Lesson 4—Skills 11-15

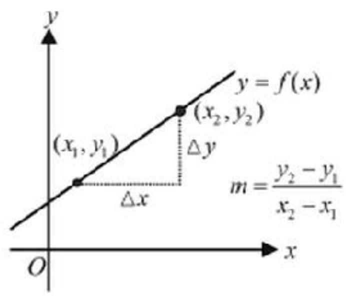
Skill 11: Linear Function

These functions are called linear because they are the functions whose graph in *xy*-plane is a straight line. Such a function can be written as

$$f(x) = mx + b$$

m is the slope and *b* is the *y*-intercept. The slope between any two points on the line is constant.

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$



Example 11:

(a) For a linear function *g*, $g(1) = 3$, $g(5) = 8$. If $k = g(-1)$, what is the value of *k*?

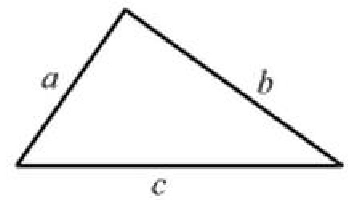
(b) For a linear function *f*, $f(-2) = -3$, $f(1) = 6$. What are the *x*- and *y*-intercepts of the graph of *f*?

(c) A linear function is given by $ax + by + c = 0$. If $b > 0$ and the graph of the line has a negative slope and a positive *y*-intercept, what are the signs of *a* and *c* in the equation of the line?

(d) In the function $M = \frac{5}{3}N - 20$, if *M* increases by 10, by how much does *N* increase?

Skill 12: Triangle Inequality

The Triangle Inequality Theorem states that the sum of the measures of any two sides of any triangle is greater than the measure of the third side. That means that in a triangle, you can pick any two sides' measures and, when you add them together, the sum will be greater than the measure of the third side.



$$a+b>c\,,\ b+c>a\,,\ c+a>b$$

or

$$|a-b|<c<a+b$$

Example 12:
If the lengths of the sides of a triangle are $2x+3$, 6, and 11, what values can x take on?

Skill 13: Permutations and Counting

Permutation, also called an **arrangement number** or **order**, is a rearrangement of the elements of an ordered list.

$${}_nP_r = P(n,r) = \frac{n!}{(n-r)!}\text{, where } n! = n\cdot(n-1)\cdot(n-2)\cdot\cdots\cdot 3\cdot 2\cdot 1 = n\cdot(n-1)!$$

Example 13:

(a) How many arrangements of the letters of the word CHEMISTRY are possible with R as the middle letter?

(b) How many different distinct arrangements of the word MISSISSIPPI are possible?

(c) If a fair die is thrown twice, what is the probability that a 4 is rolled first and an odd number is rolled second?

(d) A jar contains 5 green marbles and 3 red marbles, all the same size. What is the probability that one green and one red marble are drawn if i) the marbles are replaced? ii) not replaced?

Skill 14: Handshake

The typical problem involves several (distinct) people at a party. If everyone at the party shakes everyone else's hand exactly once, how many handshakes take place. These can be solved my drawing a star graph and counting lines, but drawing is not ideal for large groups. For large number, use the law of **combinations** (where, unlike a permutation, **order does not matter**).

$$ {}_nC_r = C(n,r) = \frac{n!}{(n-r)!r!} $$

Or

If n people shake hands, there will be $(n-1)+(n-2)+\cdots+2+1$ handshakes.

Example 14:

(a) If there are 5 people in a room and they shake each other's hands once and only once, how many handshakes are there altogether? Use the star graph and the combination formula.

(b) If there are 8 lines on a plane surface, what is the maximum number of intersection points?

Skill 15: Percent of a Solution

The percent of a solution is expressed as the percentage of solute over the total amount of solution.

p% of a solution is $$\frac{\text{Solute}}{\text{Total amount of solution}} = \frac{p}{100}$$

Or $$\frac{\text{Solute}}{\text{Total amount of solution}} \times 100 = p\%$$

Example 15:

(a) How many gallons of water must be added to 50 gallons of 20% alcohol solution to produce a 5% alcohol solution?

(b) How many gallons of a 30% salt solution must be added to 20 gallons of a 70% salt solution to produce a 50% salt solution?