Lesson 11—Skills 46-50

Skill 46: Average Speed

Average speed is the total distance travelled by the total time taken.

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

and

$$\text{Distance} = \text{Rate} \times \text{Time} \quad \text{so} \quad \text{Time} = \frac{\text{Distance}}{\text{Rate}}$$

It's helpful in these problems to let the one-way distance be anything, say D, so that the total two-way trip is $2D$.

Example 46:

(a) If you travel from city A to city B at 40 miles per hour, and then you travel back at 50 miles per hour, what is the average speed for the whole trip?

The avg will be less than $\frac{40+50}{2}=45$ since more time is spent driving at the 5 speed.

$\frac{2D}{\frac{D}{40}+\frac{D}{50}}\left(\frac{2000}{2000}\right)=\frac{4000D}{50D+40D}=\frac{4000D}{90D}=\frac{400}{9}$

≈ 44.444 mph

(b) If you travel from city A to city B in 6 hours, but in the first two hours you drove a constant speed of 50 miles per hour, and in the last 4 hours you kept your speed at 60 miles per hour, what is the average speed of your trip?

Dist driving 50 = 2(50) = 100

Dis driving 60 = 4(60) = 240

Total distance = 100+240 = 340

Avg speed = $\frac{\text{Dist}}{\text{time}}=\frac{340}{6}=56.667$ mph

Skill 47: Factoring

Factoring is to write an expression as a product of factors.
For SAT questions, the following factorings are needed

- $a^2+2ab+b^2=(a+b)^2$
- $a^2-2ab+b^2=(a-b)^2$
- $a^2-b^2=(a+b)(a-b)$
- $a^2-2a-3=(a-3)(a+1)$ *or similar "target sum/target product" problem

Example 47:

(a) If $(2x-8)(3x+5)=a$, then $(12-3x)(15x+25)=$ what?

$(2x-8)(3x+5)=a$

$2(x-4)(3x+5)=a$

$(12-3x)(15x+25)$

$=-3(x-4)(5)(3x+5)$

$(-15)(x-4)(3x+5)=\frac{1}{2}a(-15)$

$=\frac{-15}{2}a$ or $-7.5a$

(b) If $x^2-y^2=24$, where x and y are positive integers and $x>y$, what is one possible value of x?

$x^2=24+y^2$

$x=+\sqrt{24+y^2}$

x, y are pos integers

Let $y=1$: $x=\sqrt{24+1^2}=5$ ✓

Let $y=2$: $x=\sqrt{24+2^2}=\sqrt{28}$ ✗

Let $y=3$: $x=\sqrt{24+3^2}=\sqrt{33}$ ✗

∴ So x could be 5

Skill 48: Prime and Divisibility

To determine if a number is prime or composite

1. Find all the factors of the number.
2. If the number has only two factors, 1 and itself, then it is prime.
3. If the number as more than two factors, then it is composite.

A number x is divisible by another number y, if y is a factor of x. That is $\frac{x}{y}$ is an integer or $x \div y$ has no remainder.

Example 48:

(a) Determine if 323323 is divisible by 2, 3, & 5.

Divisible by 2 if ends in even

Divisible by 5 if ends in 0 or 5

Divisible by 3 if sum of digits is divisible by 3

So 323323 is not divisible by 2, 3, or 5

(b) If a number n is divisible by 3, 4 ($=2\cdot2$), and 7, which of the following is also divisible by these numbers?

i) $n+21$ ✓ ($n+21$ is a multiple of 21)

n is divisible by all combos of 3, 4, 7

factors: 3, 4, 7, 12, 21, 28, 84, 6, 8, 14, 24, 56, 42

ii) $n+84$ ✓ ($n+84$ is a multiple of 84)

iii) $21n$ ✓ ($21n$ is a multiple of 21)

iv) $6n$ ✓ ($6n$ is a multiple of 6)

v) $6n+252$ ✓ $6n$ is a multiple of 6, and 252 is a multiple of $84=3\cdot84$

So all are divisible by 3, 4, & 7

Skill 49: Rate of Work

Let's assume we have two workers: A and B.

1) Worker A can finish **1** job in a hours when working alone at a rate of $\frac{1}{a}$.

2) Worker B can finish **1** job in b hours when working alone at a rate of $\frac{1}{b}$.

If two workers are working together, the number of hours they need to complete the job is given by

Worker	Rate	Combined Rate	Combined Time
A	$\frac{1}{a}$	$\frac{1}{a}+\frac{1}{b}$	$\frac{1}{\frac{1}{a}+\frac{1}{b}}$
B	$\frac{1}{b}$		

For these types of problems where **1** job is done and to be done, **Rate** and **Time** are reciprocals!!

$$\text{Rate}=\frac{1}{\text{Time}} \quad \text{and} \quad \text{Time}=\frac{1}{\text{Rate}}$$

So the combined time is

$$Time=\frac{1}{\frac{1}{a}+\frac{1}{b}}=\frac{ab}{a+b}$$

Example 49:

(a) Worker A can do a job in 8 hours. Worker B can do a job in 6 hours. How quickly can the job be done if they both work together?

$\frac{1}{\frac{1}{8}+\frac{1}{6}}=\frac{1}{\frac{3}{24}+\frac{4}{24}}=\frac{1}{7/24}=\frac{24}{7}$ = 3.429 hrs

(b) Tom can finish a job in 10 hours. When Buford works together with Tom, they can finish the job in 5 hours. How long does it take Buford to do the job if he works alone?

$\frac{1}{\frac{1}{10}+\frac{1}{B}}=5$

$\frac{1}{\frac{B+10}{10B}}=5$

$\frac{10B}{B+10}=5 \rightarrow \frac{10B}{5}=B+10 \rightarrow 2B=B+10$

$B=10$ hrs

(c) If it takes 6 dogs 5 minutes to mark 10 fire hydrants, how long will it take 2 dogs to mark 12 fire hydrants?

6 dogs per 5 min per 10 hydrant

= 6 dogs per $\frac{5}{10}$ min per 1 hydrant

= 6 dogs per $\frac{1}{2}$ min per 1 hydrant

= 3 Dog·Min per 1 hydrant

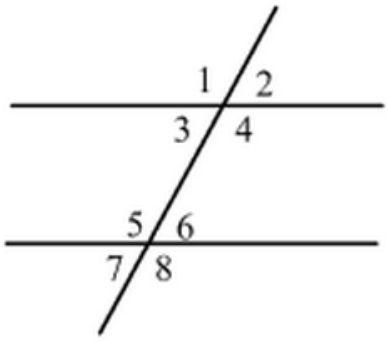
if 2 dogs work, it will take

$\frac{3}{2}$ min per 1 hydrant

Now to mark 12 hydrants, 2 dogs take $12\left(\frac{3}{2}\right)=6(3)=$ 18 minutes

Skill 50: Parallel Lines

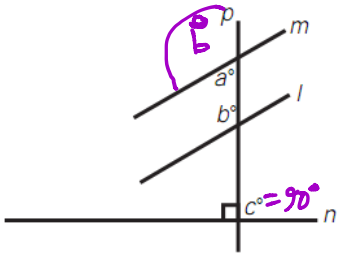
If a set of parallel lines are cut by a transversal, each of the parallel lines has 4 angles surrounding the intersections.



$\angle 1 \cong \angle 4$ and $\angle 2 \cong \angle 3$: Vertical Angles
$\angle 2 \cong \angle 6$ and $\angle 4 \cong \angle 8$: Corresponding Angles
$\angle 3 \cong \angle 6$ and $\angle 4 \cong \angle 5$: Alternate Angles

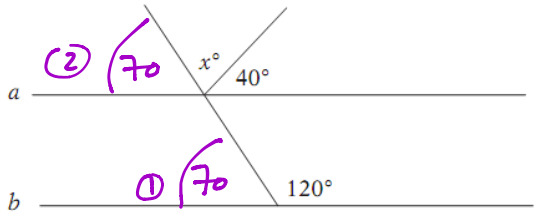
$\angle 3 + \angle 5 = 180^\circ$ and $\angle 4 + \angle 6 = 180^\circ$: Sum of interior angles on same side is 180°

Example 50:



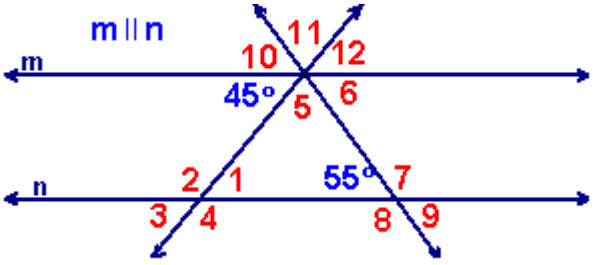
(a) In the figure above, m is parallel to l and p is perpendicular to n. Find the value of $a+b+c$.

$a+b = 180^\circ$, $c = 90^\circ$
So $a+b+c = 180^\circ + 90^\circ = 270^\circ$



(b) In the figure above, if $a \parallel b$, what is the value of x?

$x = 180 - 70 - 40 = 180^\circ - 110^\circ$
$= 70^\circ$



(c) Find the size of all the numbered angles.

(12) $= 125 - 80 = 45 =$ (1) $=$ (3)
(10) $= 180 - 45 - 80 = 55$
(6) $= 180 - 45 - 80 = 55$
(2) $= 180 - 45 = 135 =$ (4)

(7) $= 180 - 55 = 125 =$ (8)
(9) $= 55$
(5) $= 125 - 45 = 80 =$ (11)