Lesson 10—Skills 41-45

Skill 41: Functions as Models

A function can serve as a simple kid of mathematical model, or a simple piece of a larger model. Remember that a function is just a rule. We can think of the rule (given in our model as a graph, a formula, or a table of values) as a representation of some natural cause and effect relationship.

Example 41:

(a) A bookstore is selling a particular book for \$15 per copy. At this price it has been selling 20 copies of the book each day. The store owner estimates that for every dollar reduction in the selling price of the book, daily sales will increase by 20 copies. What is the daily sales, S, as a function of the price, p?

(A) $S(p) = -20p + 320$

(B) ~~$S(p) = 15p + 20$~~

(C) ~~$S(p) = \frac{3}{5}p$~~

(D) ~~$S(p) = -20p - 15$~~

(E) ~~$S(p) = p + 5$~~

$p = 15 \rightarrow 20$ books
$S(15) = -20(15) + 320 = 20$
$p = 14 \rightarrow 40$ books
$S(14) = -20(14) + 320 = 40$ ✓

$$h(t) = 24.5t^{4/3}$$

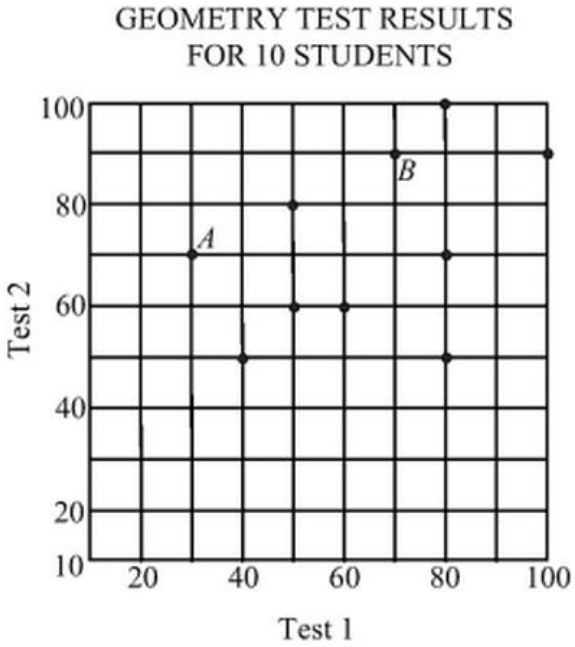
(b) The function above can be used to calculate the maximum height (in meters) that a rocket achieves during flight, where t is the time in seconds needed for the fuel to burn completely. If it takes 64 seconds for the fuel to burn completely, what is the maximum height (in meters) achieved by the rocket?

$h(64) = 24.5(64)^{4/3}$
$= 24.5(\sqrt[3]{64})^4$
$= 24.5(4^4)$
$= 6272$ meters

Skill 42: Data Interpretation

Data interpretation problems usually require two basic steps. First, you have to read a chart or graph in order to obtain certain information. Then, second, you have to apply or manipulate the information in order to obtain an answer. Be sure to read all notes related to the data and pay attention to units.

Example 42:



In the scatter plot above, student *A* got a 30 on Test 1 and a 70 on Test 2. Student *B* got a 70 on Test 1 and a 90 on Test 2.

(a) What is the median score on Test 1 and Test 2? (b) What is the average (arithmetic mean) on Test 1?

30, 40, 50, 50, 60, 70, 80, 80, 80, 100

1: median = $\frac{60+70}{2}$ = 65

2: 50, 50, 60, 60, 70, 70, 80, 90, 96, 100

median = 70

Mean = $\frac{\text{Sum}}{10}$

= $\frac{640}{10}$

= 64

Skill 43: Expected Value

The probability of an event is the ratio of the number of ways that an event can occur to the number of possible outcomes when each outcome is equally likely to occur.

If

$n(E)$ = the number of ways that even *E* can occur.

$n(S)$ = the nubmer of possible outcomes in the sample space *S*.

$P(E)$ = the probability of the event *E*.

Then

$$P(E) = \frac{n(E)}{n(S)}$$

Expected value is merely an average. If an event has a probability *p* of occuring in a single trial, then the expected number of times you'd see it happen in *n* trials is $p \times n$.

Example 43:

(a) If we were to flip a coin 20 times, how many times can you expect the coin to land on heads?

Prob of Heads $= \frac{1}{2}$
Number of flips $= 20$
Expected Number of Heads $= \left(\frac{1}{2}\right)(20) = \boxed{10}$

(b) A game is played rolling a single die. If a 3 or a 5 is rolled, you lose \$50. If any other number is rolled, you win \$10. What outcome would you expect from this game in terms of winnings or losses?

$P(3 \text{ or } 5) = \frac{2}{6}$
$P(1,2,4,6) = \frac{4}{6}$

Expected Value (\$)
$= \left(\frac{2}{6}\right)(-50) + \left(\frac{4}{6}\right)(10)$
$= -\frac{50}{3} + \frac{20}{3} = -\frac{30}{3}$
$= \boxed{-10 \text{ dollars}}$
*I would not play this game

Skill 44: Counting Digits

For these types of problems, you will be given criteria for creating a number of a certain size. You will have to determine how many different possibilities meet the given criteria.

Example 44:

How many 3-digit positive numbers have odd integer digits?

$5 \cdot 5 \cdot 5 = \boxed{125 \text{ numbers}}$

odd integer digits must be 1, 3, 5, 7, or 9 → 5 possibilities

Skill 45: Counting Multiples

Again, you will be given some criteria and a range of numbers. From this, you will have to determine how many of those numbers are multiples of one or more given numbers. For this, simply divide the number by the desired multiple, and round down. $\lfloor n \rfloor$ is floor function or "round down" function

Example 45:

(a) In the first 1000 positive integers, how many numbers are multiples of 7?

$\frac{1000}{7} = 142.857\ldots$
Always round Down
So $\boxed{142}$
or $\left\lfloor \frac{1000}{7} \right\rfloor = 142$

(b) Between 100 and 700, how many integers are multiples of 2 **and** 3? 2 **or** 3?

$n(2 \text{ AND } 3) = n(6) = \left\lfloor \frac{699}{6} \right\rfloor - \left\lfloor \frac{100}{6} \right\rfloor$
$= 116 - 16$
$= 100$

$n(2 \text{ or } 3) = n(2) + n(3) - n(6)$
$\left[\left\lfloor \frac{699}{2} \right\rfloor - \left\lfloor \frac{100}{2} \right\rfloor\right]$
$+ \left[\left\lfloor \frac{699}{3} \right\rfloor - \left\lfloor \frac{100}{3} \right\rfloor\right] - 100$

$$= [349 - 50]$$
$$+ [233 - 33] - 100$$

$$= \boxed{399}$$