Lesson 9—Skills 36-40

Skill 36: Odd and Even Numbers

For these types of questions, you will be given either an even or an odd representation of a number, then you will have to determine if this representation, when altered by way of algebraic manipulation, is even or odd. For these types of SAT math questions, the "**plug-in-the-answer-choices**" method works very, very well.

- $odd \times odd = odd$
- $even \times even = even$
- $odd \pm even = odd$
- $even \pm even = even$
- $odd \pm odd = even$

Example 36:

(a) If n is an odd number, which of the following must be even? Let $n=3$

(A) $5n$ → $5(3)=15$ → odd

(B) n^2 → $3^2=9$ → odd

(C) $2n-n$ → $2(3)-3=6-3=3$ → odd

(D) $n+2$ → $3+2$ → 5 → odd

(E) $(n+1)(n-2)$ → $(3+1)(3-1)=4(2)=8$ → even

E (for even!)

Let $a=2$ so that $2+3=5$ is odd

(b) If $a+3$ is an odd integer, which of the following must be an even integer?

(A) $2a+1$ → $2(2)+1=5$ → odd

(B) $4a$ → $4(2)=8$ → $\boxed{\text{EVEN}}$ B

(C) $\frac{a}{2}$ → $\frac{2}{2}=1$ → odd

(D) $a-1$ → $2-1=1$ → odd

(E) $3a+1$ → $3(2)+1=6+1=7$ → odd

Skill 37: Inequalities

An **inequality** says that two values are not equal.
$a \neq b$ says that a is not equal to b.

There are other special symbols that show in what way things are not equal.
$a < b$ says that a is less than b
$a > b$ says that a is greater than b
(these two are known as **strict inequalities**.)

$a \leq b$ says that a is less than or equal to b
$a \geq b$ says that a is greater than or equal to b

Here are the properties of inequality:

- If $a > b$ and $b > c$, then $a > c$
- If $a > b$, then $a \pm c > b \pm c$
- If $a > b$ and $c > 0$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$
- If $a > b$ and $c < 0$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$
- If $a > 0$ and $x^2 < a^2$, then $-a < x < a$
- If $a > 0$ and $x^2 > a^2$, then $x < -a$ or $x > a$

Example 37:

$b<a$

$a > b$ $2c>b, b<2c$
$b < c$
$a = 2c$

(a) If a, b, and c represent different integers in the statements above, which of the following statements must be true?

I. $a > c$ → maybe
II. $2c > b$ ✓
III. $ac > b^2$ → $2c^2>b^2$ → maybe

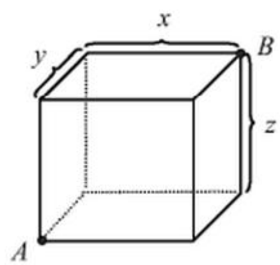
(A) I only $2c>b$, $2(2)>-5$ $c=2, b=-5$
(B) II only but $2(2^2)\not>(-5)^2$
(C) I and II only $8\not>25$
(D) II and III only
(E) I, II, and III

(b) If $a > b$ and $b(b-a) > 0$, which of the following must be true?

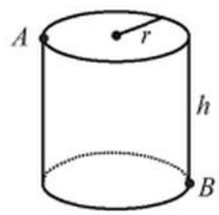
I. $b < 0$ → maybe if $a>b$ the $b-a<0$
maybe II. $a < 0$
→III. $ab < 0$ So if $b(b-a)>0$

(A) I only then $b<0$ too!
(B) II only (since neg·neg=pos)
(C) I and II only
(D) I and III only
(E) I, II, and III

Skill 38: Solids



- Surface Area $= 2(xy + yz + zx)$
- Volume $= xyz$
- Length of Diagonal $= \sqrt{x^2 + y^2 + z^2}$

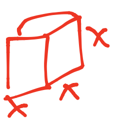


- Surface Area $= 2\pi r^2 + 2\pi rh = 2\pi r(r+h)$
- Volume $\pi r^2 h$
- Length of $\overline{AB} = \sqrt{(2r)^2 + h^2}$

diam

Example 38:

(a) What is the surface area of a cube that has a volume of 64 cubic centimeters?



$x^3 = 64, x = 4$

So $A = 6(x^2)$

$6(16) = \boxed{96 \text{ cm}^2}$

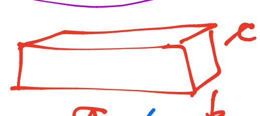
(b) The length, width, and height of a rectangular box, in centimeters, are a, b, and c are all integers. The total surface area of the box, in square centimeters, is s, and the volume of the box, in cubic centimeters, is v. Which of the following must be true?

All are true

I. v is an integer ✓

II. s is an even integer ✓

III. The greatest distance between any two vertices of the box is $\sqrt{a^2 + b^2 + c^2}$ ✓



$S = 2ab + 2bc + 2ac$

$V = abc$

$S = 2(ab + bc + ac)$

So S is a multiple of 2 → even

max vertice distance $= \sqrt{a^2 + b^2 + c^2}$

Skill 39: Sequences and Series

An **arithmetic sequence** (or arithmetic progression) is a sequence of terms, such as $1,5,9,13,17$ or $12,7,2,-3,-8,-13,-18$, which has a constant difference between consecutive terms.

- The first term is a_1
- The common difference is d
- The number of terms is n
- The nth term is $a_n = a_1 + (n-1)d$

An **arithmetic series** is a series (sum) of terms, such $3+7+11+15+\cdots+99$ or $10+20+30+\cdots+1000$, which has a constant difference between consecutive terms.

- The first term is a_1
- The common difference is d
- The number of terms is n
- The sum of an arithmetic series is found by multiplying the number of terms times the average of the first and last terms. Sum of first n terms $= S_n = n\left(\frac{a_1+a_n}{2}\right) = \frac{n\left[2a_1+(n-1)d\right]}{2}$

An **geometric sequence** (or geometric progression) is a sequence of terms, such as 2,6,18,54,162 or $3,1,\frac{1}{3},\frac{1}{9},\frac{1}{27},\frac{1}{81}$, which has a constant ratio (multiplier) between consecutive terms.

- The first term is a_1
- The common ratio is r
- The number of terms is n
- The nth term is $a_n = a_1 r^{n-1}$

An **geometric series** is a series (sum) of terms, such as $2+6+18+54+162$ or $3+1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\frac{1}{81}$, which has a constant ratio (multiplier) between consecutive terms.

- The first term is a_1
- The common ratio is r
- The number of terms is n
- The sum of a the first n terms in a geometric series $= S_n = \frac{a_1\left(1-r^n\right)}{1-r}$

An **infinite geometric series** is a geometric series with an infinite number of terms. In this case, the series is said to **converge** to a sum if its common ratio r satisfies $-1<r<1$, otherwise the series grows without bound and is said to **diverge**.

The sum of an infinite, convergent, geometric series $= S = \frac{a_1}{1-r}$, as long as $-1<r<1$

Example 39:
There are 24 red marbles, 24 blue marbles, 24 green marbles, 24 white marbles, 24 black marbles, and 24 gray marbles in a box.

$$-1, 4, -16, \ldots$$

(a) In the geometric sequence above, what is the sum of the first 10 terms of the sequence?

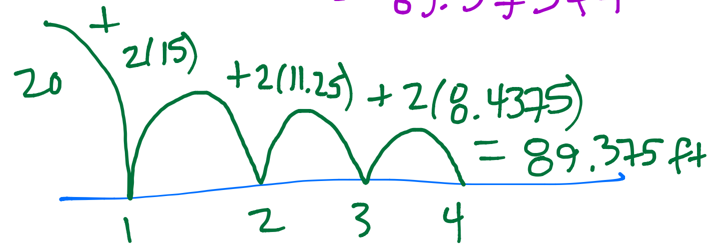
$-1, 4, -16, 64, -256, 1024,$ $r=-4$, $a_1=-1$
$\times -4$ $\times -4$ $-4096, 16384, -65536, 262144$
Sum $= 209715$

or $\dfrac{-1(1-(-4)^{10})}{1-(-4)} = \dfrac{1048575}{5} = 209715$

(b) Assume a ball bounces to a height of $\frac{3}{4}$ of the height from which it falls. If the ball is dropped from a height of 20 feet, how many feet has the ball traveled up and down when it hits the ground for the ~~10~~ 4th time?

After 1st bounce: $\frac{3}{4}(20)=15$
2nd $= \frac{3}{4}(15) = 11.25$
3rd: 8.4375
4th: 6.328125

Total $= 20 + 2(15) + 2(11.25) + 2(8.4375) = 89.375$ ft



(c) Tom is given a penny on day 1, half a penny on day two, 1/4 a penny on day three, 1/8 a penny on day four, etc. If this process continues indefinitely (and Tom lives forever), how much money will Tom have many, many, many, years from now?

1¢ + 1/2, 1/4, 1/8, 1/16 → he is getting closer & closer to 2 cents.

Skill 40: Defined Operations

A defined operation is a mathematical situation of a certain situation. ~~It~~ uses a novel symbol to represent an operation between two or more numbers.

Example 40:
If the operation ▲ is defined by $\blacktriangle a = a^a$, what is the value of ▲8 / ▲4?

$$\frac{\blacktriangle 8}{\blacktriangle 4} = \frac{8^8}{4^4} = \frac{(4\cdot 2)^8}{4^4} = \frac{4^8\cdot 2^8}{4^4} = 4^4\cdot 2^8 = (2^2)^4\cdot 2^8 = 2^8\cdot 2^8 = \boxed{2^{16}}$$

or $\boxed{65,536}$