Lesson 6—Skills 21-25

Skill 21: Sum, Numbers of Consecutive Integers

These questions will ask you to find the number of integers in a set or the first or last number in a set of consecutive integers given the sum. Remembering that integers can be positive, zero, or negative, and using the symmetry of the number line can make these problems rather easy.

Example 21:

(a) The smallest integer of a set of consecutive integers is -10. If the sum of these integers is 23, how many integers are in this set?

$\underbrace{-10, -9, -8, \ldots, 0, \ldots 8, 9, 10}_{\text{Add to } 0}, \underbrace{11, 12}_{\text{Add to } 23}$

-10 to $-1 \rightarrow 10$
1 to $12 \rightarrow 12$
zero $\rightarrow 1$
ADDS to 23 numbers

(b) If the sum of the consecutive integers from -65 to x, inclusive, is 201, what is the value of x?

$\underbrace{-65, -64 \ldots, 0, \ldots, 64, 65}_{\text{Add to } 0}, \underbrace{66, 67, 68}_{\text{add to } 201}$

so $x = 68$

Skill 22: No Solution

A system of linear equations means two or more linear equations. If two linear equations intersect, that point of intersection is called the **solution to the system of linear equations**. There are three possibilities:

1) **The system has exactly one solution.** When two lines have different slopes, the system has one and only one solution.
2) **The system has no solution.** When two lines are parallel and have different y-intercepts, the system has no solution.
3) **The system has infinite solutions.** When two lines are parallel and the lines have the same y-intercept, then the two lines are actually the same line, intersect everywhere, and there are infinitely many solutions.

Here's the easy way to determine the above without actually solving the system. For the system of linear equations:

$$a_1x + b_1y = c_1$$
$$a_2x + b_2y = c_2$$

- If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, then the system has ONE SOLUTION
- If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, then the system has NO SOLUTION
- If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then the system has INFINITE SOLUTIONS

Example 22:

$$2x - 7y = 4$$
$$-8x + 28y = -16$$

(a) How many solutions does the system of linear equations above have?

$\frac{2}{-8}$ $\frac{-7}{28}$ $\frac{4}{-16}$

$-\frac{1}{4} = -\frac{1}{4} = -\frac{1}{4}$

So same line

$\rightarrow$ ∞ many solutions

$$5x + by = 4$$
$$-15x + 6y = 11$$

(b) For what values of b willl the system of equations above have no solution?

$\frac{5}{-15} = \frac{b}{6} \neq \frac{4}{11}$ want parallel lines

$-\frac{1}{3} = \frac{b}{6}$

$b = -\frac{6}{3} = \boxed{-2}$ since $\frac{-2}{6} = -\frac{1}{3} \neq \frac{4}{11}$

Skill 23: Identical Equations

The two expressions (LHS and RHS) are always equal for any value we give to the variable. Equations that are true for any value of the variable are called **identical equations**, or simply called an **identity**.

Example 23:

(a) If $2(x+5) = ax + b$ for any value of x, what are the values of a and b?

Distribute:

$2x + 10 = ax + b$

$2x = ax$, $\boxed{10 = b}$

$\boxed{a = 2}$

(b) If $ax^2 + bx + 36 = (3x+n)^2$ for all values of x, where $n < 0$, what is the value of $a + b$?

expand right side:

$ax^2 + bx + 36 = 9x^2 + 6nx + n^2$

So $\boxed{a = 9}$, $n^2 = 36 \rightarrow n = \pm 6$, $n < 0$ so $\boxed{n = -6}$

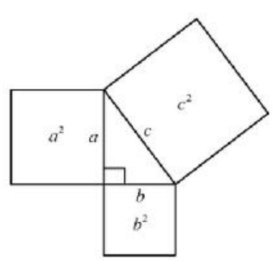
$b = 6n$, so $b = 6(-6)$, $\boxed{b = -36}$

$a + b = 9 - 36 = \boxed{-27}$

Skill 24: Pythagorean Theorem

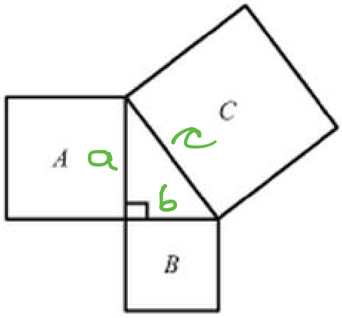
In mathematics, the Pythagorean Theorem is a relation in Euclidean geometry among the three sides of a right triangle. The theorem is named after the Greek mathematician Pythagoras, who by tradition is credited with its discovery and proof, although knowledge of the theorem almost certainly predates him. The theorem is as follows:

In any right triangle, the area of the square whose side is the hypotenuse (the side opposite the right angle) is equal to the sum of the areas of the squares whose sides are the two legs (the two sides that meet at a right angle).



$$a^2 + b^2 = c^2$$

Example 24:



(a) In the figure above, the area of the square A is 50 and the area of the square B is 31. What is the length of a side of the square C?

$a^2+b^2=c^2$

$50+31=c^2$

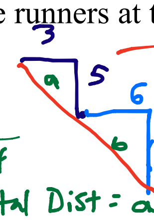
$c=\sqrt{81}=\boxed{9}$ (c can't be neg)

(b) Two joggers start at the same point. One runs 5 miles north, then 3 miles west. The other jogger runs 6 miles east, then 10 miles south. How far apart are these runners at the end of their run?

Triangles are similar, So
$b=2a$

$a=\sqrt{3^2+5^2}=\sqrt{34}$

$b=2\sqrt{34}$

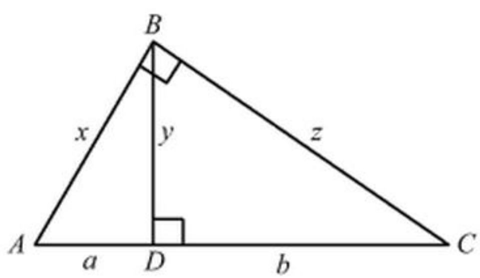


$\to$ slope $=\frac{5}{3}$

$\to$ slope $=\frac{10}{6}=\frac{5}{3}$ (SAME!)

Total Dist $=a+b=\sqrt{34}+2\sqrt{34}$
$=3\sqrt{34}$

Skill 25: Similar in Right Triangle



$\Delta ABD \sim \Delta BCD$ Similar triangles

$\frac{BD}{CD}=\frac{AD}{BD}$, that is $BD^2=AD\times CD$ resulting in

$y^2=ab$

Similarly,

$x^2=a(a+b)$

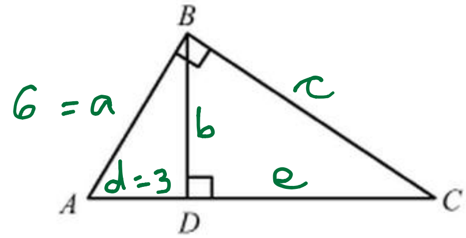
$y^2=ab$

$z^2=b(b+a)$

$xz=y(a+b)$

This might be too much to memorize. Instead, use Proportions.

Example 25:



Note: Figure not drawn to scale.

In ΔABC above, $AB = 6$ and $AD = 3$. What is the length of $\overline{CD}$? $e = ?$

label parts for easier reference

$\frac{\Delta \text{ on Right}}{\Delta \text{ on Left}}$: $\frac{e}{b} = \frac{b}{3}$

but $b = \sqrt{36 - 9}$

$b = \sqrt{27} = 3\sqrt{3}$

so $\frac{e}{3\sqrt{3}} = \frac{\cancel{3}\sqrt{3}}{\cancel{3}}$

$e = (\sqrt{3})(3\sqrt{3})$

$e = 3(3) = \boxed{9}$