Lesson 11—Skills 46-50

Skill 46: Average Speed

Average speed is the total distance travelled by the total time taken.

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

and

$$\text{Distance} = \text{Rate} \times \text{Time} \quad \text{so} \quad \text{Time} = \frac{\text{Distance}}{\text{Rate}}$$

It's helpful in these problems to let the one-way distance be anything, say D, so that the total two-way trip is $2D$.

Example 46:

(a) If you travel from city A to city B at 40 miles per hour, and then you travel back at 50 miles per hour, what is the average speed for the whole trip?

(b) If you travel from city A to city B in 6 hours, but in the first two hours you drove a constant speed of 50 miles per hour, and in the last 4 hours you kept your speed at 60 miles per hour, what is the average speed of your trip?

Skill 47: Factoring

Factoring is to write an expression as a product of factors.
For SAT questions, the following factorings are needed

- $a^2 + 2ab + b^2 = (a+b)^2$
- $a^2 - 2ab + b^2 = (a-b)^2$
- $a^2 - b^2 = (a+b)(a-b)$
- $a^2 - 2a - 3 = (a-3)(a+1)$ *or similar "target sum/target product" problem

Example 47:

(a) If $(2x-8)(3x+5)=a$, then $(12-3x)(15x+25)=$ what?

(b) If $x^2-y^2=24$, where x and y are positive integers and $x>y$, what is one possible value of x?

Skill 48: Prime and Divisibility

To determine if a number is prime or composite
1. Find all the factors of the number.
2. If the number has only two factors, 1 and itself, then it is prime.
3. If the number as more than two factors, then it is composite.

A number x is divisible by another number y, if y is a factor of x. That is $\frac{x}{y}$ is an integer or $x \div y$ has no remainder.

Example 48:

(a) Determine if 323323 is divisible by 2, 3, & 5.

(b) If a number n is divisible by 3, 4, and 7, which of the following is also divisible by these numbers?

i) $n+21$

ii) $n+84$

iii) $21n$

iv) $6n$

v) $6n+252$

Skill 49: Rate of Work

Let's assume we have two workers: A and B.

1) Worker A can finish **1** job in a hours when working alone at a rate of $\frac{1}{a}$.

2) Worker B can finish **1** job in b hours when working alone at a rate of $\frac{1}{b}$.

If two workers are working together, the number of hours they need to complete the job is given by

Worker	**Rate**	**Combined Rate**	**Combined Time**
A	$\frac{1}{a}$	$\frac{1}{a}+\frac{1}{b}$	$\frac{1}{\frac{1}{a}+\frac{1}{b}}$
B	$\frac{1}{b}$		

For these types of problems where **1** job is done and to be done, **Rate** and **Time** are reciprocals!!

$$\text{Rate} = \frac{1}{\text{Time}} \quad \text{and} \quad \text{Time} = \frac{1}{\text{Rate}}$$

So the combined time is

$$Time = \frac{1}{\frac{1}{a}+\frac{1}{b}} = \frac{ab}{a+b}$$

Example 49:

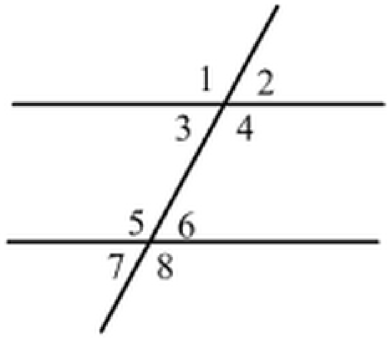
(a) Worker A can do a job in 8 hours. Worker B can do a job in 6 hours. How quickly can the job be done if they both work together?

(b) Tom can finish a job in 10 hours. When Buford works together with Tom, they can finish the job in 5 hours. How long does it take Buford to do the job if he works alone?

(c) If it takes 6 dogs 5 minutes to mark 10 fire hydrants, how long will it take 2 dogs to mark 12 fire hydrants?

Skill 50: Parallel Lines

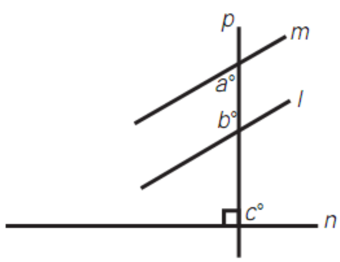
If a set of parallel lines are cut by a transversal, each of the parallel lines has 4 angles surrounding the intersections.



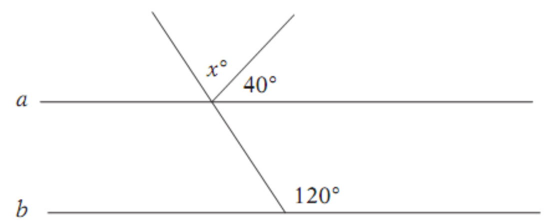
$\angle 1 \cong \angle 4$ and $\angle 2 \cong \angle 3$: Vertical Angles
$\angle 2 \cong \angle 6$ and $\angle 4 \cong \angle 8$: Corresponding Angles
$\angle 3 \cong \angle 6$ and $\angle 4 \cong \angle 5$: Alternate Angles

$\angle 3 + \angle 5 = 180^\circ$ and $\angle 4 + \angle 6 = 180^\circ$: Sum of interior angles on same side is 180°

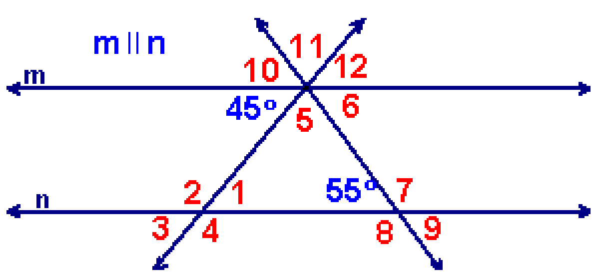
Example 50:



(a) In the figure above, m is parallel to l and p is perpendicular to n. Find the value of $a + b + c$.



(b) In the figure above, if $a \parallel b$, what is the value of x?



(c) Find the size of all the numbered angles.