Lesson 7—Skills 26-30

Skill 26: Coordinates of a Circle

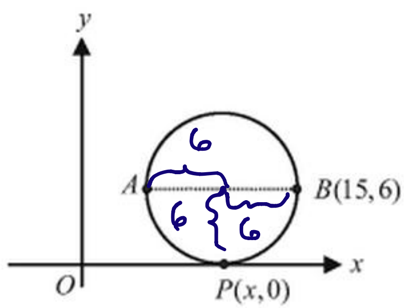
When given questions regarding coordinates of a circle, tools such as the Pythagorean Theorem, Midpoint formula and/or Distance formulas can be used. Remember that the **diameter** is the width of the circle and runs through the center of the circle. The **radius** is half the diameter. Being **tangent** to a line means touching it once.

Pythagorean Theorem: $a^2 + b^2 = c^2$

Midpoint formula: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Example 26:



So radius
$r = 6$

In the figure above, AB is a diameter of the circle and parallel to the x-axis. What is the value of x?

So $x = 15 - \text{radius}$
$x = 15 - 6$
$\boxed{x = 9}$

Skill 27: Paths in a Grid

For these types of problems, you will be given a square or rectangular grid with two or more points labeled on the grid. You will be asked to determine how many different paths are possible between these points using vertical and/or horizontal sequences.

From earlier, we can use our combination formula for arrangements when order does NOT matter:

$${}_nC_r = C(n,r) = \frac{n!}{r!(n-r)}$$

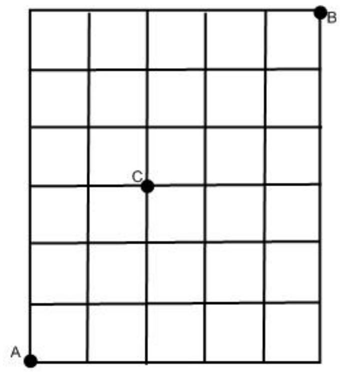
Where n is the minimum number of moves required along any path and r is **either** the number of vertical **or** horizontal moves (it works out the same either way).
Also, remember to multiply independent events together to get a total.

*if you must pass through or avoid a pt, treat the paths as separate independent paths!

Example 27:

* the factorial key (!) is found under

MATH → PRB → #4



In the figure above, a path from point *A* to point *B* is determined by moving upward or to the right along the grid lines.

(a) How many different paths can be drawn from *A* to *B*?

min number of moves = 6 up + 5 right = 11 moves = n

I will choose $r = 6$ (vert moves)

$$C(11,6) = \frac{11!}{6!(11-6)!} = \frac{11\cdot 10\cdot 9\cdot 8\cdot 7\cdot 6!}{5\cdot 4\cdot 3\cdot 2\cdot 1\cdot 6!} = 77(6) = \boxed{462}$$

(b) How many different paths can be drawn from *A* to *B* that must include point *C*?

A TO C: $C(5,3) = \frac{5!}{3!2!} = 10$

C TO B: $C(6,3) = \frac{6!}{3!3!} = 20$

A TO C TO B $= 10 \cdot 20 = \boxed{200}$

(c) How many different paths can be drawn from *A* to *B* that do not include point *C*?

A To B excluding C

= Total ways from (a) — Total ways from (b)

$= 462 - 200$

$= \boxed{262}$

Skill 28: Transformations

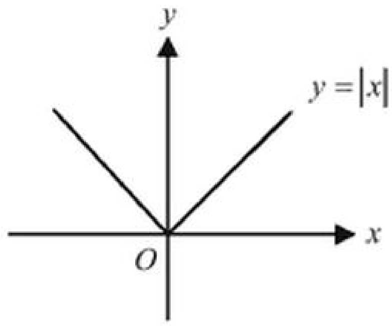
The types of transformations are

- Translations: involves "sliding" or "shifting" the object from one position to another. Shape and orientation are preserved.
- Reflections: involves "flipping" the object over a line called the line of reflections. Preserves shape, changes orientation.
- Rotation: involves "turning" the object about a pint called the center of rotation.
- Dilation: involves a "stretching" or "compressing" of an object. It changes the shape and/or size of the object, getting bigger or smaller (or narrower or wider).

If the graph of $y = f(x)$ is translated c units horizontally and d units vertically, then the equation of the translated graph is

$$y - d = f(x - c) \text{ or } y = f(x - c) + d$$

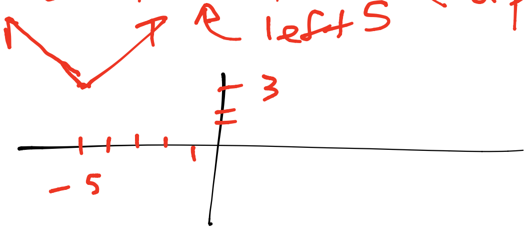
Example 28:



(a) If the graph of $y = |x|$ is given above, what would the graph of $y - 3 = |x+5|$ look like?

$y = |x+5| + 3$ up 3

left 5



(b) If the graph of $y = f(x)$ is given above, what would the graph of $y = -|3-x| - 5$ look like?

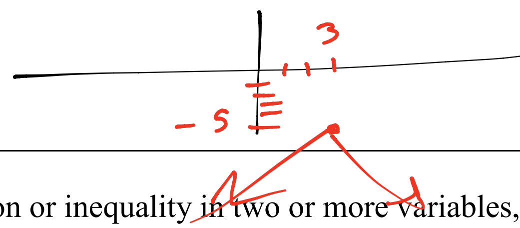
$|3-x| = |x-3|$

$y = -|x-3| - 5$

x-axis reflect

right 3

DWN 5



Skill 29: The Least/Greatest Number

For these types of problems, you will be given an algebraic equation or inequality in two or more variables, then asked to predict an upper or lower bound for one of the variables. This requires a bit of algebra, thought, and practice.

Example 29:

(a) If $0 \le x \le y$ and $(x+y)^2 - (x-y)^2 \ge 64$, what is the least possible value of y?

expand:

$\cancel{x^2} + 2xy + \cancel{y^2} - \cancel{x^2} + 2xy - \cancel{y^2} \ge 64$

$4xy \ge 64$

$xy \ge 16$

*To get the LEAST value of y, the greatest value of x is needed. So $x = y$. $4xy$ becomes $4y^2$

So $4y^2 \ge 64$

$y^2 \ge 16$

$y \le -4$ or $y \ge 4$

but $y \ge 0$, so the least value of $\boxed{y = 4}$

(b) If $x^2 - y^2 \ge 77$ and $x + y = 11$, what is the greatest possible value of y?

$x^2 - y^2 \ge 77$

$(x-y)(x+y) \ge 77$, $x+y = 11$, so

$11(x-y) \ge 77$ $x = 11 - y$

$x - y \ge 7$ (sub in)

$11 - y - y \ge 7$

$11 - 2y \ge 7$

$-2y \ge 7 - 11$

$-2y \ge -4$

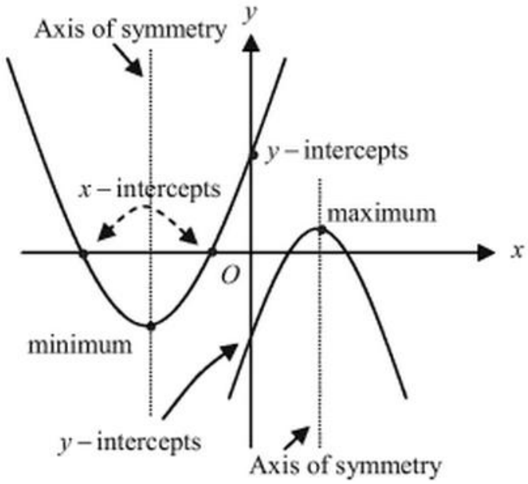
$y \le 2$ flip sign!

So max y-value is $\boxed{y = 2}$

Skill 30: Maximum & Minimum

For the quadratic function $f(x) = ax^2 + bx + c$

- The axis of symmetry is the vertical line $x = \frac{-b}{2a}$
- If $a > 0$ (opens up), the minimum is $y = f\left(\frac{-b}{2a}\right)$
- If $a < 0$ (opens down), the maximum is $y = f\left(\frac{-b}{2a}\right)$
- The x-intercepts are the solutions to $f(x) = 0$



Example 30:

(a) Given the equation $f(x) = x^2 - 2x - 3$, find the equation of the axis of symmetry, the max/min value, and the x-intercept(s).

Opens up. X-coord o vertex:

$X = -\frac{(-2)}{2(1)} = 1$

Axis of symm: $\boxed{X=1}$

Min value: $f(1) = 1-2-3 = \boxed{-4}$

X-int: $x^2-2x-3=0$

$(x-3)(x+1)=0$

$X=3$ & $X=-1$

(b) Given the equation $f(x) = -x^2 + 2x + 3$, find the equation of the axis of symmetry, the max/min value, and the x-intercept(s).

Opens down

Axis: $X = -\frac{2}{2(-1)} = \boxed{1 = X}$

Max value: $f(1) = -1+2+3 = \boxed{4}$

X-int: $-x^2+2x+3=0$

$-(x^2-2x-3)=0$

$-(x-3)(x+1)=0$

$\boxed{X=3 \ \& \ X=-1}$