Lesson 11

Glencoe Geometry Chapter 5.1

Special Triangle Segments

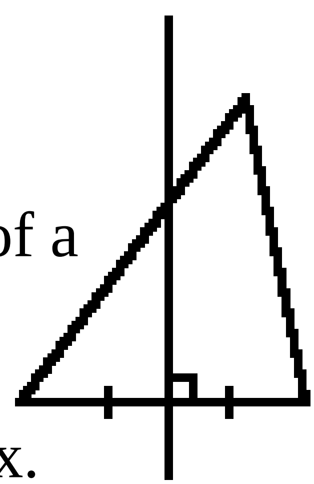
By the end of this lesson, you should be able to

1. Identify and use medians, altitudes, angle bisectors, and perpendicular bisectors in a triangle.
2. Find measures of segments and angles using algebra.

This week we are AGAIN sticking with our topic from the last THREE weeks: **TRIANGLES**

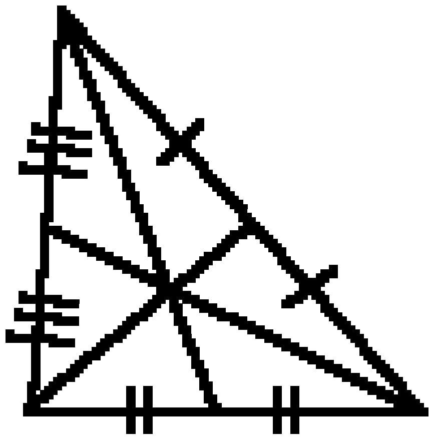
Today, we are going to focus primarily on the four types of special segments that each triangle has. This means more memorization of definitions, and learning how to apply those definitions properly.

Let’s Define our first special segment.



PERPENDICULAR BISECTOR - a line or segment that passes through the midpoint of a side of a triangle and is also perpendicular to that side. Perpendicular bisectors do not always pass through the opposite vertex.

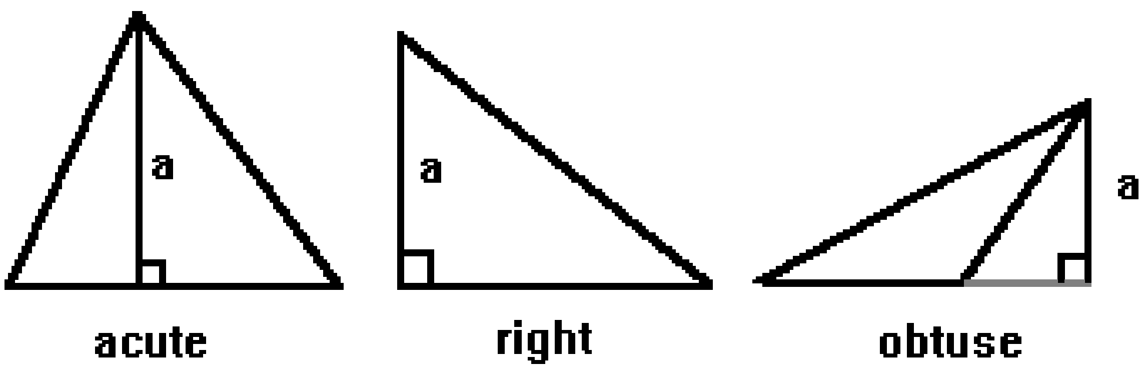
Here's another special segment:



MEDIAN - a special segment that connects a vertex of a triangle to the midpoint of the opposite side. Every triangle has three medians.

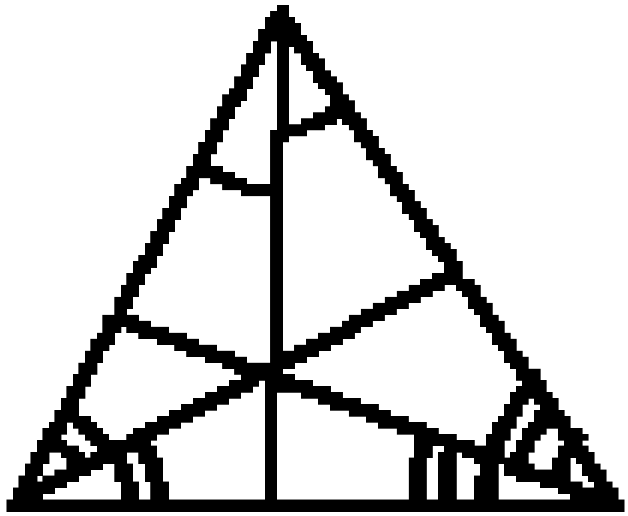
Here's yet another special segment.

ALTITUDE - a segment that is perpendicular to a side of a triangle, and it intersects with the vertex opposite that side. Most altitudes rise from the base and their length is also the height of the triangle, but any side can have an altitude.



And finally, the fourth special segment is . . . .

ANGLE BISECTOR - a special segment of a triangle that bisects an angle of the triangle and then intersects the opposite side of the triangle.

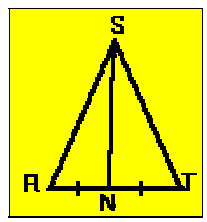


Got them all straight???

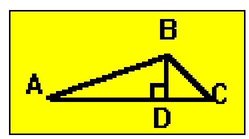
Pencils ready...

Draw an example of the following . . .

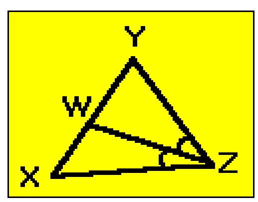
1) Triangle $\triangle RST$ with **median** $\overline{SN}$



2) Triangle $\triangle ABC$ with **altitude** $\overline{BD}$



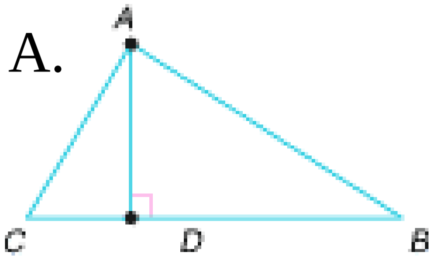
3) Triangle $\triangle XYZ$ with **angle bisector** $\overline{WZ}$



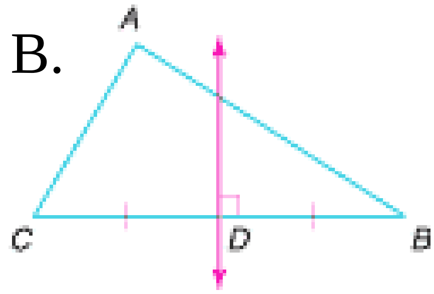
Example:

In which picture is $\overline{AD}$ an angle bisector?

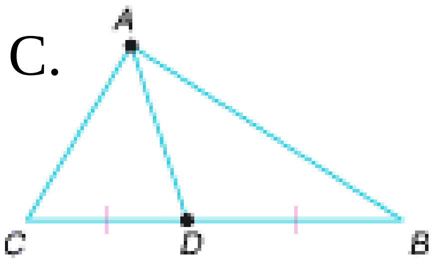
A.



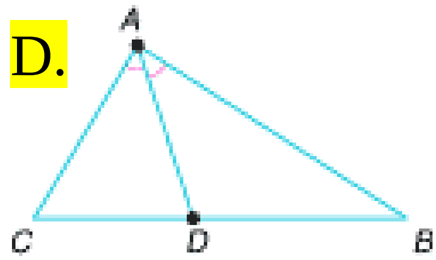
B.



C.



D.



Example:

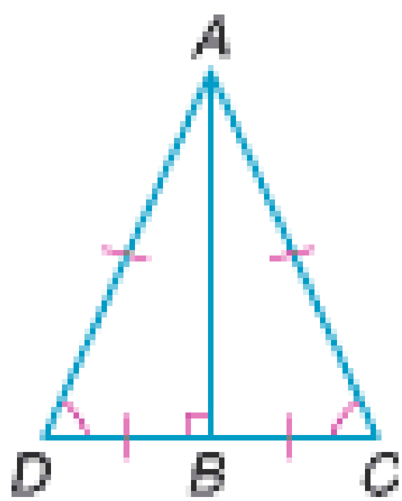
In $\triangle ADC$, $\overline{AB}$ is the

A. median

B. altitude

C. perpendicular bisector

D. all of these

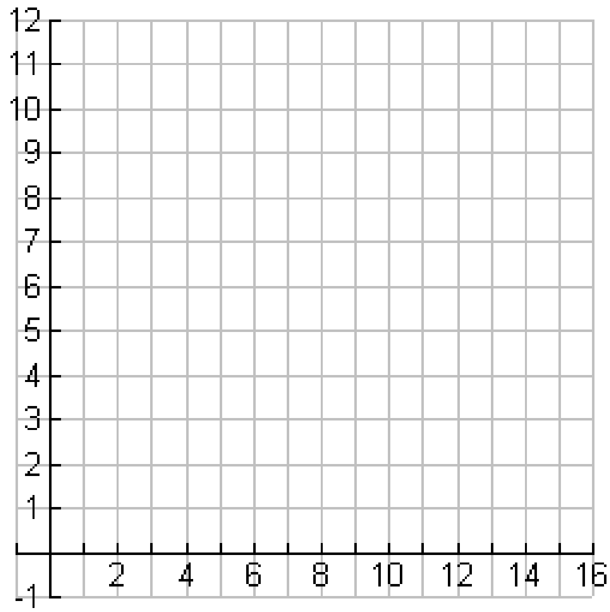


Example:

$\triangle CPR$ has vertices $C(15, 1)$, $P(9, 11)$, and $R(2, 1)$. Determine the coordinates of point A on $\overline{CP}$ so that $\overline{RA}$ is the median of $\triangle CPR$.

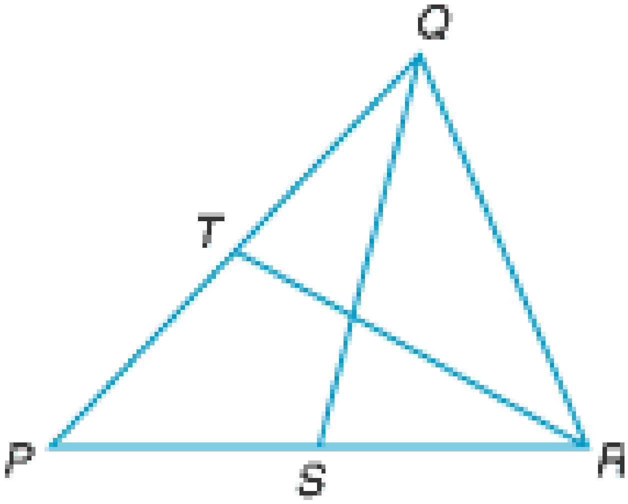
Point A is the midpoint between points P and C. Using the midpoint formula:

$$A = \left(\frac{9+15}{2}, \frac{11+1}{2}\right) = (12, 6)$$



Example:

In $\triangle PQR$, $\overline{QS}$ and $\overline{RT}$ are medians. If $PT = 3x - 1$, $PS = 4x - 2$, and $SR = 2x + 4$, find TQ.



We first find x by setting PS and SR equal to each other.

$PS = SR$

$4x-2 = 2x+4$

$2x = 6$

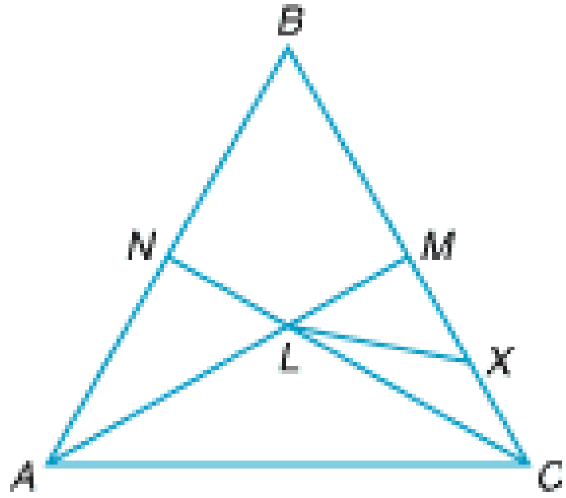
$x = 3$

Since PT and TQ are equal, we can plug $x = 3$ into PT to find TQ:

$TQ = PT = 3(3)-1 = 8$

Example:

$\overline{AM}$ and $\overline{CN}$ are medians of $\triangle ABC$, and $\overline{LX}$ is a median of $\triangle LMC$. Find XM if $BC = 18$.



Since $\overline{AM}$ is a median, $BM = MC = BC/2 = 18/2 = 9$.

Since $\overline{LX}$ is a median, $CX = XM = MC/2 = 9/2 = 4.5$

So $XM = 4.5$ or $\frac{9}{2}$

Example:

In $\triangle MOP$, $\angle O \cong \angle MPO$, $m\angle M = 40^\circ$, and $\overline{PN}$ is an altitude. Find $m\angle NPO$.

We first need to find $m\angle O$:

$40 + m\angle O + m\angle MPO = 180$

$40 + 2m\angle O = 180$ (since $\angle O \cong \angle MPO$)

$2m\angle O = 140$

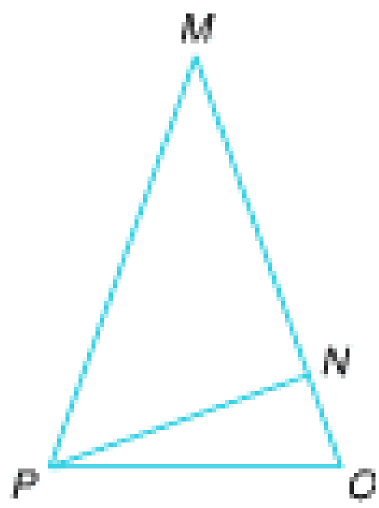
$m\angle O = 70$

Since $\overline{PN}$ is an altitude, $\angle ONP$ is a right angle (90 degrees):

$m\angle NPO + 90 + 70 = 180$

$m\angle NPO + 160 = 180$

$m\angle NPO = 20$



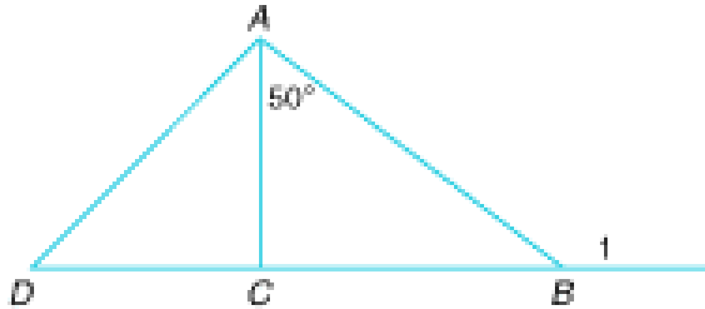
Example:

$\overline{AC}$ is an altitude of $\triangle ABD$, find $m\angle 1$.

Since $\overline{AC}$ is and altitude, $\angle ACB$ is a right angle and its measure equals 90 degrees.

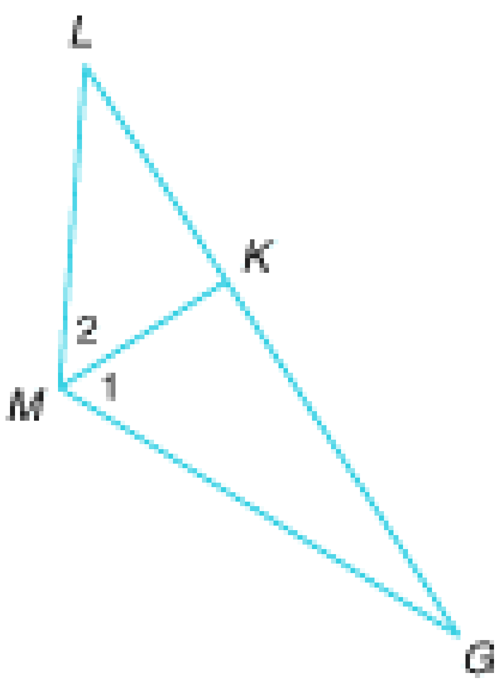
$m\angle ABC = 180 - 50 - 90 = 40$
$\angle 1$ is supplementary to $\angle ABC$, so

$m\angle 1 = 180 - m\angle ABC = 180 - 40 = 140$
or
$m\angle 1 =$ sum of two remote interior angles
$m\angle 1 = m\angle CAB + m\angle ACB = 50 + 90 = 140$



Example:

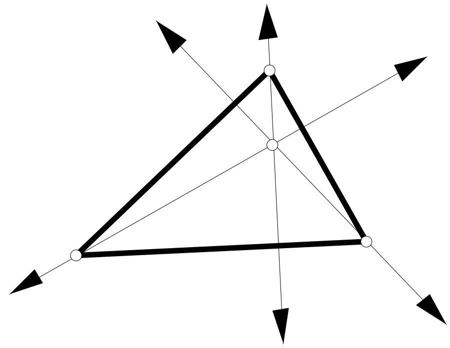
In $\triangle LMG$, $\overline{MK}$ is an angle bisector, $m\angle 1 = 2n + 10$, $m\angle 2 = 4n - 32$ and $m\angle L = 60$. Find $m\angle G$.



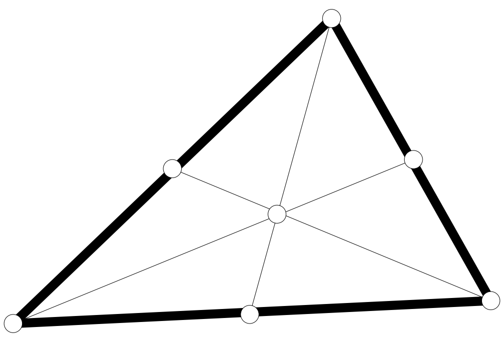
$m\angle 1 = m\angle 2 \rightarrow 2n + 10 = 4n - 32$
$42 = 2n \rightarrow n = 21$
so
$m\angle 1 = m\angle 2 = 2(21) + 10 = 42 + 10 = 52$
So $m\angle LMG = 2(52) = 104$
And finally,
$m\angle L + m\angle LMG + m\angle G = 180$
$60 + 104 + m\angle G = 180$
$164 + m\angle G = 180$
$m\angle G = 180 - 164 = 16$

Say What??!!

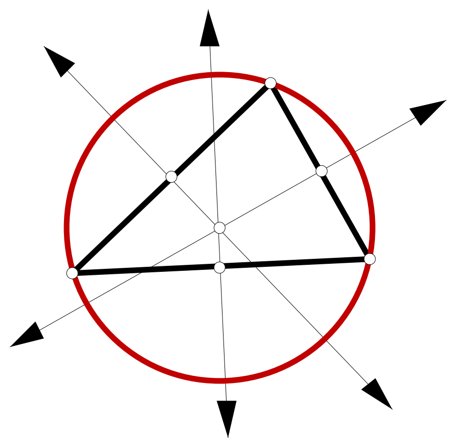
When drawn from each of the three sides, the intersection of these special segments form some unique "centers" of the triangle.



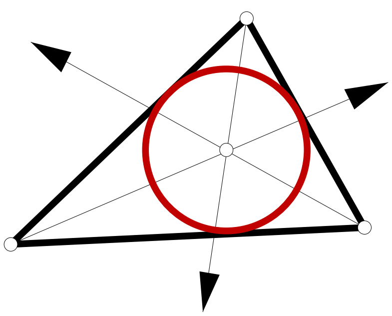
ORTHOCENTER — the intersection of the three altitudes of a triangle.



CENTROID — the intersection of the three medians of a triangle. If a triangle could have constant area mass density, the centroid would be the center of mass, or balancing point.



CIRCUMCENTER — the intersection of the perpendicular bisectors of the three sides of a triangle. The circumcenter is the center of the circumscribed circle.



INCENTER — the intersection of the bisectors of the three interior angles of a triangle. The incenter is the center of the inscribed circle.

*all graphics and many examples are from www.glencoe.com or http://whistleralley.com/orthocenter/orthocenter.htm