WS 9—Skills 36-40

Directions: For this section, solve each problem and decide which is the best of the choices given. Circle the corresponding capital letter. You may use any available space for scratchwork.

Notes:
1. The use of a calculator is permitted.
2. All numbers used are real numbers.
3. Figures that accompany problems in this test are intended to provide information useful in solving the problems. They are drawn as accurately as possible EXCEPT when it is stated in a specific problem that the figure is not drawn to scale. All figures lie in a plane unless otherwise indicated.
4. Unless otherwise specified, the domain of any function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

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1. If exactly two of the three integers a, b, and c are even, which of the following must be odd?

    I. $a+b+c$
    II. $ab+c$
    III. $ab(2c)$

(A) I only
(B) II only
(C) I and II only
(D) I and III only
(E) I, II, and III

2. If n is a positive integer such that n^2 is odd, then which of the following must be an odd integer?

(A) $\dfrac{n}{2}$

(B) $2n+n$

(C) $2(n+1)$

(D) $\dfrac{n+3}{2}$

(E) $(n+1)(n-1)$

3. If a is an odd integer, which of the following is an even integer?

(A) $a-2$
(B) a^2
(C) a^2-2
(D) $(a-2)^2$
(E) a^2-a

4. If a and b are even integers, which of the following must be even?

    I. ab
    II. $a+b$
    III. $a(a^2-1)$

(A) I only
(B) II only
(C) III only
(D) I and II only
(E) I, II, and III

5. If $p+q$ is an even integer, which of the following must be even?

(A) pq
(B) $2p+q$
(C) $(p+1)(q+1)$

(D) $\dfrac{p}{q}$

(E) p^2-q^2

6. If k is a positive even integer, then $k(k+1)(k+2)$ could equal which of the following?

(A) 48
(B) 50
(C) 60
(D) 120
(E) 210

7. If $a < b$, which of the following must be true?
   (A) $b < 0$
   (B) $a > 0$
   (C) $ab > 0$
   (D) $ab < 0$
   (E) $a - b < 3$

8. If $a > b > 0$, which of the following must be greater than $\dfrac{a}{b}$?
   (A) 1
   (B) $\dfrac{b}{a}$
   (C) $a - b$
   (D) $\dfrac{a}{2b}$
   (E) $\dfrac{2a}{b}$

9. If $s^3 t^4 u^3 w > 0$ and $w < 0$, which of the following must be true?
   (A) $s > 0$
   (B) $u < 0$
   (C) $su > 0$
   (D) $su < 0$
   (E) $t > 0$

10. If $b > a > 1$, which of the following must be true?
   (A) $\dfrac{1}{a} < \dfrac{1}{b}$
   (B) $a - b > 0$
   (C) $b^2 < ab$
   (D) $a + 2b > 3b$
   (E) $a^2 < b^2$
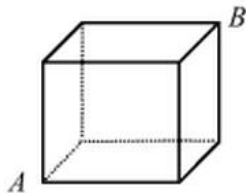
11. In the figure above, if the volume of the cube is 64, what is the length of $\overline{AB}$ (not shown)?
   (A) 4
   (B) $4\sqrt{2}$
   (C) $4\sqrt{3}$
   (D) 8
   (E) 12

12. If the surface area of a cube is 96, what is the volume of the cube?
   (A) 8
   (B) 27
   (C) 64
   (D) 81
   (E) 125
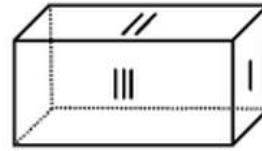
Note: figure not drawn to scale.

13. In the rectangular solid above, the area of region I (side) is 8, the area of region II (top) is 10, and the area of region III (front) is 20. What is the volume of the solid?
   (A) 40
   (B) 60
   (C) 80
   (D) 100
   (E) 200
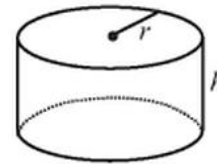
14. The cylinder shown above has a radius of r and a height of h. If $r = h$, what is the surface area of the cylinder?
   (A) $2\pi r^2$
   (B) $2\pi r^3$
   (C) $4\pi r^2$
   (D) $4\pi r^3$
   (E) $2\pi r^2 + \pi r$

$$-2, -1, 0, 1, 2$$

15. A sequence is formed by repeating the five numbers above in the same order indefinitely. What is the sum of the first 124 terms of the sequence?
   (A) -2
   (B) 0
   (C) 4
   (D) 124
   (E) 248

$$9, 27, 81, 243, \ldots$$

16. In the sequence above, the first term is 9 and each term after the first is 3 times the term before it. Which of the following is the expression for the 300th term of the sequence?
(A) $3(299)$
(B) $3(300)$
(C) 3^{299}
(D) 3^{300}
(E) 3^{301}

$$5, -10, 20, \ldots$$

17. In the geometric sequence above, what is the sum of the first eight terms of the sequence?
(A) -425
(B) -160
(C) 120
(D) 160
(E) 425

18. If a certain salesman's salary increases 10% each year, approximately what is the percent increase in salary after 4 years?
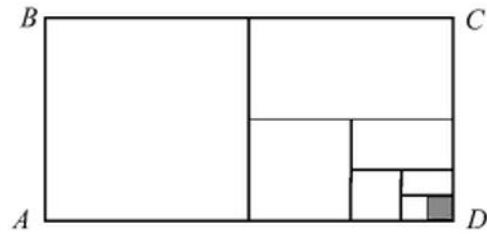(A) 40
(B) 44
(C) 46
(D) 50
(E) 146

19. Assume a ball bounces to a height of $\frac{3}{5}$ of the height from which it falls. If the ball is dropped from a height of 30 feet, after which bounce will the rebounded height be less than 4 feet?
(A) 3
(B) 4
(C) 5
(D) 6
(E) 7

$$a, 4a, 16a, 64a, \ldots$$

20. In the geometric sequence above, the first term is a, and the sum of the first 6 terms is 4095. What is the value of a?
(A) 3
(B) 4
(C) 5
(D) 6
(E) 9

21. In the rectangle above, $BC = 2AB$, and it has been repeatedly divided in half resulting in the figure. What fraction of the area of rectangle $ABCD$ is the small shaded square?
(A) $\dfrac{1}{16}$
(B) $\dfrac{1}{32}$
(C) $\dfrac{1}{64}$
(D) $\dfrac{1}{128}$
(E) $\dfrac{1}{256}$

22. Le the operation $\odot$ be defined for all numbers by $a \odot b = \dfrac{a+b}{a-b}$. If $p \odot q = 3$, what is the value of $\dfrac{p}{q}$?
(A) $\dfrac{1}{2}$
(B) 1
(C) $\dfrac{3}{2}$
(D) 2
(E) $\dfrac{5}{2}$

23. Let the operation Δ be defined by $a \Delta b = \dfrac{a}{b}$ for all positive numbers. If $4\Delta(k\Delta 6) = 3$, what is the value of k?
(A) 4
(B) 8
(C) 12
(D) 20
(E) 36

24. Let the operation

$n^{\Theta} = n(n-1)(n-2)(n-3)\cdots(2)(1)$, where n is a positive integer. Which of the following is equivalent to $(n+1)^{\Theta}$?

(A) $n(n^{\Theta})$

(B) $(n+1)(n+1)^{\Theta}$

(C) $n(n-1)^{\Theta}$

(D) $(n+1)n^{\Theta}$

(E) $(n+1)(n-1)^{\Theta}$

Problem Number	Correct Answer	Skill Number
1	A	36
2	B	36
3	E	36
4	E	36
5	E	36
6	D	36
7	E	37
8	E	37
9	D	37
10	E	37
11	C	38
12	C	38
13	A	38
14	C	38
15	A	39
16	E	39
17	A	39
18	C	39
19	B	39
20	A	39
21	D	39
22	D	40
23	B	40
24	D	40