Lesson 5—Skills 16-20

Skill 16: Slope of a Line

One of the most important properties of a straight line is its angle from the horizontal, or its rate of change. This concept is called slope. The slope between any two points on the line is constant.
To find the slope, we need two points from the line.

From two points (x_1, y_1) and (x_2, y_2), we have

$$\text{Slope} \quad m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

The slope-intercept form of the equation of the line is

$$y = mx + b$$

Where m is the slope and b is the y-intercept.

Example 16:

(a) Find the slope of the line $2x + 3y = 4$

$3y = -2x + 4$

$y = -\frac{2}{3}x + \frac{4}{3}$

$\boxed{m = -2/3}$

(b) Find the slope of the line that is i) parallel and ii) perpendicular to the line containing the points $(-3, 4)$ and $(5, -6)$

$m_{\parallel}$ (same slope)

$= \frac{-6 - 4}{5 - -3} = \frac{-10}{8} = -\frac{5}{4}$

$m_{\perp}$ (opp recip) $= \frac{4}{5}$

Skill 17: Number of Factors

The number of factors of a number can be found by adding one to all exponents of prime factors and multiplying those results together.

Example 17:

(a) Determine the number of factors 12 has. List them first, then verify the rule above.

$12: 1, 2, 3, 4, 6, 12 \rightarrow$ 6 factors

$12 = 2^2 \cdot 3^1 \rightarrow (2+1)(1+1)$

$3 \cdot 2$

$\boxed{6}$

(b) A positive integer k is defined by $k = 8m^3np^4$ where m, n, and p are distinct prime numbers greater than 2. How many factors does the number k have?

$k = 2^3 m^3 n^1 p^4$

Factors: $(4)(4)(2)(5)$

$(16)(10)$

Skill 18: Composition of Functions

Composition of functions is a process to combine two functions by adding, subtracting, multiplying, or dividing two given functions. There is another way in which we **substitute** an entire function into another function. Given two functions $f(x)$ and $g(x)$, we denote the substitution composition as

$$f\big(g(x)\big) \text{ or } (f \circ g)(x)$$

Example 18:

(a) Given $f(x)=3x-4$ and $g(x)=x^2$, find $f\big(g(x)\big)$ and $g\big(f(x)\big)$

$f(g(x)) = 3(x^2) - 4 = 3x^2 - 4$

$g(f(x)) = (3x-4)^2 = 9x^2 - 24x + 16$

(b) Given $f(x)=3x-4$ let the function g be defined by $g(x)=2f(x)+3$. If $g(k)=0$, what is the value of k?

$g(x) = 2(3x-4)+3$
$= 6x - 8 + 3$
$= 6x - 5$

$g(k) = 6k - 5 = 0$
$6k = 5$
$\boxed{k = 5/6}$

Skill 19: Consecutive Numbers

Consecutive numbers are numbers which follow each other in order, without gaps, from smallest to largest.

12, 13, 14, and 15 are consecutive integer numbers.
14, 16, and 18 are consecutive even numbers.
13, 15, and 17 are consecutive odd numbers.

For *n* consecutive integer numbers, the first term is usually labeled a_1, the second a_2, etc., and the last number is labeled a_n. For such numbers, the following are true:

$$\text{Average (Arithmetic Mean)} = \frac{a_1 + a_2 + \cdots + a_n}{n} = \frac{a_1 + a_n}{2}$$
$$\text{Median} = \text{Average} = \text{Middle number}$$

Example 19:

(a) For the sequence of consecutive numbers 2, 3, 4, 5, 6, 7, 8, 9, 10, find the average and the median.

$\text{Avg} = \frac{2+10}{2} = \frac{12}{2} = 6$

$\underbrace{2,3,4,5}_{4},6,\underbrace{7,8,9,10}_{4}$

middle = Avg

(b) What is the sum of 15 consecutive integers if the middle one is 50?

middle = 50 = Avg, So

$50 = \frac{a_1 + \cdots + a_{15}}{15} = \frac{\text{Sum}}{15}$

$\text{Sum} = 50(15) = \boxed{750}$

Skill 20: Must be true/Could be true

These questions require some analysis. It's also helpful to remember the **zero-product property**, which states

If $ab = 0$, then **either** $a = 0$ **or** $b = 0$

Here's what a question might ask:

$$(a+b)^2 = (a-b)^2$$

The questions can be as follows:

1) If the statement above is true, which of the following must also be true (always true)?

or

2) If the statement above is true, which of the following could be true (possibly true)?

Example 20:

(a) Given $(a+b)^2 = (a-b)^2$ to be true, disscus whether each of the following must be true and/or could be true:

I. $ab = 0$
II. $a = 0$
III. $b = 0$

must be → always
could be → sometimes

$(a+b)^2 = a^2 + 2ab + b^2$
$(a-b)^2 = a^2 - 2ab + b^2$
so $a^2 + 2ab + b^2 = a^2 - 2ab + b^2$
$2ab = -2ab$
$4ab = 0$
$ab = 0$

so $ab = 0 \rightarrow a = 0$, $\rightarrow b = 0$ → could be true

$ab = 0$ ↓ must be true

(b) If $a > b$ and $a(a-b) = 0$, which of the following must be true?

I. $a = 0$
II. $b < 0$
III. $a + b < 0$

$a(a-b) = 0$
So either $\boxed{a = 0}$ or $a - b = 0$
$a = b$
but $a > b$ so $a \neq b$ so

I is true

if $a = 0$ and $a > b$, then b is neg, so $b < 0$ → II is true

if $a = 0$, then $a + b < 0$ means $b < 0$, which is true
so III is true.

All choices I, II, & ~~III~~ are "must be true"