Lesson 13

Glencoe Geometry Chapter 5.4 & 5.5

The Triangle Inequality

By the end of this lesson, you should be able to

1. Recognize and apply relationships between sides and angles in a triangle.
2. Apply the Triangle Inequality Theorem

We learned previously that if sides in a triangle were congruent, then the angles opposite those sides are also congruent (and vice-versa).

There are also important relationships that deal with **unequal** quantities. Today, we will examine two of these relationships.

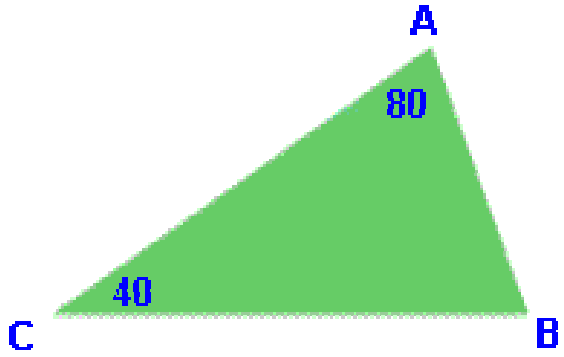
The first relationship involves the lengths of the sides of a triangle in relation to the triangle's angles.

Theorem:

In a triangle, the longest side is across from the largest angle. The shortest side is across from the shortest angle. The "middle" side is across from the "middle" angle.

Example:

Suppose we want to know which side of this triangle is the longest.



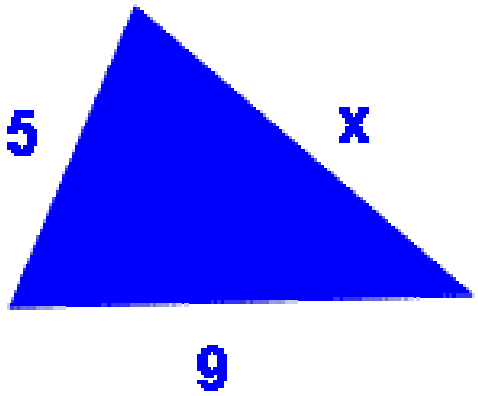
The second relationship involves the lengths of the sides of a triangle.

Theorem: *The Triangle Inequality*

The sum of the lengths of any two sides of a triangle must be greater than the third side.

Example:

Suppose we know the lengths of two sides of a triangle, and we want to find the possible lengths of the third side.



While there are other inequality relationships in a triangle, these two relationships are the ones most commonly used. Be sure that you learn these two relationships and you'll be set !

RAPID FIRE!!

1. Which of the following could represent the lengths of the sides of a triangle ?

A. 1, 2, 3 B. 6, 8, 15 C. 5, 7, 9

2. Two sides of an isosceles triangle measure 3 and 7. Which of the following could be the measure of the third side ?

A. 9 B. 7 C. 3

3. In triangle ABC, m<A=30 and m<B=50. Which is the longest side of the triangle?

A. $\overline{AB}$ B. $\overline{AC}$ C. $\overline{BC}$

4. In triangle DEF, an exterior angle at D measures 170, and m<E=80. Which is the longest side of the triangle?

A. $\overline{EF}$ B. $\overline{DF}$ C. $\overline{DE}$

5. In triangle ABC, m<C=55, and m<C > m<B. Which is the longest side of the triangle?

A. $\overline{AB}$ B. $\overline{AC}$ C. $\overline{BC}$

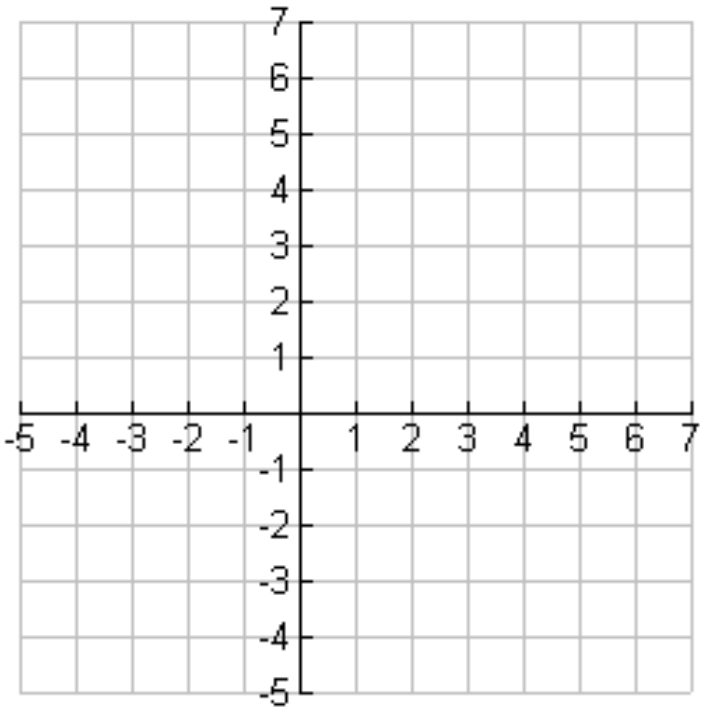
Challenging problems.

1. In △GHI, m<G=6x-3, m<H=10x+8, and m<I=49-2x. Which inequality shows the relationship between the lengths of the sides of the triangle?

A. GH>GI>HI B. GI>GH>HI

C. GH<HI<GI D. GH<GI<HI

2. △PQR has vertices at *P*(-4, 6), *Q*(4, 5), *R*(-2, -3). Which angle has the smallest measure?



A. Not enough information

B. <P C. <Q D. <R

Say What??!!

The early Egyptians used to make triangles by using a rope with knots tied at equal intervals. Each vertex of the triangle had to occur at a knot. Suppose you had a rope with exactly **10** knots making **9** equal lengths as shown below. How many different triangles could you make?

A. 2 B. 3 C. 5 D. 4

x	*y*	*z*	*Triangle?*
1	1	7	
1	2	6	
1	3	5	
1	4	4	
2	2	5	
2	3	4	
3	3	3	

PLAN: Let x, y, and z be the length of each side. Check every possible combination of $x + y + z = 9$ to see how many can be made into triangles. A table can help us keep track of the combinations.

*all graphics and many examples are from www.glencoe.com or from http://regentsprep.org/Regents/Math/math-a.cfm