Lesson 13

Glencoe Geometry Chapter 5.4 & 5.5

The Triangle Inequality

By the end of this lesson, you should be able to

1. Recognize and apply relationships between sides and angles in a triangle.
2. Apply the Triangle Inequality Theorem

We learned previously that if sides in a triangle were congruent, then the angles opposite those sides are also congruent (and vice-versa).

There are also important relationships that deal with **unequal** quantities. Today, we will examine two of these relationships.

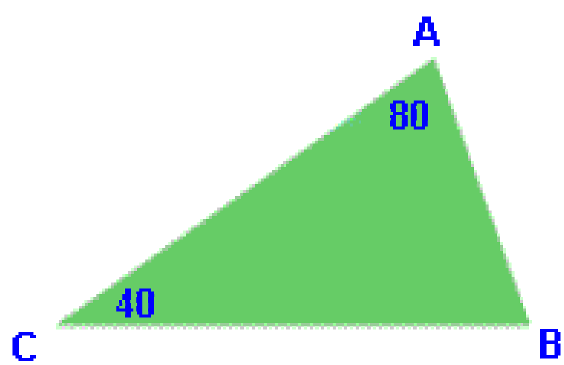
The first relationship involves the lengths of the sides of a triangle in relation to the triangle's angles.

Theorem:

In a triangle, the longest side is across from the largest angle. The shortest side is across from the shortest angle. The "middle" side is across from the "middle" angle.

Example:

Suppose we want to know which side of this triangle is the longest.



Before we can utilize our theorem, we need to know the size of <B. We know that the 3 angles of the triangle add up to 180.

$$80 + 40 + x = 180$$
$$120 + x = 180$$
$$x = 60$$

We have now found that <B measures 60. According to our theorem, the longest side will be across from the largest angle.

Now that we know the measures of all 3 angles, we can tell that <A is the largest. This means the side across from <A, side CB, is the longest.

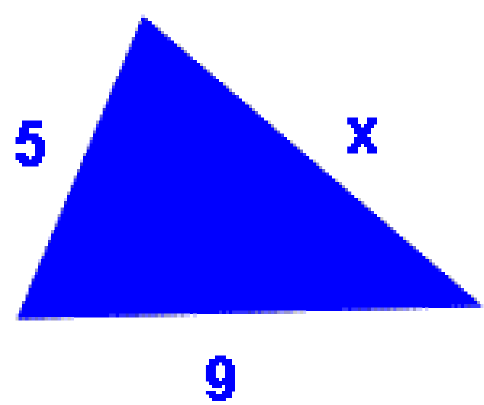
The second relationship involves the lengths of the sides of a triangle.

Theorem: *The Triangle Inequality*

The sum of the lengths of any two sides of a triangle must be greater than the third side.

Example:

Suppose we know the lengths of two sides of a triangle, and we want to find the possible lengths of the third side.



According to our theorem, the following 3 statements must be true:

5+x > 9 So, **x > 4**	5+9 > x So, **14 > x**	x+9 > 5 So, **x > -4** (no real information gained here since the lengths of the sides must be positive.)

Putting these statements together we get that **x** must be **greater than 4**, but **less than 14.** So any number in the range 4 < x < 14 can represent the length of the missing side of our triangle.

While there are other inequality relationships in a triangle, these two relationships are the ones most commonly used. Be sure that you learn these two relationships and you'll be set !

RAPID FIRE!!

1. Which of the following could represent the lengths of the sides of a triangle ?

A. 1, 2, 3 B. 6, 8, 15 C. 5, 7, 9

2. Two sides of an isosceles triangle measure 3 and 7. Which of the following could be the measure of the third side ?

A. 9 B. 7 C. 3

3. In triangle ABC, m<A=30 and m<B=50. Which is the longest side of the triangle?

A. $\overline{AB}$ B. $\overline{AC}$ C. $\overline{BC}$

4. In triangle DEF, an exterior angle at D measures 170, and m<E=80. Which is the longest side of the triangle?

A. $\overline{EF}$ B. $\overline{DF}$ C. $\overline{DE}$

5. In triangle ABC, m<C=55, and m<C > m<B. Which is the longest side of the triangle?

A. $\overline{AB}$ B. $\overline{AC}$ C. $\overline{BC}$

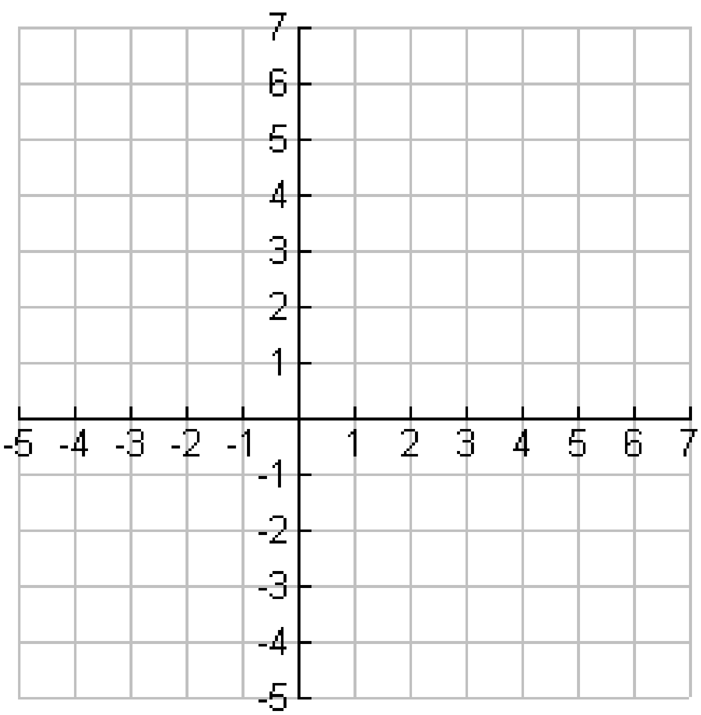
Challenging problems.

1. In △GHI, m<G=6x-3, m<H=10x+8, and m<I=49-2x. Which inequality shows the relationship between the lengths of the sides of the triangle?

A. GH>GI>HI B. GI>GH>HI

C. GH<HI<GI D. GH<GI<HI

2. △PQR has vertices at *P*(-4, 6), *Q*(4, 5), *R*(-2, -3). Which angle has the smallest measure?



A. Not enough information

B. <P C. <Q D. <R

Say What??!!

The early Egyptians used to make triangles by using a rope with knots tied at equal intervals. Each vertex of the triangle had to occur at a knot. Suppose you had a rope with exactly **10** knots making **9** equal lengths as shown below. How many different triangles could you make?

A. 2 B. 3 C. 5 D. 4

x	*y*	*z*	*Triangle?*
1	1	7	No
1	2	6	No
1	3	5	No
1	4	4	Yes
2	2	5	No
2	3	4	Yes
3	3	3	Yes

PLAN: Let x, y, and z be the length of each side. Check every possible combination of $x + y + z = 9$ to see how many can be made into triangles.

A table can help us keep track of the combinations.