Lesson 8—Skills 31-35

Skill 31: Percentage

In mathematics, a percentage is a way of expressing a number as a fraction of 100 (per cent means "per hundred"). It is often denoted using the percent sign "%." For example, 45% (read as "forty-five percent) is equal to $\dfrac{45}{100} = \dfrac{9}{10} = 0.45$.

When you learned how to translate simple English statements into mathematical expressions, you learned that "**of**" can indicate "**times**" or multiplication. Also, "**is**" indicates "**equals**." This frequently comes up when using percentages.

x percent of y is z

translates mathematically as

$$\frac{x}{100} \times y = z$$

- % increase = $\dfrac{\text{amount of increase}}{\text{original amount}} \times 100$

- % decrease = $\dfrac{\text{amount of decrease}}{\text{original amount}} \times 100$

Example 31:

(a) What percent of 20 is 30?

(b) 22% of what number is 11?

(c) The enrollment at a university increased from 12,000 students to 15,000 students over a period of 5 years. What is the percent increase in enrollment during this time?

(d) What is the percent decrease on a DVD recorder that is marked down from $400 to $350?

Skill 32: Proportion & Ratios

A **ratio** is a comparison of two numbers. We can write this as, for example, 8:12 or as a fraction $\frac{8}{12}$, and we say the ratio is "eight to twelve."

A **proportion** is an equation with a ratio on each side. It is a statement that two ratios are equal, for example, $\frac{3}{4} = \frac{6}{8}$ is an example of a proportion.

A **rate** is a ratio that expresses how long it takes to do something, such as traveling a certain distance or a price per unit.   There are units both in the numerator and denominator. For example, to walk 3 miles in one hour is to walk at a rate of 3 m/h. A rate **may be flipped** without loss of meaning, only a new interpretation, for example, from above, 3 m/h is the same as 1/3 hour per mile.
When using a rate in a calculation, have the desired final units in the numerator of you given rate!!

Example 32:
(a) Juan runs 4 km in 30 minutes. At that rate, how far could he run in 45 minutes?

(b) Oranges cost $2.25 per pound. If $10.20 is spent on oranges, how many pounds were purchased?

(c) If *a* is divisible by 2, *b* is divisible by 5, and $\frac{a}{b} = \frac{7}{9}$, where *a* and *b* are positive numbers, and $a + b < 400$, what is one possible value of $a + b$?

Skill 33: Probability

Consider a random sampling process in which all the outcomes solely depend on chance, i.e., each outcome is equally likely to happen. If the collection of all possible outcomes is U and the collection of desired outcomes is A, the **probability** of the desired outcomes is

$$P(A) = \frac{\text{number of } A}{\text{number of } B} = \frac{n(A)}{n(B)}, \ 0 \le P(A) \le 1$$

Accordingly, the probability of an unwanted outcome $\overline{A}$ is $P(\overline{A}) = 1 - P(A)$

Law of Addition
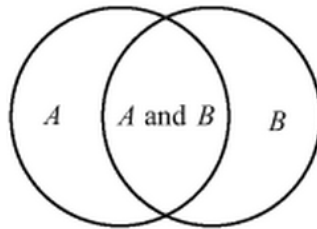
The probability of event A or event B (or both) occuring is given by

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Or

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

the probabilty of the intersection, $P(A \cap B)$, is often zero, but make sure!

Example 33:

(a) A bag contains 4 red marbles and 7 blue marbles. What is the probability that you do not select any blue marbles if you draw two marbles from the bag one after the other i) with replacement? ii) without replacement?

(b) A bag contains 3 red marbles and 4 blue marbles. What is the probability that you select one red marble and one blue, at random, one after the other, from the bag without replacement?

Skill 34: Number of guarantee

These types of questions are related to probability, but ask you to determine the number of trial needed to achieve a certain goal. Listing out scenarios will help you answer these.
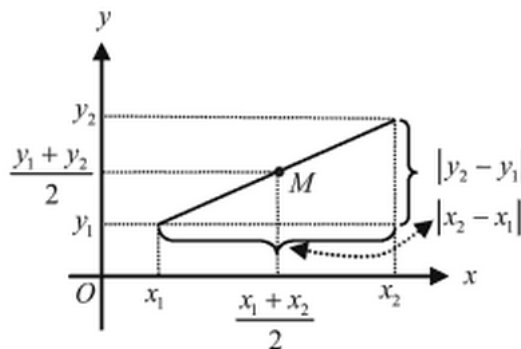
Example 34:
There are 24 red marbles, 24 blue marbles, 24 green marbles, 24 white marbles, 24 black marbles, and 24 gray marbles in a box.

(a) Tom wants to select at least two marbles of the same color. What is the least number of marbles he must select?

(b) Tom wants to select at least four marbles of the same color. What is the least number of marbles he must select?

Skill 35: Midpoints & Distances

The Midpoint Formula: Given the two points (x_1, y_1) and (x_2, y_2), the midpoint of the segment between these two points is $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2} \right)$

The Distance Forumla: Given the two points (x_1, y_1) and (x_2, y_2), the distance between these two points is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Example 35:

(a) In the xy-plane, the midpoint of $\overline{EF}$ is $(-3, 5)$. If the coordinates of point F are $(2, 2)$, then what are the coordinates of point E?

(b) The distance between $(-1, n)$ and $(6, m)$ it 20, what is the value of $|m - n|$?