Are you ready for calculus: KEY

1. a) $\frac{x^3-9x}{x^2-7x+12} = \frac{x(x^2-9)}{(x-3)(x-4)} = \frac{x\cancel{(x-3)}(x+3)}{\cancel{(x-3)}(x-4)} = \boxed{\frac{x(x+3)}{x-4} \text{ or } \frac{x^2+3x}{x-4}}$

b) $\frac{x^2-2x-8}{x^3+x^2-2x} = \frac{(x-4)(x+2)}{x(x^2+x-2)} = \frac{(x-4)\cancel{(x+2)}}{x\cancel{(x+2)}(x-1)} = \boxed{\frac{x-4}{x(x-1)} \text{ or } \frac{x-4}{x^2-x}}$

c) $\frac{\frac{1}{x}-\frac{1}{5}}{\frac{1}{x^2}-\frac{1}{25}} \cdot \frac{25x^2}{25x^2} = \frac{25x-5x^2}{25-x^2} = \frac{5x\cancel{(5-x)}}{\cancel{(5-x)}(5+x)} = \boxed{\frac{5x}{5+x}}$

d) $\frac{9-x^{-2}}{3+x^{-1}} = \frac{9-\frac{1}{x^2}}{3+\frac{1}{x}} \cdot \frac{x^2}{x^2} = \frac{9x^2-1}{3x^2+x} = \frac{(3x-1)\cancel{(3x+1)}}{x\cancel{(3x+1)}} = \boxed{\frac{3x-1}{x} \text{ or } 3-\frac{1}{x} \text{ or } 3-x^{-1}}$

2. a) $\frac{2}{\sqrt{3}+\sqrt{2}} \cdot \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{2(\sqrt{3}-\sqrt{2})}{3-2} = \boxed{2(\sqrt{3}-\sqrt{2}) \text{ or } 2\sqrt{3}-2\sqrt{2}}$

b) $\frac{4}{1-\sqrt{5}} \cdot \frac{1+\sqrt{5}}{1+\sqrt{5}} = \frac{4(1+\sqrt{5})}{1-5} = \frac{\cancel{4}(1+\sqrt{5})}{\cancel{-4}} = \boxed{-(1+\sqrt{5}) \text{ or } -1-\sqrt{5}}$

c) $\frac{1}{1+\sqrt{3}-\sqrt{5}} \cdot \frac{1-(\sqrt{3}-\sqrt{5})}{1-(\sqrt{3}-\sqrt{5})} = \frac{1-\sqrt{3}+\sqrt{5}}{1-(\sqrt{3}-\sqrt{5})^2} = \frac{1-\sqrt{3}+\sqrt{5}}{1-(3-2\sqrt{15}+5)} = \frac{1-\sqrt{3}+\sqrt{5}}{2\sqrt{15}-7} \cdot \frac{2\sqrt{15}+7}{2\sqrt{15}+7}$

$= \frac{(1-\sqrt{3}+\sqrt{5})(2\sqrt{15}+7)}{4(15)-49} = \boxed{\frac{(1-\sqrt{3}+\sqrt{5})(2\sqrt{15}+7)}{11} \text{ or } \frac{7+3\sqrt{3}+\sqrt{5}+2\sqrt{15}}{11}}$

3. a) $\frac{(2a^2)^3}{b} = \frac{8a^6}{b} = \boxed{8a^6b^{-1}}$ b) $\sqrt{9ab^3} = \sqrt{9} \cdot a^{1/2} \cdot (b^3)^{1/2} = \boxed{3a^{1/2}b^{3/2}}$

c) $\frac{a(2/b)}{3/a} = \frac{2a}{b} \cdot \frac{a}{3} = \boxed{\frac{2}{3}a^2b^{-1}}$ d) $\frac{ab-a}{b^2-b} = \frac{a\cancel{(b-1)}}{b\cancel{(b-1)}} = \boxed{ab^{-1}}$

e) $\frac{a^{-1}}{(b^{-1})\sqrt{a}} = a^{-1} \cdot b^{1} \cdot a^{-1/2} = \boxed{a^{-3/2}b}$ f) $\left(\frac{a^{2/3}}{b^{1/2}}\right)^2 \left(\frac{b^{2/3}}{a^{1/2}}\right) = a^{4/3} \cdot a^{-1/2} \cdot b^{-1} \cdot b^{2/3} = \boxed{a^{5/6}b^{-1/3}}$

4. a) $5^{x+1} = 25 \rightarrow 5^{x+1} = 5^2 \rightarrow x+1 = 2 \rightarrow \boxed{x=1}$

b) $\frac{1}{3} = 3^{2x+2} \rightarrow 3^{-1} = 3^{2x+2} \rightarrow -1 = 2x+2 \rightarrow 2x = -3 \rightarrow \boxed{x = \frac{-3}{2}}$

c) $\log_2 x = 3 \rightarrow 2^3 = x \rightarrow \boxed{x = 8}$

Are you ready for Calculus: KEY

4) d) $\log_3 x^2 = 2\log_3 4 - 4\log_3 5$

$\log_3 x^2 = \log_3 16 - \log_3 625$

$\log_3 x^2 - \log_3 16 + \log_3 625 = 0$

$\log_3\left(\frac{625x^2}{16}\right) = 0$

$3^0 = \frac{625}{16}x^2$

$\frac{16}{625} = x^2$

$\boxed{x = \pm\frac{4}{25}}$

5) a) $\log_2 5 + \log_2 (x^2-1) - \log_2 (x-1) = \log_2\left(\frac{5(x^2-1)}{(x-1)}\right) = \boxed{\log_2(5(x+1))}$

b) $2\log_4 9 - \log_2 3 = \log_4 (3^2)^2 - \log_2 3 = \frac{\log_2 3^4}{\log_2 4} - \log_2 3$

$= \frac{4\log_2 3}{\log_2 2^2} - \log_2 3 = \frac{4\log_2 3}{2} - \log_2 3 = 2\log_2 3 - \log_2 3 = \boxed{\log_2 3}$

c) $3^{2\log_3 5} = 3^{\log_3 5^2} = 5^2 = \boxed{25}$

6) a) $\log_{10} 10^{1/2} = \boxed{\frac{1}{2}}$ b) $\log_{10}\left(\frac{1}{10^x}\right) = \log_{10} 10^{-x} = \boxed{-x}$

c) $2\log_{10}\sqrt{x} + 3\log_{10} x^{1/3} = \log_{10} x + \log_{10} x = \boxed{2\log_{10} x = \log_{10} x^2}$; $x \geq 0$

either

7) a) $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ for a

$\frac{x}{a} = 1 - \frac{y}{b} - \frac{z}{c}$

$\frac{x}{a} = \frac{bc - cy - bz}{bc}$

$\boxed{a = \frac{bcx}{bc - cy - bz}}$

b) $V = 2(ab + bc + ca)$ for a

$V = 2ab + 2bc + 2ca$

$V = a(2b + 2c) + 2bc$

$a(2b + 2c) = V - 2bc$

$\boxed{a = \frac{V - 2bc}{2b + 2c}}$

7 c) $A = 2\pi r^2 + 2\pi r h$, for r^+

$$2\pi r^2 + 2\pi h r - A = 0$$

$$r = \frac{-2\pi h \pm \sqrt{(2\pi h)^2 - 4(2\pi)(-A)}}{2(2\pi)}$$

$$r = \frac{-2\pi h + \sqrt{4\pi^2 h^2 + 8\pi A}}{4\pi}$$

$$\boxed{r = \frac{-\pi h + \sqrt{\pi^2 h^2 + 2\pi A}}{2\pi}}$$

d) $A = P + xrP$, for P

$$P(1 + xr) = A$$

$$\boxed{P = \frac{A}{1 + xr}}$$

e) $2x - 2yd = y + xd$, for d

$$xd + 2yd = 2x - y$$

$$d(x + 2y) = 2x - y$$

$$\boxed{d = \frac{2x - y}{x + 2y}}$$

f) $\frac{2x}{4\pi} + \frac{1-x}{2} = 0$, for x

$$\frac{4x + 4\pi - 4\pi x}{8\pi} = 0$$

$$x(4 - 4\pi) = -4\pi$$

$$\boxed{x = \frac{-\pi}{1 - \pi} \text{ or } \frac{\pi}{\pi - 1}}$$

8 a) $y = x^2 + 4x + 3$

$$y = x^2 + 4x + 4 - 4 + 3$$

$$y = (x + 2)^2 - 1$$

$$\boxed{y + 1 = (x + 2)^2}$$

b) $3x^2 + 3x + 2y = 0$

$$3(x^2 + x + 1/4) = -2y + 3/4$$

$$3(x + 1/2)^2 = -2(y - 3/8)$$

$$\boxed{y - 3/8 = -3/2\,(x + 1/2)^2}$$

c) $9y^2 - 6y - 9 - x = 0$

$$9(y^2 - \tfrac{2}{3}y + 1/9) = x + 9 + 1$$

$$9(y - 1/3)^2 = x + 10$$

$$\boxed{x + 10 = 9(y - 1/3)^2}$$

(9) a) $x^6-16x^4=0 \rightarrow x^4(x^2-16) \rightarrow \boxed{x^4(x-4)(x+4)}$

b) $4x^3-8x^2-25x+50$

$= (x-2)(4x^2-25)$

$= \boxed{(x-2)(2x-5)(2x+5)}$

$$\begin{array}{r|rrrr} & 4 & -8 & -25 & 50 \\ 2 & & 8 & 0 & -50 \\ \hline & 4 & 0 & -25 & \underline{|0} \end{array}$$

c) $8x^3+27$

$=(x+3/2)(8x^2-12x+18)$

$= 2(x+3/2)(4x^2-6x+9)$

$= \boxed{(2x+3)(4x^2-6x+9)}$

$$\begin{array}{r|rrrr} & 8 & 0 & 0 & 27 \\ -3/2 & & -12 & 18 & -27 \\ \hline & 8 & -12 & 18 & \underline{|0} \end{array}$$

d) $x^4-1 = (x^2-1)(x^2+1) = \boxed{(x-1)(x+1)(x^2+1)}$

(10) a) $x^6-16x^4=0 \rightarrow x^4(x-4)(x+4)=0 \rightarrow \boxed{x=0, 4, -4}$

b) $4x^3-8x^2-25x+50=0 \Rightarrow (x-2)(2x-5)(2x+5)=0 \rightarrow \boxed{x=2, \pm\frac{5}{2}}$

c) $8x^3+27=0 \rightarrow (2x+3)(4x^2-6x+9)=0 \rightarrow \boxed{x=-\frac{3}{2}}$ + 2 imaginary

(11) a) $3\sin^2 x=\cos^2 x$, $0 \le x \le 2\pi$

$3\sin^2 x-\cos^2 x=0$

$3\sin^2 x-(1-\sin^2 x)=0$

$3\sin^2 x-1+\sin^2 x=0$

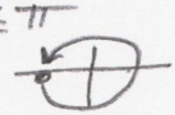
$4\sin^2 x=1$

$\sin^2 x=\frac{1}{4}$

$\sin x=\pm\frac{1}{2}$, $0 \le x \le 2\pi$

$\boxed{x=\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}}$

b) $\cos^2 x-\sin^2 x=\sin x$, $-\pi < x \le \pi$



$\sin^2 x+\sin x-(1-\sin^2 x)=0$

$2\sin^2 x+\sin x-1=0$

$(2\sin x-1)(\sin x+1)=0$

$\sin x=\frac{1}{2}$, $\sin x=-1$

$\boxed{x=\frac{\pi}{6}, \frac{5\pi}{6}, -\frac{\pi}{2}}$

c) $\tan x+\sec x=2\cos x$, $-\infty < x < \infty$

$\frac{\sin x}{\cos x}+\frac{1}{\cos x}-\frac{2\cos^2 x}{\cos x}=0$

$\frac{\sin x+1-2(1-\sin^2 x)}{\cos x}=0$

$2\sin^2 x+\sin x-1=0$, $\cos x \ne 0 \Rightarrow \underline{\underline{x \ne \pi/2+\pi n}}$

from 11b)

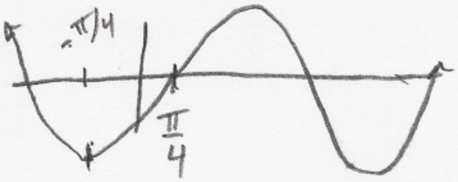
$\boxed{x=\frac{\pi}{6}+2\pi n \text{ or } \frac{5\pi}{6}+2\pi n,\ n\in\mathbb{Z}}$

(12) a) $\cos 210° = \boxed{-\frac{\sqrt{3}}{2}}$ b) $\sin \frac{5\pi}{4} = \boxed{-\frac{\sqrt{2}}{2}}$ c) $\tan^{-1}(-1) = -\frac{\pi}{4}$ (Principal Value)

d) $\arcsin(-1) = -\frac{\pi}{2}$ e) $\cos\frac{9\pi}{4} = \cos\frac{\pi}{4} = \boxed{\frac{\sqrt{2}}{2}}$ f) $\sin^{-1}\frac{\sqrt{3}}{2} = \boxed{\frac{\pi}{3}}$

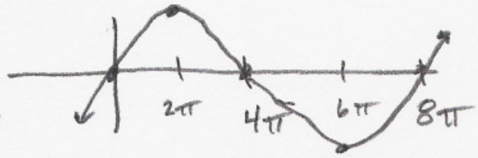
g) $\tan\frac{7\pi}{6} = \boxed{\frac{\sqrt{3}}{3}}$ h) $\cos^{-1}(-1) = \boxed{\pi}$

(13) a)

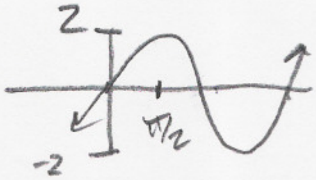


b) $y = \sin\left(\frac{1}{4}x\right)$

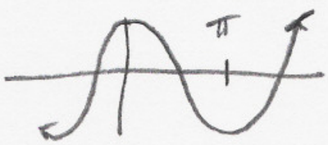
per $= \frac{2\pi}{1/4} = 8\pi$



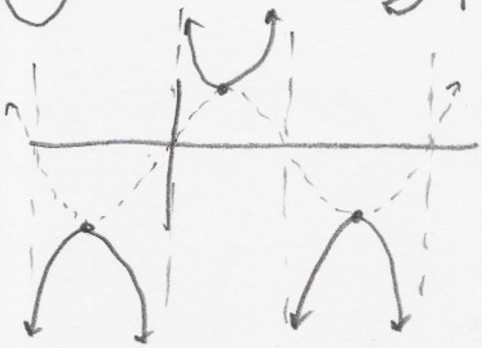
c) $y = 2\sin x$



d) $y = \cos x$



e) $y = \frac{1}{\sin x} = \csc x$



(14) a) $4x^2 + 12x + 3 = 0$

$x = \frac{-12 \pm \sqrt{144 - 48}}{8}$

$\boxed{x = \frac{-3 \pm \sqrt{6}}{2}}$

b) $2x + 1 = \frac{5}{x+2}$

$2x^2 + 4x + x + 2 = 5$

$2x^2 + 5x - 3 = 0$

$(2x-1)(x+3) = 0$

$\boxed{x = \frac{1}{2}, -3}$

c) $\frac{x+1}{x} - \frac{x}{x+1} = 0$

$\frac{x^2 + 2x + 1 - x^2}{x(x+1)} = 0$

when $2x + 1 = 0$, $x \neq 0, x \neq -1$

$\boxed{x = -\frac{1}{2}}$

(15) a) $x^5 - 4x^4 + x^3 - 7x + 1$ by $x + 2$

	1	-4	1	0	-7	1
-2		-2	12	-26	52	-90
	1	-6	13	-26	45	-89

$\boxed{\text{Remainder} = -89}$

b) $x^5 - x^4 + x^3 + 2x^2 - x + 4$ by $x^3 + 1$

$$
\begin{array}{r}
x^2 - x + 1 \\
x^3+1 \overline{)\, x^5 - x^4 + x^3 + 2x^2 - x + 4} \\
-x^5 \qquad\qquad + x^2 \qquad\qquad \\
\hline
-x^4 + x^3 + x^2 - x + 4 \\
\pm x^4 \qquad\qquad\quad \mp x \quad \\
\hline
x^3 + x^2 \quad + 4 \\
-x^3 \qquad\quad + -1 \\
\hline
\boxed{x^2 + 3 = R}
\end{array}
$$

(16) a) $12x^3 - 23x^2 - 3x + 2 = 0$, $x = 2$

$\rightarrow (x-2)(12x^2 + x - 1) = 0$

$(x-2)(4x-1)(3x+1) = 0$

$\boxed{x = 2, \tfrac{1}{4}, -\tfrac{1}{3}}$

$$\begin{array}{r|rrrr} & 12 & -23 & -3 & 2 \\ 2 & & 24 & 2 & -2 \\ \hline & 12 & 1 & -1 & \boxed{0} \end{array}$$

b) $12x^3 + 8x^2 - x - 1 = 0$

$\rightarrow (x - \tfrac{1}{3})(12x^2 + 12x + 3) = 0$

$(x - \tfrac{1}{3})(3)(4x^2 + 4x + 1) = 0$

$3(x - \tfrac{1}{3})(2x+1)^2 = 0$

$\boxed{x = \tfrac{1}{3}, -\tfrac{1}{2} \text{ multiplicity } 2}$

$$\begin{array}{r|rrrr} & 12 & 8 & -1 & -1 \\ \tfrac{1}{3} & & 4 & 4 & 1 \\ \hline & 12 & 12 & 3 & \boxed{0} \end{array}$$

(17) a) $x^2 - 2x - 3 \le 0$

$(x-3)(x+1) = 0$

$x = 3, -1$ are critical values

test: Let $x = 0$: $0^2 - 2(0) - 3 \overset{?}{\le} 0$

$-3 \le 0$ ✓

so $\boxed{-1 \le x \le 3 \text{ or } [-1, 3]}$

b) $\dfrac{2x-1}{3x-2} \le 1 \rightarrow \dfrac{2x-1}{3x-2} - 1 \le 0$

$\rightarrow \dfrac{2x-1-3x+2}{3x-2} \le 0 \rightarrow \dfrac{-x+1}{3x-2} \le 0$

$-x + 1 = 0 \rightarrow x = 1$

$3x - 2 = 0 \rightarrow x = \tfrac{2}{3}$

test: Let $x = 0$: $\dfrac{-1}{-2} = \dfrac{1}{2} \le 1$ ✓

So $x < \tfrac{2}{3}$ or $x \ge 1$ (since $x \ne \tfrac{2}{3} \rightarrow \frac{1/3}{0}$)

-or- $(-\infty, \tfrac{2}{3}) \cup [1, \infty)$

c) $x^2 + x + 1 > 0$

(imaginary roots)

$\boxed{\text{so true } \forall x \text{ (for all } x)}$

(18) a) $\dfrac{2x-1}{3x-1} \le 1 \rightarrow \dfrac{2x-1}{3x-1} - 1 \le 0$

$\rightarrow \dfrac{2x-1-3x+1}{3x-1} \le 0 \rightarrow \dfrac{-x}{3x-1} \le 0$

$x \le 0$ or $x > \tfrac{1}{3}$

-or-

$(-\infty, 0] \cup (\tfrac{1}{3}, \infty)$

b) $|5x - 2| = 8$

so $5x - 2 = 8 \rightarrow \boxed{x = 2}$

or

$5x - 2 = -8 \rightarrow \boxed{x = -\tfrac{6}{5}}$

c) $|2x + 1| = x + 3$

$2x + 1 = x + 3$

or

$2x + 1 = -x - 3$

$\boxed{x = 2 \text{ or } -\tfrac{4}{3}}$

(19) a) $(-1, 3), (2, -4)$: $m = \dfrac{3 - -4}{-1-2} = \dfrac{7}{-3}$

$y = mx + b$

$3 = (-\tfrac{7}{3})(-1) + b$

$3 - \tfrac{7}{3} = \tfrac{2}{3} = b$

so $\boxed{y = -\tfrac{7}{3}x + \tfrac{2}{3}}$

or $\boxed{7x + 3y = 2}$

19 (b) $(-1,2)$, Normal ($\perp$) to $2x-3y+5=0$

$-3y=-2x-5$

$y=\frac{2}{3}x+\frac{5}{3} \Rightarrow m=\frac{2}{3}$ so $m_\perp=-\frac{3}{2}$.

$y=mx+b$

$2=(-\frac{3}{2})(-1)+b$

$2-\frac{3}{2}=\frac{1}{2}=b$

so $\boxed{y=-\frac{3}{2}x+\frac{1}{2}}$

or $\boxed{3x+2y=1}$

(c) $(2,3)$, mdpt $(-1,4)$ to $(3,2)$

mdpt: $\left(\frac{-1+3}{2},\frac{4+2}{2}\right)=(1,3)$, slope, $m=\frac{3-3}{2-1}=\frac{0}{1}=0 \Rightarrow$ horizontal line

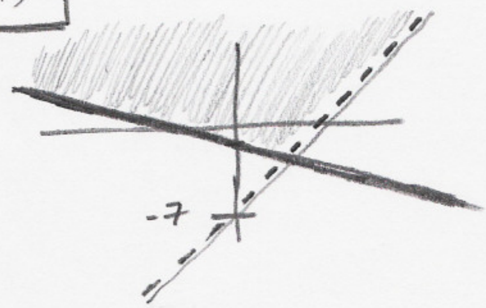
eq: $\boxed{y=3}$

20 a) $3x-y-7=0$ and $x+5y+3=0$

$y=3x-7$ by sub: $x+5(3x-7)+3=0 \to x+15x-35+3=0 \to 16x=32$, $\boxed{x=2}$

so point is $(2,3(2)-7)=\boxed{(2,-1)}$

b) $\begin{cases} 3x-y-7<0 \\ x+5y+3\geq 0 \end{cases} \to \begin{cases} y>3x-7 \\ y\geq -\frac{1}{5}x-\frac{3}{5} \end{cases} \to$



21 a) $C(1,2)$, $(-2,-1)$: $(x-h)^2+(y-k)^2=r^2$

$(-2-1)^2+(-1-2)^2=r^2$

$9+9=r^2=18$

eq: $\boxed{(x-1)^2+(y-2)^2=18}$

b) $(0,0)\,(1,0)\,(0,2)$; $(x-h)^2+(y-k)^2=r^2$

$\begin{cases}(0-h)^2+(0-k)^2=r^2\\(1-h)^2+(0-k)^2=r^2\\(0-h)^2+(2-k)^2=r^2\end{cases} \to \begin{cases}h^2+k^2=r^2\\1-2h+h^2+k^2=r^2\\h^2+4-4k+k^2=r^2\end{cases}$

systems: $1-2h+h^2+k^2=r^2$

$h^2+k^2=r^2$

$1-2h=0 \Rightarrow h=\frac{1}{2}$

& $4-4k+h^2+k^2=r^2$

$h^2+k^2=r^2$

$4-4k=0 \Rightarrow k=1$

$h^2+k^2=\frac{1}{4}+1=\frac{5}{4}=r^2$

So eq:

$\boxed{(x-\frac{1}{2})^2+(y-1)^2=\frac{5}{4}}$

22 $x^2+y^2+6x-4y+3=0$

a) $x^2+6x+\underline{9}+y^2-4y+\underline{4}=-3+\underline{9}+\underline{4}$

$(x+3)^2+(y-2)^2=10$

$\boxed{C(-3,2),\ r=\sqrt{10}}$

b) $C(-3,2)$, $P(-2,5)$; slope $m=\frac{5-2}{-2--3}=\frac{3}{1}=3$ so tan slope $=-\frac{1}{3}$

$y=mx+b$

$5=(-\frac{1}{3})(-2)+b$

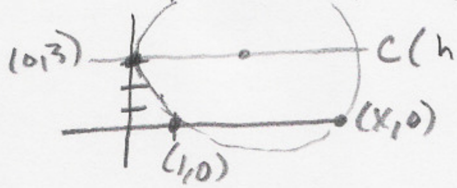
$5-\frac{2}{3}=\frac{13}{3}=b$

$\boxed{y=-\frac{1}{3}x+\frac{13}{3} \text{ or } x+3y=13}$

AYRFC [p.8]

(23) tan to y-axis @ $y=3$, $(1,0)$

a)



$(x-h)^2+(y-3)^2=r^2$

$(0,3)$: $(0-h)^2+(3-3)^2=r^2$

$\underline{h^2=r^2}$

$(1,0)$: $(1-h)^2+(0-3)^2=r^2$

$1-2h+h^2+9=r^2$

by sub: $1-2h+\cancel{h^2}+9=\cancel{h^2}$

$-2h=-10$

$\underline{h=5}$

x int: $y=0$: $(x-5)^2+(0-3)^2=5^2$

$(x-5)^2=25-9$

$x-5=\pm 4$

$\boxed{x=1,9}$

b) $\boxed{(x-5)^2+(y-3)^2=25}$

(24) $P(x,y)$: $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=D$

$\sqrt{(x--1)^2+(y-1)^2}=3\sqrt{(x-2)^2+(y--1)^2}$

$x^2+2x+1+y^2-2y+1=9(x^2-4x+4+y^2+2y+1)$

$\boxed{8x^2-38x+8y^2+20y+43=0}$ (a circle!)

(25) a) $f(x)=\dfrac{3x+1}{\sqrt{x^2+x-2}}$; Domain $x^2+x-2\neq 0$ and $\underline{x^2+x-2>0}$ (stronger argument)

$(x+2)(x-1)=0$

$x=-2, x=1$

test: $x=0$ $\quad -2\overset{?}{>}0$ No!

Domain: $x<-2$ or $x>1$

-or-

$(-\infty,-2)\cup(1,\infty)$

b) i) $f(x)=7$; $D: \mathbb{R}$, $R: \{y \mid y=7\}$

-or-

$D: (-\infty,\infty)$, $R: \{7\}$

ii) $g(x)=\dfrac{5x-3}{2x+1}$, $2x+1\neq 0$

$2x\neq -1$

$x\neq -\frac{1}{2}$

Horizontal Asymptote @ $y=\frac{5}{2}=\lim\limits_{x\to\infty} g(x)$

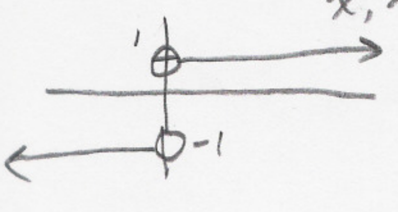
So $D: \{x \mid x\neq -\frac{1}{2}\}$, $R: \{y \mid y\neq \frac{5}{2}\}$

or

$D: (-\infty,-\frac{1}{2})\cup(-\frac{1}{2},\infty)$

$R: (-\infty,\frac{5}{2})\cup(\frac{5}{2},\infty)$

(26) $f(x)=\dfrac{|x|}{x}=\begin{cases}\dfrac{x}{x}, & x>0\\ \dfrac{-x}{x}, & x<0\end{cases}=\begin{cases}1, & x>0\\ -1, & x<0\end{cases}$



$D: \{x \mid x\neq 0\}$

$R: \{y \mid y=\pm 1\}$

27) $\dfrac{f(x+h)-f(x)}{h}$

a) $f(x)=2x+3$: $\dfrac{(2(x+h)+3)-(2x+3)}{h} = \dfrac{\cancel{2x}+2h+\cancel{3}-\cancel{2x}-\cancel{3}}{h}$

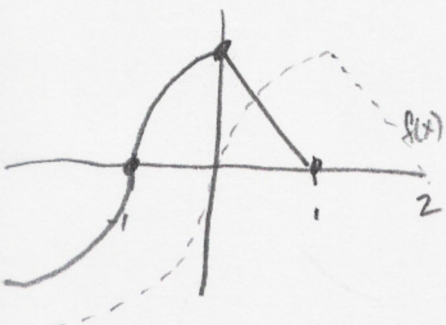
$= \dfrac{2h}{h} = \boxed{2}$

b) $f(x)=\dfrac{1}{x+1}$: $\dfrac{\frac{1}{x+h+1}-\frac{1}{x+1}}{h}\cdot\dfrac{(x+h+1)(x+1)}{(x+h+1)(x+1)}$

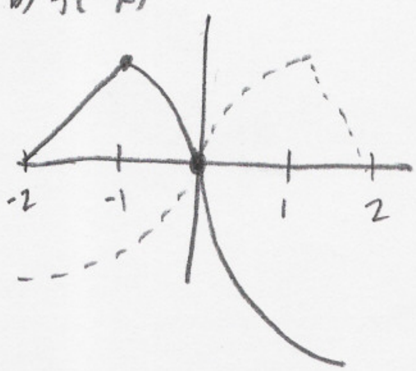
$= \dfrac{(x+1)-(x+h+1)}{h(x+h+1)(x+1)} = \dfrac{\cancel{x}+\cancel{1}-\cancel{x}-h-\cancel{1}}{h(x+h+1)(x+1)} = \dfrac{-h}{h(x+h+1)(x+1)} = \boxed{\dfrac{-1}{(x+h+1)(x+1)}}$

c) $f(x)=x^2$: $\dfrac{(x+h)^2-x^2}{h} = \dfrac{\cancel{x^2}+2xh+h^2-\cancel{x^2}}{h} = \boxed{2x+h}$

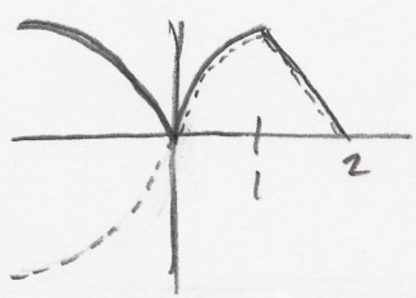
28) a) $f(x+1)$



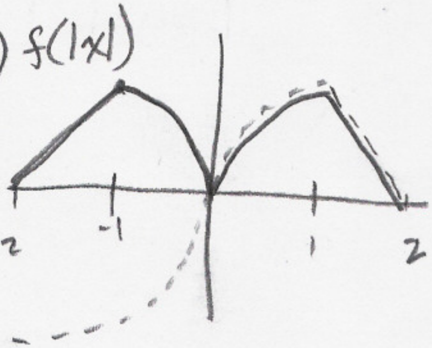
b) $f(-x)$



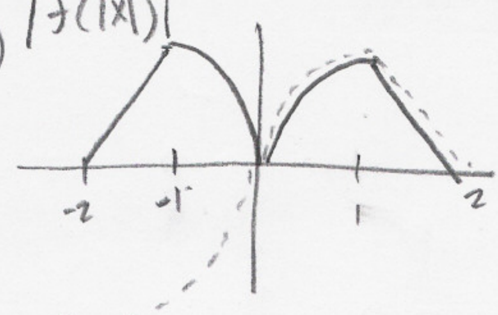
c) $|f(x)|$



d) $f(|x|)$

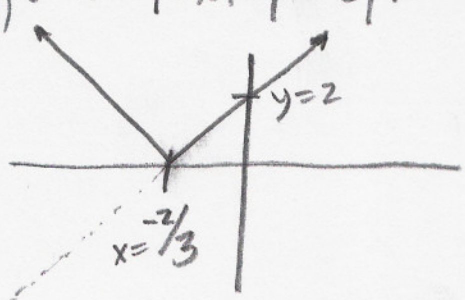


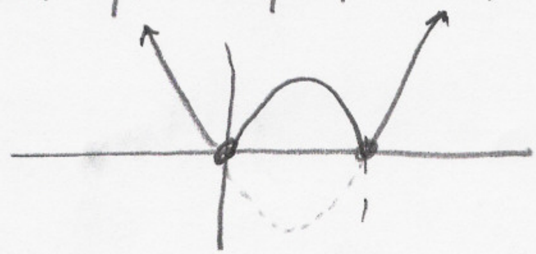
e) $|f(|x|)|$



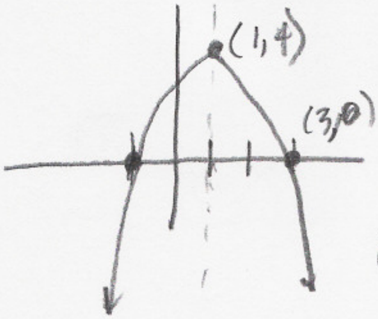
Same as d)

29) a) $g(x)=|3x+2| = 3\left|x+\frac{2}{3}\right|$ b) $h(x)=|x(x-1)| = |x^2-x|$





30 a) x int: $(-1,0)\,(3,0)$, R: $\{y \mid y \le 4\}$



$$(y-k) = \frac{1}{4p}(x-h)^2$$

$$y-4 = \frac{1}{4p}(x-1)^2$$

$$(3,0):\ -4 = \frac{1}{4p}(4) \rightarrow -4 = \frac{1}{p} \rightarrow p = -\frac{1}{4}$$

$$y-4 = -(x-1)^2 \rightarrow y-4 = -x^2+2x-1$$

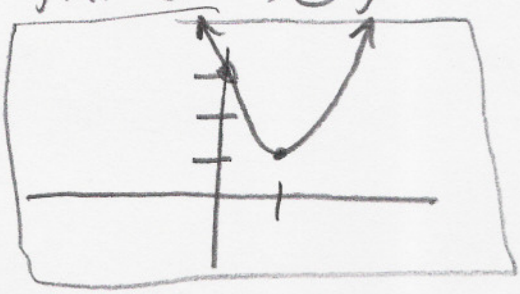
$$\boxed{y = -x^2+2x+3}$$

b) $y = 2x^2-4x+3$

y int: $(x=0)$ @ $y=3$

x coord Vertex $= \frac{-b}{2a} = \frac{4}{4} = 1$

Vertex $(1,1)$



31 a) $\begin{cases} x = t+1 \\ y = t^2-t \end{cases} \xrightarrow{\text{sub}} t = x-1$ so $y = (x-1)^2-(x-1) \rightarrow y = x^2-2x+1-x+1$

$$\boxed{y = x^2-3x+2}$$

b) $\begin{cases} x = \sqrt[3]{t} - 1 \\ y = t^2-t \end{cases} \rightarrow t = (x+1)^3$ so $\boxed{y = (x+1)^6-(x+1)^3}$

c) $\begin{cases} x = \sin t \\ y = \cos t \end{cases} \rightarrow \cos^2 x + \sin^2 x = 1$ so $\boxed{x^2+y^2=1}$

32 a) $f(x) = 2x+3$

$y = 2x+3$

$x = 2y+3$

$2y = x-3$

$y = \frac{1}{2}x - \frac{3}{2}$

$\boxed{f^{-1}(x) = \frac{1}{2}x - \frac{3}{2}}$

b) $f(x) = \frac{x+2}{5x-1}$

$x = \frac{y+2}{5y-1}$

$5xy - x = y+2$

$y(5x-1) = x+2$

$\boxed{f^{-1}(x) = \frac{x+2}{5x-1}}$

c) $f(x) = x^2+2x-1$, $x>0$ $(y>-1)$

$y = (x^2+2x+1)-1-1$

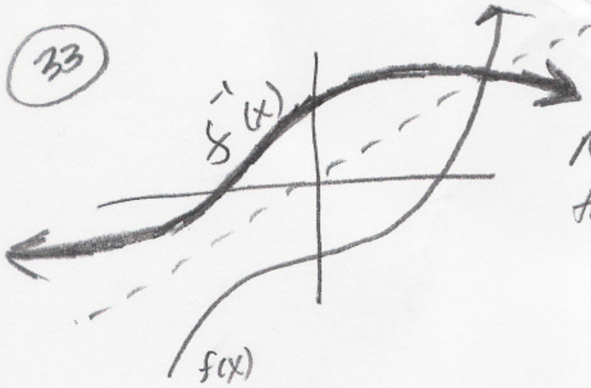
$y = (x+1)^2-2 \rightarrow x = (y+1)^2-2$

$x+2 = (y+1)^2$

$y+1 = \sqrt{x+2}$

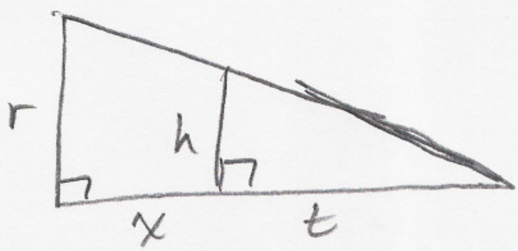
$\boxed{f^{-1}(x) = -1+\sqrt{x+2},\ x>-1}$

33



Reflect across line $y=x$ for graph of inverse

34 a)

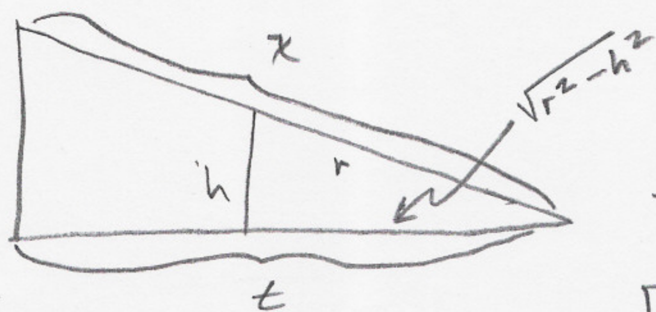


by similar triangles

$$\frac{r}{h} = \frac{x+t}{t} \rightarrow x+t = \frac{rt}{h}$$

$$\boxed{x = \frac{rt}{h} - t \quad \text{or} \quad x = \frac{t(r-h)}{h}}$$

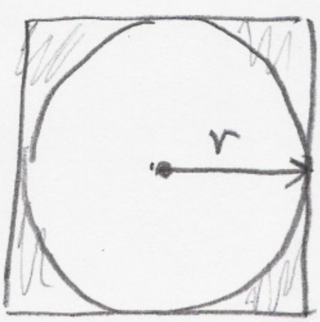
b)



$$\frac{x}{t} = \frac{r}{\sqrt{r^2-h^2}}$$

$$\boxed{x = \frac{rt}{\sqrt{r^2-h^2}}}$$

35 a)



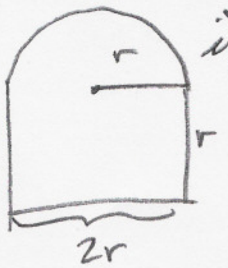
$A_{square} = (2r)^2 = 4r^2$

$A_{circle} = \pi r^2$

$A_{\blacksquare} = A_{sq.} - A_{cir} = 4r^2 - \pi r^2$

$$\text{Ratio} = \frac{4r^2 - \pi r^2}{4r^2} = \boxed{1 - \frac{\pi}{4}}$$

b)



i) $P = 2r + r + r + \frac{2\pi r}{2}$

$\boxed{P = 4r + \pi r}$

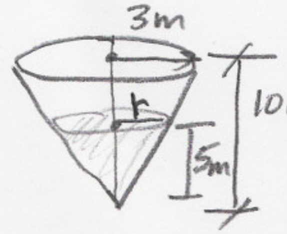
ii) $A = r(2r) + \frac{\pi r^2}{2}$

$= 2r^2 + \frac{\pi r^2}{2}$

$= \frac{4r^2 + \pi r^2}{2}$

$\boxed{A = \frac{r^2}{2}(4+\pi)}$

(35) c)

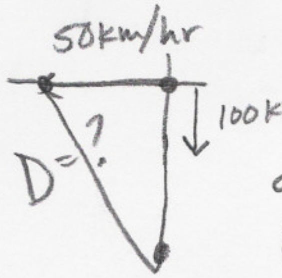


$SA = \text{circle} = \pi r^2$

by similar triangles: $\frac{r}{3} = \frac{5}{10} \rightarrow r = 1.5\text{m} = \frac{3}{2}\text{m}$

$SA = \pi\left(\frac{3}{2}\right)^2 = \boxed{\frac{9\pi}{4}\ \text{m}^2}$

d)

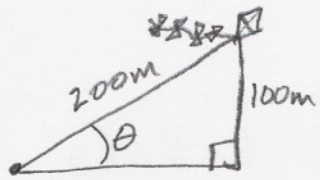


after 2 hrs, they have travelled $100(2) = 200\text{km}$ and $50(2) = 100\text{km}$

by Pythag. thm: $D = \sqrt{200^2 + 100^2} = \sqrt{40000 + 10000}$

$= \sqrt{50000} = \boxed{100\sqrt{5}\ \text{km}}$

e)



$\sin\theta = \frac{\text{opp}}{\text{hyp}} \rightarrow \sin\theta = \frac{100}{200} = \frac{1}{2}$

$\boxed{\theta = \pi/6 \text{ or } 30^\circ}$

(36) a) $\sin^2 x + \cos^2 x = 1$: from b) + d) and $y = 0$ (working on left side)

$(\sin x \cos y + \cos x \sin y)^2 + (\cos x \cos y - \sin x \sin y)^2$, Let $y = 0$

$(\sin(x)(1) + \cos x(0))^2 + (\cos x(1) - \sin x(0))^2$

$\sin^2 x + \cos^2 x$

$\frac{1}{2}(1 - \cos 2x) + \frac{1}{2}(1 + \cos 2x)$

$\frac{1}{2} - \frac{1}{2}\cos 2x + \frac{1}{2} + \frac{1}{2}\cos 2x$

$\boxed{1}$ QED ("that which has been demonstrated")

b) from d) let $x = y$

$\sin(x+y) = \sin x \cos y + \cos x \sin y$

$\sin(x+x) = \sin x \cos x + \cos x \sin x$

$\boxed{\sin 2x = 2\sin x \cos x}$

c) from b) Let $x = y$

$\cos(x+y) = \cos x \cos y - \sin x \sin y$

$\cos(x+x) = \cos x \cdot \cos x - \sin x \sin x$

$\boxed{\cos 2x = \cos^2 x - \sin^2 x}$

d) from previous. let $\sin^2 x = 1 - \cos^2 x$

$\cos 2x = \cos^2 x - (1 - \cos^2 x)$

$= \cos^2 x - 1 + \cos^2 x$

$= \boxed{2\cos^2 x - 1}$

e) from previous c), let $\cos^2 x = 1 - \sin^2 x$

$\cos 2x = (1 - \sin^2 x) - \sin^2 x$

$= \boxed{1 - 2\sin^2 x}$

f) from previous, solve for $\sin^2 x$

g) from previous d), solve for $\cos^2 x$

(37) a) $\left(\frac{1}{2}\right)\left(\frac{5}{6}\right)\left(\frac{8}{5}\right) = \boxed{\frac{2}{3}}$

b) $\left(\frac{1}{2}\right)+\left(\frac{5}{6}\right)+\left(\frac{8}{5}\right) = \left(\frac{1}{2}\right)\left(\frac{15}{15}\right)+\frac{5}{6}\left(\frac{5}{5}\right)+\frac{8}{5}\left(\frac{6}{6}\right)$

$= \frac{15+25+48}{30} = \frac{88}{30} = \boxed{\frac{44}{15}}$

c) $\left(\frac{1}{2}\right)\div\left(\frac{5}{6}\right)\div\left(\frac{8}{5}\right) = \left(\frac{1}{2}\right)\left(\frac{6}{5}\right)\div\frac{8}{5} = \frac{3}{5}\cdot\frac{5}{8} = \boxed{\frac{3}{8}}$

(38) $F(x) = x^{1/2}x^{-2/3} = x^{-1/6}$, $F(2^6) = (2^6)^{-1/6} = 2^{-1} = \boxed{\frac{1}{2}}$

(39) x^2-2x+3; $x_{\text{coord of vertex}} = -\frac{b}{2a} = \frac{2}{2} = 1$ $V(1,2)$

Min value @ $x=1$ is $\boxed{y=2}$

(40) $\ln\left(\frac{e^3}{e^4}\right) = \ln e^{-1} = -1\ln e = (-1)(1) = \boxed{-1}$

(41) $\lim_{x\to 2}\frac{4-x^2}{x^2-3x+2} = \lim_{x\to 2}\frac{(2-x)(2+x)}{(x-1)(x-2)} = \lim_{x\to 2}\frac{2+x}{1-x} = \frac{4}{-1} = \boxed{-4}$

(42) a) $y = \frac{x}{x^2-1}$ (Replace x with $-x$) $\rightarrow \frac{-x}{(-x)^2-1} = \frac{-x}{x^2-1} = -\left(\frac{x}{x^2-1}\right) = -y$

$\boxed{\text{ODD function (origin symmetry)}}$

b) $y = x^3+2x$: $(-x)^3+2(-x) = -x^3-2x = -(x^3+2x) = -y$

$\boxed{\text{ODD function}}$

c) $y = x^3+2x+1$: $(-x)^3+2(-x)+1 = -x^3-2x+1 \rightarrow \boxed{\text{Neither}}$

d) $y = x^3+2x^2+x-1$: $-x^3+2x^2-x-1 \rightarrow \boxed{\text{Neither}}$

e) $y = x^4-x^2+1$; $(-x)^4-(-x)^2+1 = x^4-x^2+1 = y$

$\boxed{\text{EVEN function (y-axis symmetry)}}$

f) $y = \frac{x^2}{x^4-2x^2+1}$; $\frac{(-x)^2}{(-x)^4-2(-x)^2+1} = \frac{x^2}{x^4-2x^2+1} = y \rightarrow \boxed{\text{EVEN}}$

(42) g) $y = \frac{3x}{x^3-2x}$; $\frac{3(-x)}{(-x)^3-2(-x)} = \frac{-3x}{-x^3+2x} = \frac{-3x}{-(x^3-2x)} = \frac{3x}{x^3-2x} = y$

$\rightarrow$ $\boxed{\text{EVEN}}$

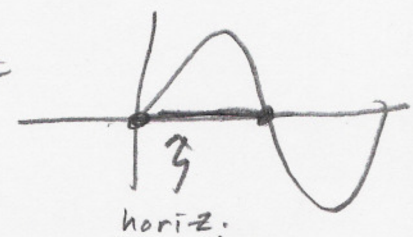
(43) $s(t) = 2\sin t, t \geq 0$ a) Avg vel $[0,5]$: $\frac{s(5)-s(0)}{5-0} = \frac{2\sin 5 - 2\sin 0}{5}$

$= \boxed{\frac{2}{5}\sin 5 \text{ ft/sec}} \approx -0.384$ ft/sec

radians

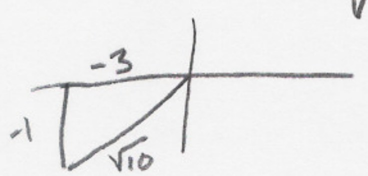
b) Avg velocity = slope of secant line. $s(t) = 2\sin t$

So we want slope to equal zero, i.e. a horizontal secant line. $\boxed{t = \pi \text{ sec}}$



(44) $\pi \text{ rad} \times \frac{180°}{\pi \text{ rad}} = \boxed{180°}$

(45) $\sin\theta = -\frac{1}{\sqrt{10}}$ and $\cos\theta < 0$



$\tan\theta = \frac{-1}{-3} = \boxed{\frac{1}{3}}$

(46) $\sin\left(\frac{\pi}{6}\right) + \cos\left(\frac{2\pi}{3}\right) = \frac{1}{2} + \left(-\frac{1}{2}\right) = \boxed{0}$

(47) $c = \frac{1}{3}, d = 5$, $\frac{c}{d-c} = \frac{1/3}{5-1/3} \cdot \frac{3}{3} = \frac{1}{15-1} = \boxed{\frac{1}{14}}$

(48) $\frac{3}{4}$ of $\frac{1}{5} = \frac{3}{4}\left(\frac{1}{5}\right) = \boxed{\frac{3}{20} = .15}$

(49) a) \$50(1.1) = \$55 b) \$55(1 − .1) = \$55(.9) = $\boxed{\$49.50}$

(50) $3\sqrt{3} - 2\sqrt{27} + 4\sqrt{12} = 3\sqrt{3} - 6\sqrt{3} + 8\sqrt{3} = \boxed{5\sqrt{3}}$

(51) a) $\sqrt{7x-11} = 0$

$7x - 11 = 0$

$\boxed{x = 11/7}$

test: $\sqrt{7(\frac{11}{7})-11} = 0$ ✓

b) $\sqrt{7x-11} = 6$

$7x - 11 = 36$

$7x = 47$

$\boxed{x = 47/7}$

test: ✓

c) $\sqrt{7x-11} = -11$

$7x - 11 = 121$

$7x = 132$

$x = 132/7$

test: X, doesn't work

$\boxed{\text{No Solution}}$

* d) $\sqrt{7x-11} + \sqrt{x} = 6$

$\sqrt{7x-11} = 6 - \sqrt{x}$

$7x - 11 = 36 - 12\sqrt{x} + x$

$6x - 47 = -12\sqrt{x}$

$36x^2 - 564x + 2209 = 144x$

$36x^2 - 708x + 2209 = 0$

$x = \frac{708 \pm \sqrt{708^2 - 4(36)(2209)}}{2(36)}$

$\rightarrow x = \frac{708 \pm \sqrt{183168}}{72}$

$x = 15.778$ or 3.889

of these 2, only

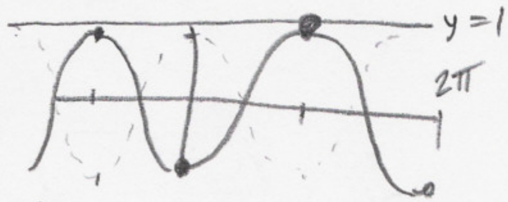
$\boxed{x = 3.889 \text{ works}}$

the other is extraneous

(52) $A = l \cdot w = \sqrt{12}(\sqrt{8} + \sqrt{18}) = 2\sqrt{3}(2\sqrt{2} + 3\sqrt{2})$

$= 4\sqrt{6} + 6\sqrt{6} = \boxed{10\sqrt{6}\ ft^2}$

(53) $\begin{cases} y = \sin^2 x + \cos^2 x \\ x + \pi = \cos^{-1} y \end{cases} \rightarrow \begin{cases} y = 1 \\ y = \cos(x + \pi) \end{cases}$



$x = \pi \rightarrow \boxed{(\pi, 1)}$

*be sure final answer is within orignal specified domain $[0, 2\pi)$

(54) a) i) $A = \{0, 1, 2, 3\}$ ii) $A = \{n \mid n < 4, n \in \mathbb{W}\}$

b) i) $F = \{15, 16, 17, \ldots\}$ ii) $F = \{n \mid n \geq 15, n \in \mathbb{Z}\}$

c) i) $N = \{-4, -3, -2, -1\}$ ii) $N = \{n \mid n > -5, n \in \mathbb{Z}^-\}$

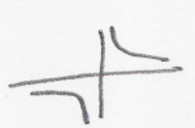
d) i) $Q =$ can't list all #s (impossible) ii) $Q = \{n \mid 4 < n < 8, n \in \mathbb{R}\}$

(55) a) $\boxed{(3,12]}$ b) $\boxed{[0,\infty)}$ c) $\boxed{(-\infty,0]}$ d) $\boxed{[-5,5]}$

e) $\boxed{\{\ \}}$ can't be <2 AND >3 simultaneously (or $\varnothing$)

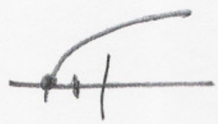
f) $\boxed{(-\infty,2)\cup(3,\infty)}$

(56) a) $f(x)=\frac{1}{x}$



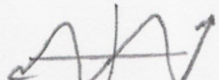
i) $\boxed{D: (-\infty,0)\cup(0,\infty) \text{ ii) } R: (-\infty,0)\cup(0,\infty)}$

b) $g(x)=\sqrt{x+2}$



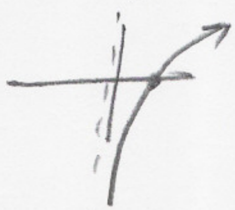
i) $\boxed{D: [-2,\infty) \text{ ii) } R: [0,\infty)}$

c) $s(\theta)=\sin\theta$



i) $\boxed{D: (-\infty,\infty) \text{ ii) } R: [-1,1]}$

d) $t(x)=\ln x$



i) $\boxed{D: (0,\infty) \text{ ii) } R: (-\infty,\infty)}$

(57) $f(x)=x-2$, $g(x)=5x+3$ a) $f(g(2))=f(13)=\boxed{11}$

b) $g(f(-4))=g(-6)=\boxed{-27}$ c) $f(f(1))=f(-1)=\boxed{-3}$

d) $f(g(x))=(5x+3)-2=\boxed{5x+1}$

(58) $f(x)=[x]$ (Greatest Integer Function/Floor Function)

$g(x)=2x$, $h(x)=\frac{2}{x}$

a) $f(g(-3.1))=f(-6.2)=\boxed{-7}$ b) $f(g(x))=\boxed{[2x]}$

c) $f(h(x))=\boxed{\left[\frac{2}{x}\right]}$ d) $h(f(x))=\boxed{\frac{2}{[x]}}$ e) $f(g(h(x)))=f\left(\frac{4}{x}\right)=\boxed{\left[\frac{4}{x}\right]}$

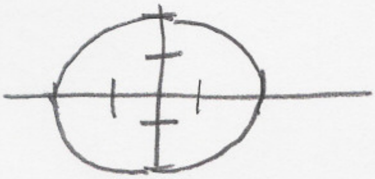
(59) a) $\sum_{n=0}^{4}\frac{n^2}{2}=0+\frac{1}{2}+2+\frac{9}{2}+8=\boxed{15}$ b) $\sum_{n=1}^{3}\frac{1}{n^3}=1+\frac{1}{8}+\frac{1}{27}=\boxed{\frac{251}{216}}$

(60) $\vec{v} = -2\vec{i} + 5\vec{j} \quad \vec{w} = 3\vec{i} + 4\vec{j}$

a) $\frac{1}{2}\vec{v} = \boxed{-\vec{i} + \frac{5}{2}\vec{j}}$ b) $\vec{w} - \vec{v} = \boxed{5\vec{i} - \vec{j}}$ c) $\|\vec{w}\| = \sqrt{9+16} = 5$

d) $\|\vec{v}\| = \sqrt{4+25} = \sqrt{29}$ $\boxed{\vec{u}_v = -\frac{2}{\sqrt{29}}\vec{i} + \frac{5}{\sqrt{29}}\vec{j}}$

(61) a) $r = 2$



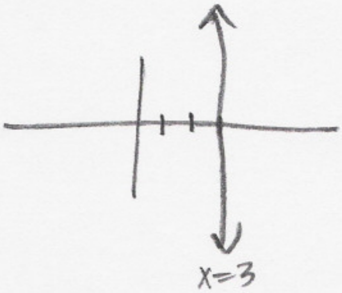
Circle w/radius = 2

b) $r = 3\sec\theta$

$x = r\cos\theta$

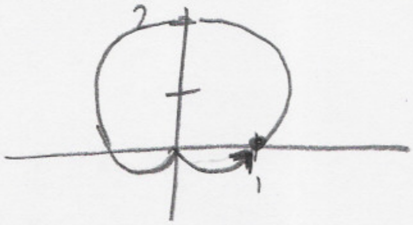
$x = (3\sec\theta)\cos\theta$

$x = 3$



c) $r = 1 + \sin\theta$

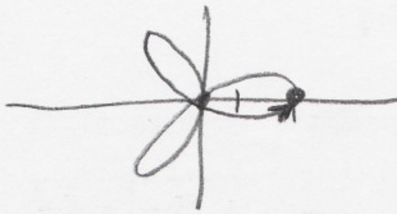
θ	r
0	1
$\frac{\pi}{6}$	1.5
$\frac{\pi}{2}$	2
π	1
$\frac{3\pi}{2}$	0
2π	1



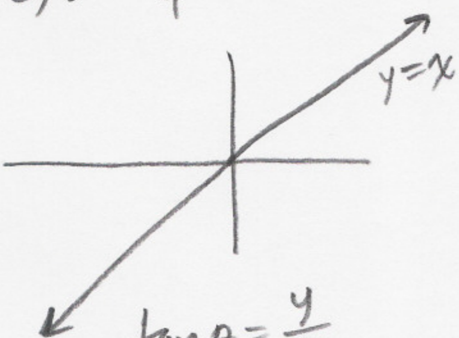
cardiod

d) $r = 2\cos 3\theta$

θ	r
0	2
$\frac{\pi}{6}$	0
$\frac{\pi}{3}$	-2
$\frac{\pi}{2}$	0
$\frac{2\pi}{3}$	2
$\frac{5\pi}{6}$	0



e) $\theta = \frac{\pi}{4}$



$\tan\theta = \frac{y}{x}$

$\tan\frac{\pi}{4} = 1 = \frac{y}{x}$

$y = x$

Done!