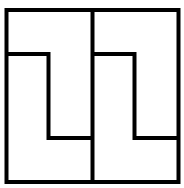
Example:

Look at the shape below.

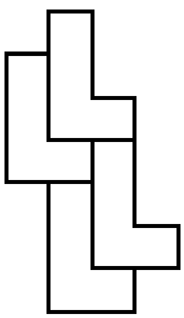


Which of these patterns can be made using only translations, rotations, or a combination of translations and rotations?

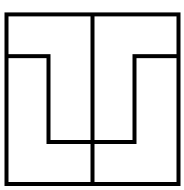
I



II



III



A I and II only

B I and III only

C II and III only

D I, II, and III

A Correct. The pattern in Figure I is formed by a shape, a rotation of the shape by 180°, and a translation of the shape and its rotated image to the right.

The pattern in Figure II is formed by a shape and three translated images of the shape.

The pattern in Figure III is formed by a shape, and a rotation of the shape by 180°, and a reflection of the shape and its rotation across a vertical line.

Only Figures I and II can be made using only translations and rotations.

Objective 7

The student will demonstrate an understanding of two- and three-dimensional representations of geometric relationships and shapes.

For this objective you should be able to

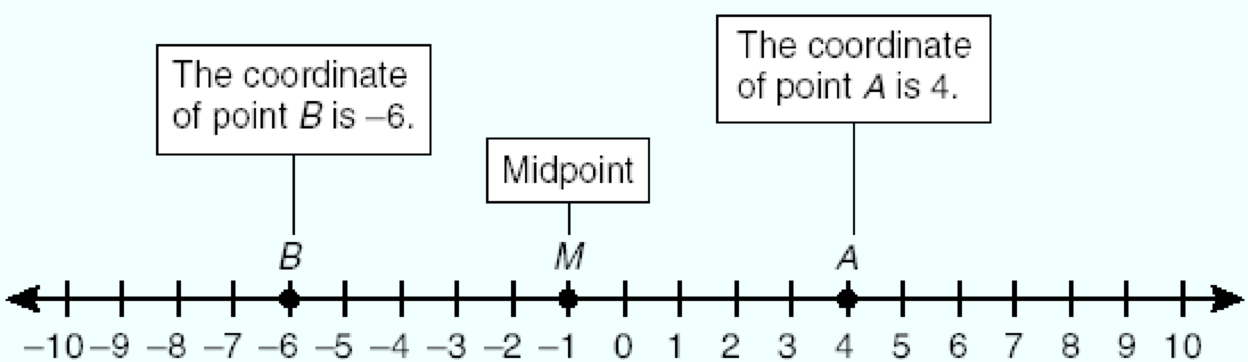
- analyze the relationship between three-dimensional objects and related two-dimensional representations and use these to solve problems;
- understand that coordinate systems provide convenient and efficient ways of representing geometric figures and use them accordingly; and
- analyze properties and describe relationships in geometric figures.

How Do You Locate and Name Points on a Line or Plane?

A number line is used to locate and name points on a line.

Example:

Find the coordinate of the midpoint of $\overline{AB}$.

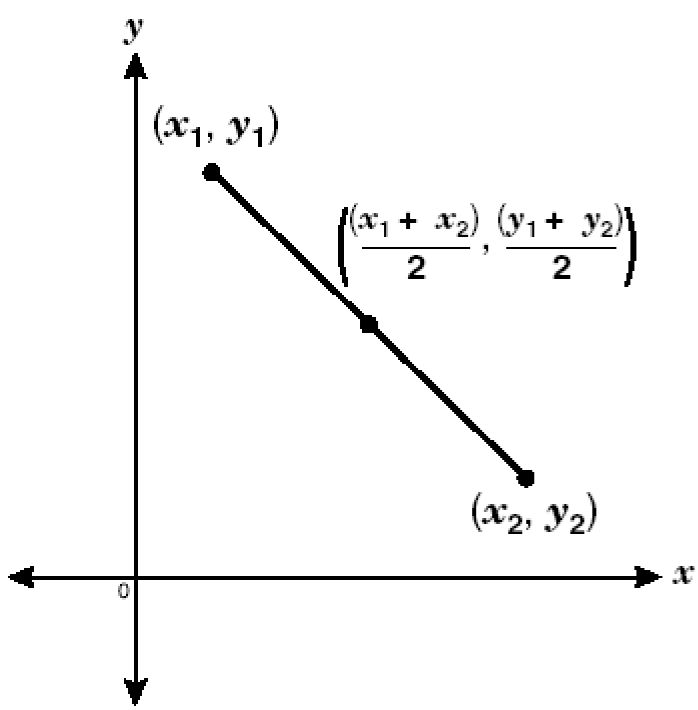


The coordinate of the midpoint of a segment is the average of the coordinates of its endpoints. The midpoint of $\overline{AB}$ has the coordinate $\frac{(4 + -6)}{2} = \frac{-2}{2} = -1$.

Midpoint Formula

For any two points (x_1, y_1) and (x_2, y_2), the coordinates of the midpoint of the line segment they determine are given by the formula below.

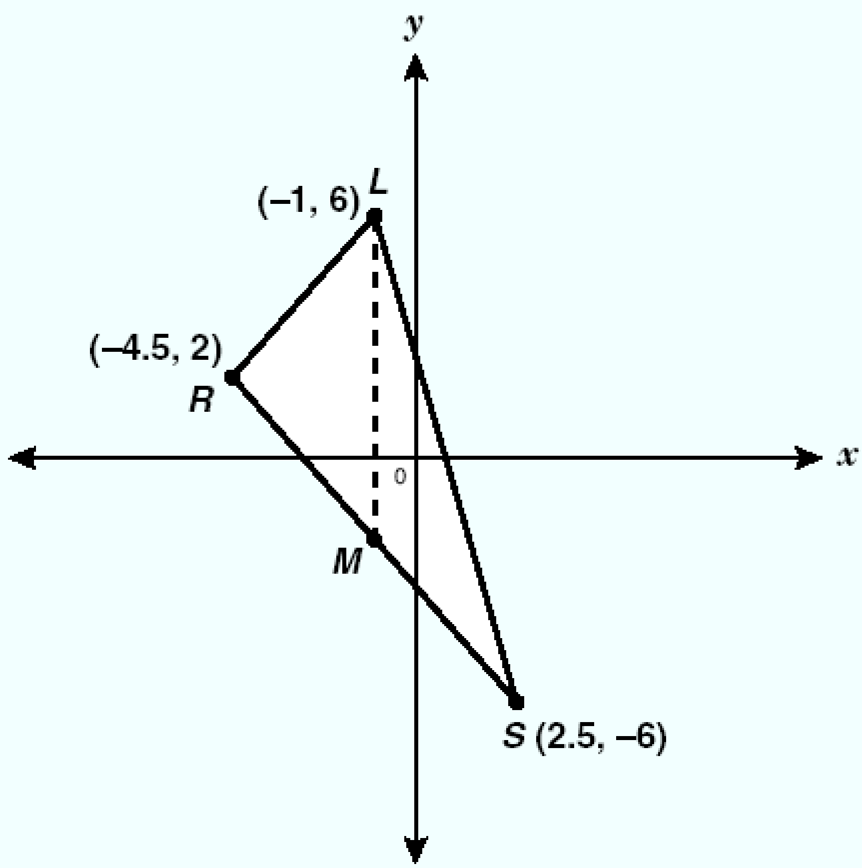
$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$



A **median** of a triangle is a line segment drawn from a vertex to the midpoint of the opposite side.

Example:

Triangle *LRS* has the following vertices: *L* (−1, 6), *R* (−4.5, 2), and *S* (2.5, −6). Find the coordinates of point *M*, the endpoint of median *LM* drawn to $\overline{RS}$.



Median *LM* is drawn from vertex *L* to point *M*, the midpoint of $\overline{RS}$.

Find the coordinates of point *M*.

- The *x*-coordinates of the endpoints of $\overline{RS}$ are −4.5 and 2.5.
- The *y*-coordinates of the endpoints of $\overline{RS}$ are 2 and −6.
- Substitute these values into the midpoint formula.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$M = \left(\frac{-4.5 + 2.5}{2}, \frac{2 + (-6)}{2}\right)$$

$$M = \left(\frac{-2}{2}, \frac{-4}{2}\right)$$

$$M = (-1, -2)$$

The coordinates of point *M* are (−1, −2).