Lesson 12

Glencoe Geometry Chapter 5.2 & 8.1

Right Triangles & The Pythagorean Theorem

By the end of this lesson, you should be able to
1. Determine if two right triangles are congruent.
2. Use the Pythagorean Theorem to solve problems.

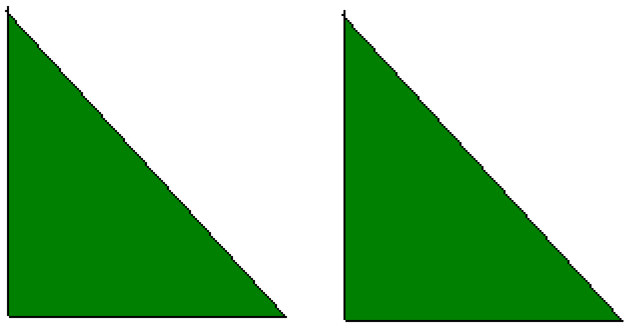
A few episodes ago, we looked at determining whether ANY two triangles were congruent. To review, there were several combinations of sides and angles we needed to draw a conclusion. They were (in no particular order):

SSS, SAS, AAS, ASA

Today, we are going to take a special look at the most famous of all triangles: **Right Triangles**

When determining if two right triangles are congruent, we are **ALWAYS** given at least one angle, the right (or 90 degree) angle. Though all of the above methods work on right triangles, there was a special case that worked **ONLY** for right triangles,

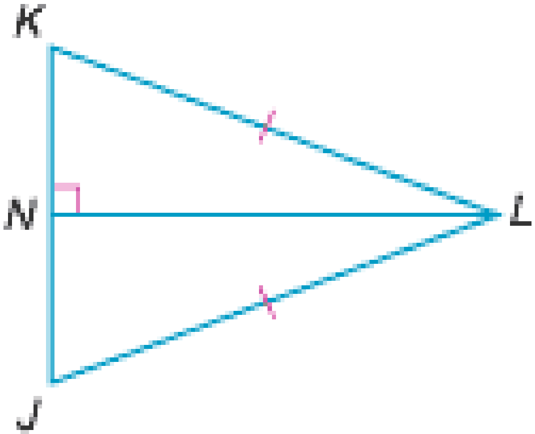
HL: If the hypotenuse and leg of one right triangle are congruent to the corresponding parts of another right triangle, the right triangles are congruent.



Remember that this was the SSA or Ambiguous case for non-right triangles. Because this only works for right triangles, we give it its own special name: HL

Example:

Determine the reason that $\triangle NKL \cong \triangle NJL$. Then, find $m\angle J$ if $m\angle KLN = 20^\circ$.



Because the two right triangles share a common side (a leg), segment *NL*, and we know that the hypotenuses are congruent, the two triangles are congruent by *HL*. Remember that means that all corresponding sides and angles are congruent.

Notice also that $\triangle JKL$ is isosceles, so the base angles, J and K are congruent. Also notice that segment NL is then a median, perpendicular bisector, altitude, and angle bisector—All 4!!! So $m\angle KLJ = 20 + 20 = 40$ leaving 180 – 40 = 140 for the two congruent base angles K and J. Therefore, they are both equal to 140/2 = 70 degrees.

Example:

Find the value of x so that the two right triangles $\triangle ABC$ and $\triangle XYZ$ are congruent by the HL postulate. Assume angle B is the right angle.

$$AC = 28, AB = 7x + 4, ZX = 9x + 1, YX = 5(x + 2)$$

AC and XZ are the hypotenuses and have equal measures. $28 = 9x + 1$. So $x = 27/9 \approx 3$
Or
AB = XY. So, $7x + 4 = 5(x + 2) \to 7x + 4 = 5x + 10 \to 2x = 6 \to x = 3$

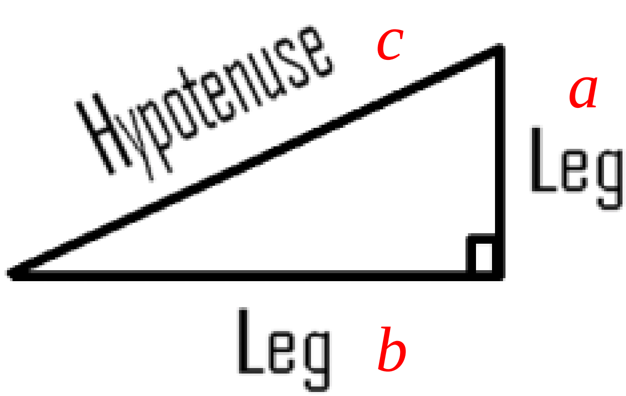
Now onto the best part about Right triangles:

THE PYTHAGOREAN THEOREM

Recall from Lesson 2:

The sum of the square of the two legs of a right triangle equals the square of the hypotenuse.

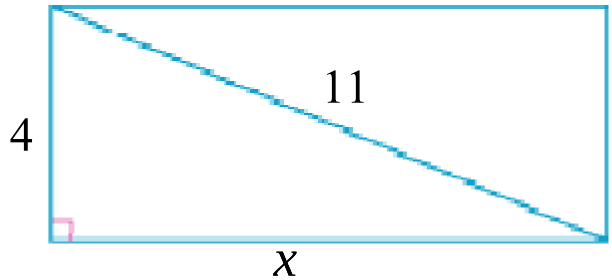
$$a^2 + b^2 = c^2$$



http://mac.ysu.edu/~jmartin/pythagorean/pythagorean1.gif

Example:

Find the value of *x*.



A. $\sqrt{137}$ C. $\sqrt{44}$

B. $\sqrt{105}$ D. 15

$x^2 + 4^2 = 11^2$

$x^2 = 121 - 16$

$x = \sqrt{105}$

There are certain combinations of three whole numbers that will always satisfy the Pythagorean Theorem. These sets of numbers are called **Pythagorean Triples**. If the measures of a right triangle are whole numbers, the measures form a Pythagorean triple.

Do the measures in the above example, $4, \sqrt{105}, 11$, form a Pythagorean triple? No, $\sqrt{105}$ is not a whole number.

Example:

$3, 4, 5$ and $7, 24, 25$ (It is customary to list the numbers in increasing order, with the measure of the hypotenuse last.

Example:

Which set of numbers is a Pythagorean triple?

A. 10, 15, 18

B. 10, 20, 30

C. 9, 40, 41

D. 8, 10, 12

A. $10^2 + 15^2 = 100 + 225 = 325 \neq 18^2 = 324$

B. $10^2 + 20^2 = 100 + 400 = 500 \neq 30^2 = 900$

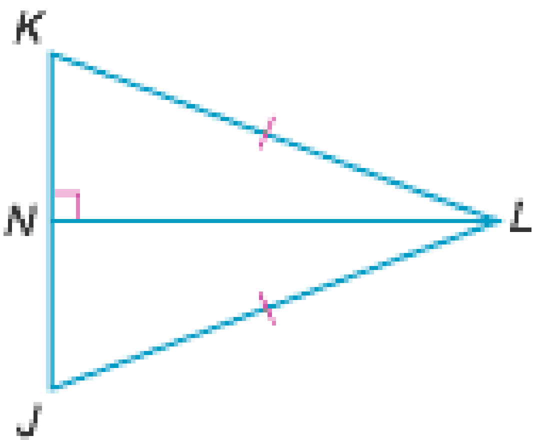
C. $9^2 + 40^2 = 81 + 1600 = 1681 = 41^2$ so this is the P-triple

Checking to be safe (and for extra practice)

D. $8^2 + 10^2 = 64 + 100 = 164 \neq 144 = 12^2$

Example:

If $\triangle NKL \cong \triangle NJL$, $NL = 8$ft. and $KL = 10$ft. , find JN .



Since segment NL is a median, KN = JN, So we find KN.
By the Pythagorean theorem,

$8^2 + (KN)^2 = 10^2$

$(KN)^2 = 100 - 64 = 36$

$\sqrt{(KN)^2} = \sqrt{36}$

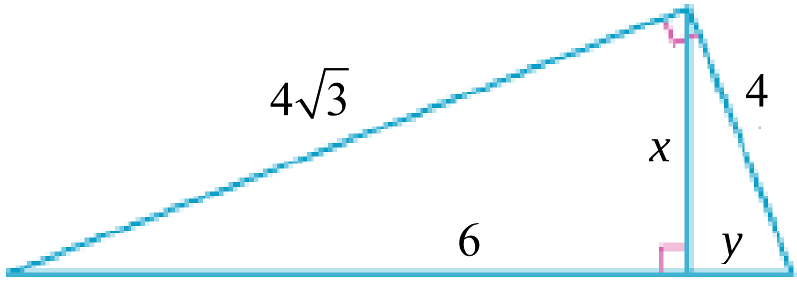
$KN = 6\,ft.$ DON'T FORGET YOUR UNITS!

So, $JN = 6$ too

This gives us another P-triple: 6,8,10

Example:

Find the value of *y*.



We must find ***y*** first. For the outer triangle:

$$\left(4\sqrt{3}\right)^2 + 4^2 = \left(6+y\right)^2$$

$$48+16=36+12y+y^2$$

$$y^2+12y+36=64$$

$$y^2+12y-28=0$$

$$\left(y+14\right)\left(y-2\right)=0$$

$y=-14$ or $y=2$

But since our side lengths cannot be negative, $y=2$

Or finding *x* first:

$$x^2+6^2=\left(4\sqrt{3}\right)^2$$

$$x^2+36=48$$

$$x=\sqrt{48-36}=\sqrt{12}=2\sqrt{3}$$

Now finding *y* from the small triangle:

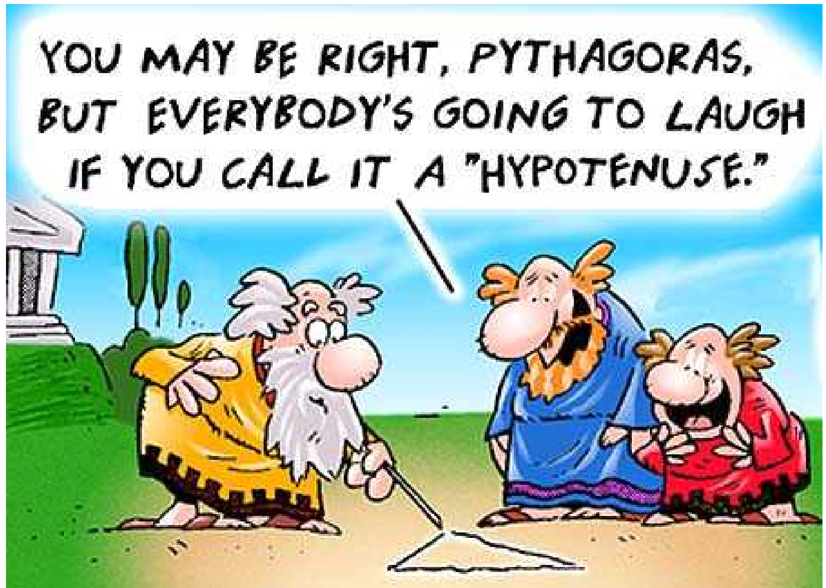
$$\left(2\sqrt{3}\right)^2+y^2=4^2$$

$$12+y^2=16$$

$$y=\sqrt{16-12}=\sqrt{4}=2$$

Say What??!!

Hypotenuse comes from the common Greek root ***hypo***(for under, as in hypodermic -under the skin) and the less common ***tein*** or ***ten***, for stretch. This last is the source of our modern word tension. The hypotenuse was the line segment "*stretched under*" the right angle.



http://sid.at/hs-strassburg/images/pythagoras.jpg and Frank and Ernest by Bob Thaves

Example: Computer Link

The actual screen of one of the newest models of flat screen computer monitors measures 19.5 inches by 12 inches. Find the measure of the diagonal of the screen. Round to the nearest tenth of an inch.