Lesson 12—Traps & Strategies

The Top 10 Most Common Math Traps (and how to avoid them)

1. Percent Increase/Decrease

Tate bought a new car in 2000. Three years later, he sold it to a dealer for 40% less than he paid for it in 2000. The dealer then added 20 percent onto the price he paid and resold it to another customer. The price the final customer paid for the care was what percent of the original price?

(A) 40% (B) 60% (C) 72% (D) 80% (E) 88%

Did You Take the Bait?

You might think the answer is (D), 80% . Afterall, $100\% - 40\% + 20\% = 80\%$.

The Trap?

You cannot blindly add/subtract percentages!

How to Avoid the Trap.

1. Assume the original price was \$100. 100 is a very nice and easy number to use when working with percentages.
2. Calculate how the price increases and decreases throughout the problem using 100 to start with.
3. Because you used 100 as your original value, it's easy to see the percent change.

Solving It!

Tate paid: \$100 in 2000

Tate sold the car: $\$100 - \$40 = \$60$

Dealer charges 20% more: $\$60 + 0.20 \times \$60 = \$60 + \$12 = \$72$

Therefore, the final price was 72% of the orignal price, choice (C)

2. **Weighted Averages**

In a class of 27 math students, the average score of the male students on the final exam was 83. If the average score of the 15 female students in the class was 92, what was the average of the whole class?

(A) 86.2 (B) 87 (C) 87.5 (D) 88 (E) 88.2

Did You Take the Bait?

You might think the answer is (C), 87.5 since $\frac{83+92}{2} = 87.5$.

The Trap?

You cannot combine averages of groups with different sizes.

How to Avoid the Trap.

1. Never assume that the average of the total will be the averages of the groups' averages, especially on harder questions.

2. There are 12 males and 15 females in the class, so the average of the females' scores is "worth more."
3. Remember the formula for averages: $\dfrac{\text{Sum of total}}{\text{Number of values}}$

Solving It!
Eliminate answers that are not possible. Since there are more females (15) than males (12), the females have to contribute more to the average, so (A) and (B) are wrong since they are below 87.5. You've got a 50/50 shot now at simply guessing, but let's not.

$$\text{Total average} = \frac{15\times 92 + 12\times 83}{27} = 88$$

USE YOUR CALCULATOR!

3. <u>Ratio:Ratio:Ratio</u>
Jenna's coin collection consists of quarters, dimes, and nickels. If the ratio of the number of quarters to the number of dimes is 5:2, and the ratio of the number of dimes to the number of nickels is 3:4, what is the ratio of the number of quarters to the number of nickels?

(A) 5:4 (B) 7:5 (C) 10:6 (D) 12:7 (E) 15:8

Did You Take the Bait?
I bet you're thinking (A), 5:4, right?!

The Trap?
Parts of different ratios don't always refer to the same whole.

How to Avoid the Trap.
1. Remember that the ratios may refer to different wholes.
2. Figure out what the "whole" is going to be.
3. Or, assign numbers to the ratios and then solve.

Solving It!
Restate the ratios so that the same number refers to the same quantity. Make sure that the common quanitity in both ratios has the same number in both, in this case, dimes.

Quarters:**Dimes** = 5;2 = 15:**6**
Dimes:Nickels = 3:4 = **6**:8

So the final ratio is 15:6:8, so Quarters to Nickels is 15:8, choice (E).

4. "<u>Least" and "Greatest"</u>
What is the least positive integer that is divisible by both 2 and 5 and leaves a remainder of 3 when divided by 11?

(A) 30 (B) 32 (C) 33 (D) 70 (E) 80

Did You Take the Bait?
You're thinking (A), 30, I bet.

The Trap?
The least is not always the smallest number, nor is it always the first one that satisfies part of the requirements. In fact, when 30 is divided by 11, the remainder is 8, not 3, even though 8 + 3 = 11.

How to Avoid the Trap.
1. When you are asked for the least, minimum, or smallest value, the smallest answer choice is rarely right.
2. You actually have to do the work, and test ALL the requirements! Don't just go with a guess.
3. Plugging in your choices is a great strategy for these questions.

Solving It!
If a number is divisible by 2 and 5, then it is a multiple of 10 also, so eliminate (B) and (C).
If it leaves a remainder of 3 when divided by 11, it has to be a multiple of 11 plus 3 more, so (E) is correct.
*Notice how, in this case, the largest number was correct for a "least" question.

5. <u>Percent "of" vs. Percent "Less" or Percent "Greater"</u>

What number is $33\frac{1}{3}\%$ less than 9? (Grid-in question)

Did You Fall in the Trap?
If you gridded-in "3," you sure did.

The Trap?
Reading the question too quickly or without care. This almost always leads to a mistake. Always pay attention, especially in percentage questions.

How to Avoid the Trap.
1. If you read to fast, you might think the question asks for $33\frac{1}{3}\%$ "of" 9.
2. Pay attention to the wording in the question. Is it asking for
   a) x percent of y
   b) x percent less than y
   c) x percent greater than y

Solving It!

$33\frac{1}{3}\% = \frac{1}{3}$

$9 \times \frac{1}{3} = 3$

$9 - 3 = 6$

6. <u>Ratio vs. Quantity</u>
The ratio of two quantities is 3:4. If each of the quantities is increased by 1, what is the ratio of these two quantities?

(A) $\frac{9}{16}$ (B) $\frac{2}{3}$ (C) $\frac{3}{4}$ (D) $\frac{4}{5}$ (E) It cannot be determined from information given

Did You Take the Bait?

$(3+1):(4+1)=4:5$, so it's got to be (D), $\frac{4}{5}$, right?!?

The Trap?

You cannot simply add and subtract from both parts. You CAN, however, multiply and divide.

How to Avoid the Trap.

1. Don't take the easy route; it's fraught with inaccuracy.
2. The question said each quantity increased by one. It said nothing about the ratios increasing by one.
3. Try subbing actual numbers into the question and see what happens to the ratio.

Solving It!

Picking some numbers and try more than one scenario,

- 3 women and 4 men $(3:4)\Rightarrow$ 4 women and 5 men $(4:5)$
- 9 women and 12 men $(3:4)\Rightarrow$ 10 women and 13 men $(10:13)$
- 30 women and 40 men $(3:4)\Rightarrow$ 31 women and 41 men $(31:41)$
- Obviously $(4:5)\neq(10:13)\neq(31:41)$
- The answer is (E). The ratios will depend on the numbers you substitute. There is no definite answer.

7. <u>Not all numbers are positive integers</u>

If $n \neq 0$, then which of the following must be true?

I. $n^2 > n$
II. $2n > n$
III. $n+1 > n$

(A) I only (B) II only (C) III only (D) I and III only (E) I, II, and III

Did You Take the Bait?

Do you think (E) is the right answer? I, II, and III are all correct??

The Trap?

Not all numbers are positive integers. Don't forget the negative numbers, fractions, and irrational numbers.

How to Avoid the Trap.

1. Don't assume that a number is a positive integer unless the question specifically says so. There are many other Real numbers.
2. Always choose a variety of numbers like negative numbers, fractional numbers & irrational numbers like $\sqrt{2}$ and π. Explore all possibilities before selecting a final answer.

Solving It!

- For positive numbers: I, II, and III are all true
- For a fraction between 0 and 1, like $\frac{1}{2}:\left(\frac{1}{2}\right)^2=\frac{1}{4}<\frac{1}{2}$, so **I is not true.**

- For a negative number, like -3: $2\times(-3)=-6<-3$, so **II is not true.**
- By process of elimination, the correct answer is (C), III only.

8. <u>Hidden Instructions</u>

At a certain restaurant, the hourly wage for a waiter is 20% more than the hourly wage for a dishwasher, and the hourly wage for a dishwasher is half as much as the hourly wage for a cook's assistant. If a cook's assistant earns $8.50 an hour, how much less than a cook's assistant does a waiter earn each hour?

(A) $2.55 (B) $3.40 (C) $4.25 (D) $5.10 (E) $5.95

Did You Take the Bait?

You want to say $5.10, choice (D), right?

Dishwasher gets: $\left(\frac{1}{2}\right)(\$8.50)=\$4.25$ per hour

Waiter gets 20% more: $4.25\times1.25=\$5.10$ per hour . . . WRONG!

The Trap?

Not reading and/or processing the simple little word "less." It's too easy to go for the first recognizable answer without actually answering the question.

How to Avoid the Trap.

1. Read and reread the question carefully.
2. Annotate the question: underline, circle, cross out, etc.
3. Begin with the end in mind. "What is the question actually asking?"

Solving It!

Same as the above calculation to get the Waiter's wage, but then subtract it from the cook's assistant's wage: $\$8.50-\$5.10=\$3.40$, choice (B).

9. <u>Average Rates</u>

A car travels from City *A* to City *B* at an average speed of 40 mph and then immediately returns from City *B* to City *A* at an average speed of 60 mph. What was the car's average speed (in mph) for the round trip?

(A) 45 (B) 48 (C) 50 (D) 52 (E) 54

Did You Take the Bait?

Obviously, $\frac{40+60}{2}=50$, so the answer must be 50??? NO!

The Trap?

To get the average speed, you don't just average the speeds, because the car spent more time going from *A* to *B* than from *B* to *A*, so that leg of the trip counts more, or carries more weight.

How to Avoid the Trap.

Look out for these weighted questions. Since the car spent more time from *A* to *B*, answer will be less than 50 (the average of the averages), giving you an immediate option of either (A) or (B).

Solving It!

Pick a number: Let the one-way Distance $=120$ miles The LCM of 40 and 60). Round trip $=240$

- Average speed $=\dfrac{\text{Total distance}}{\text{Total time}}$
- From A to B: $\dfrac{120}{40}=3$ hours
- From B to A: $\dfrac{120}{60}=2$ hours
- Round trip: $\dfrac{240}{5}=48$ mph

10. <u>Counting Numbers</u>

The tickets for a certain raffle are consecutively numbered. If Jack sold the tickets numbered from 75 to 148 inclusive, how many raffle tickets did he sell?

(A) 71 (B) 72 (C) 73 (D) 74 (E) 75

Did You Take the Bait?

$148-75=73$, choice in"C"corret.

The Trap?

Be careful with the word "inclusive."

How to Avoid the Trap.

Realize that subtracting gives the number of tickets strictly **between** 75 and 148, namely ticket numbers 76, 77, 78, . . . , 147, and 148. He sold the 75th ticket, though, so you've got to add it in.

Solving It!

$148-75+1=73+1=74$, choice (D).

Your goal is to recognize these traps (and others) and NOT fall into them!

Here are some general test-taking strategies.

1. Stop rushing.
2. Do not get hung up on one question. If after 30 seconds, you don't see the insight, mark it in the test booklet and come back to it.
3. If you're stuck, either 1) work with your answer choices or 2) plug in real numbers. Remember you can usually eliminate one or two answer choices immediately. Make educated guesses if you're not sure. You'll probably come out better in the long run.
4. Learn SAT specific math strategies:
5. Practice solving problems in several different ways:
   a) Using an SAT specific strategy
   b) The quickest way you can think of
   c) The way you would do it in school
   d) The easiest way for you

   During the actual exam you should try to solve each problem using one method, and then use a second method later on when "checking over" your answer. (see #10 below)
6. Practice 10 to 20 minutes per day.

7. Do not dismiss errors as "careless" when practicing:
   It is important that you wait at least a few days before reattempting a problem you got wrong - Redoing problems you get wrong is extremely important - more important than attempting new problems. This is what separates students that show dramatic improvement from students that show only average improvement. Remember, you learn from your mistakes, not your successes!
8. Make sure you have answered what the question was asking for.
9. Do not go with your instinct on hard questions.
10. "Check" your answers the correct way: If you are pacing yourself properly during the exam, you should wind up with about 5 to 7 minutes to check over your answers. So what is the best way to do this?
    DO NOT simply look over your work. Start the test over and redo each question from the beginning without looking at your prior work. Ideally you should try to use a different method than you used the first time (See (4) above).
    If you picked numbers the first time, then at least pick new numbers. If you can't think of a different way to solve it, that's okay. Just do it again. Then compare your two answers. If they are the same, move on. If not, then take a little time to catch your careless mistake.
11. Use your TI calculator effectively (and as little as possible).
12. Have Fun!