Lesson 3—Skills 6-10

Skill 6: Inverse Variation or Inverse Proportion

$x \times y = k$

If y is inversely proportional to x, then

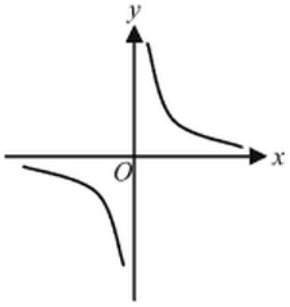
$x \times y = k$, where k is a nonzero constant

or

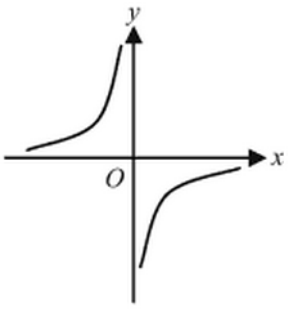
$$x \times y = x_1 \times y_1 = x_2 \times y_2 = \cdots = k$$

In the xy-plane, the graph of $y = f(x)$ is

1) $k > 0$



2) $k < 0$



Example 6:

(a) The cost of hiring a bus for a trip to Happymathland is \$400. If 25 people go on the trip, what is the cost per person?

$400 = K$

$25x = 400$

$x = \frac{400}{25} = 16$

(b) If 4 typists can complete the typing of a manuscript in 9 days, how long would it take 12 typists to complete the manuscript?

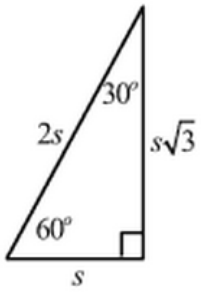
$4(9) = 36 = K = 12x$

$12x = 36$

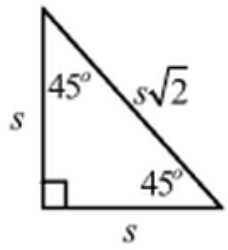
$x = 3$

Skill 7: Special Triangles

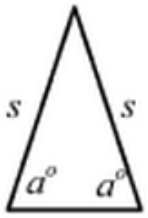
(A)



(B)

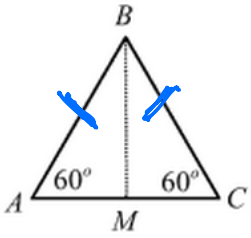


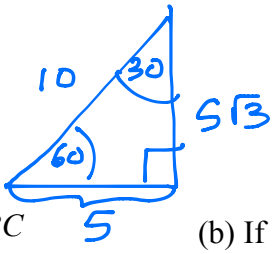
(C)



$$\text{Area} = \frac{1}{2}bh$$

Example 7:





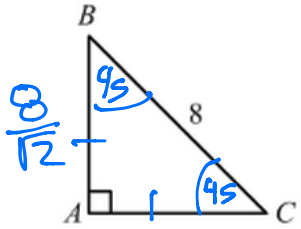
(a) If $AC = 10$, what is the area of ΔABC

AC = 10 so AM = 5

BM = $5\sqrt{3}$

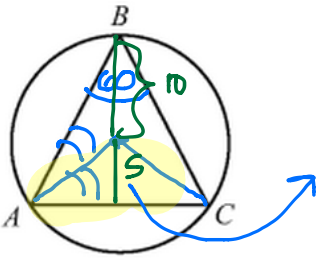
So area = $\frac{1}{2}(10)(5\sqrt{3})$

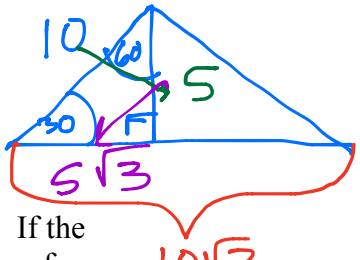
= $25\sqrt{3}$



(b) If $AB = AC$ and $\angle ABC = 45^\circ$, what is the area of ΔABC?

Area = $\frac{1}{2}\left(\frac{8}{\sqrt{2}}\right)\left(\frac{8}{\sqrt{2}}\right) = \frac{32}{2} = 16$





10√3

(c) An equilateral triangle is inside a circle. If the radius of the circle is 10, what is the area of ΔABC?

area = $\frac{1}{2}(10\sqrt{3})(5) = 25\sqrt{3}$ of 1Δ

We have 3 of these triangles, so total area = $3(25\sqrt{3})$ = $75\sqrt{3}$

OR Area of big Δ = $\frac{1}{2}(10\sqrt{3})(15)$ = $75\sqrt{3}$

Skill 8: Exponents

$5^2 = 25$ { 5 = base; 2 = exponent; 25 = power }

The Operations of Exponents

1) $a^m a^n = a^{m+n}$
2) $\left(a^m\right)^n = a^{mn}$
3) $(ab)^n = a^n b^n$
4) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
5) $a^{-n} = \frac{1}{a^n}$
6) $a^{\frac{m}{n}} = \sqrt[n]{a^m}$

Example 8:

(a) If $n = -1$, evaluate $\frac{(2n)^2}{3} \div (-2)^n$

$\frac{(2(-1))^2}{3} \div (-2)^{-1}$

$\frac{4}{3} \div \left(-\frac{1}{2}\right)$

$\frac{4}{3}\left(-\frac{2}{1}\right) = -\frac{8}{3}$

(b) If $u^{5/3} = v^{-15}$ what is v in terms of u? $v = ?$

$\left(v^{-15}\right)^{-1/15} = \left(u^{5/3}\right)^{-1/15}$

$v = u^{-5/45}$

$v = u^{-1/9}$

(c) x, y, and z are three positive integers. If $x^{-z} = \frac{1}{27}$ and $(x-y)^{1/2} = 4$, what is the value of y?

$\frac{1}{x^z} = \frac{1}{27}$

$x^z = 27$

$x = 27, z = 1$ or $x = 3, z = 3$

$\left((x-y)^{1/2}\right)^2 = (4)^2$

$x - y = 16$

$y = x - 16$

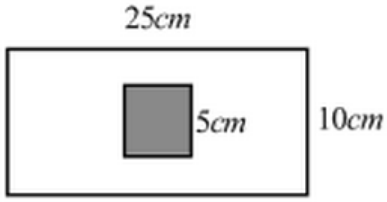
so $y = 27 - 16 = 11$ or $y = 3 - 16 = -13$ not positive

Skill 9: Geometric Probability

Geometric probability is the probability of dealing with the areas of regions instead of the number of outcomes. The equation becomes

$$\text{Probability} = \frac{\text{Favorable region}}{\text{Area of total region}}$$

Example 9:



If you are throwing a dart at the rectangular target above and are equally likely to hit any point on the target, what is the probability that you will hit the small square?

Area of small square $= 5^2 = 25\ cm^2$

Area of big rectangle $= 25(10) = 250\ cm^2$

Probability $= \frac{25}{250} = \boxed{\frac{1}{10} \text{ or } 0.1}$

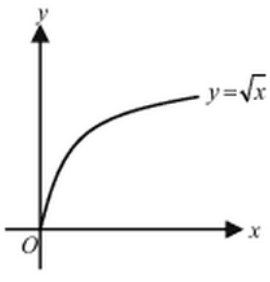
Skill 10: Domain and Range

The **domain** of a given function is the set of "input" values for which the function is defined. In a representation of a function in the *xy*-plane, the domain is represented on the <u>*x*-axis</u> (or abscissa).

The **range** of a function is the set of all "output" values produced by that function. Sometimes it is called the image of the domain of the function. Range is also occasionally used to indicated the difference between the larges and smallest numbers in a set of real-valued data. In a representation of a function in the *xy*-plane, the range is represented on the <u>*y*-axis</u> (or ordinate).

Example 10:

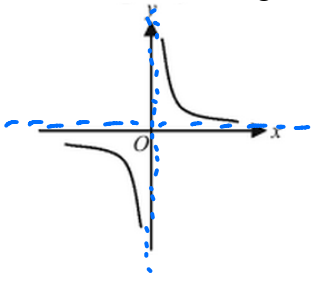
(a) Determine the domain and range for



D: $\{x \mid x \geq 0\}$ or $x \geq 0$

R: $\{y \mid y \geq 0\}$ or $y \geq 0$

(b) Determine the domain and range for



D: $\{x \mid x \neq 0\}$ or $x \neq 0$

R: $\{y \mid y \neq 0\}$ or $y \neq 0$

(c) Find the domain of $f(x)=\dfrac{\sqrt{x+3}}{x-1}$

$x+3\geq 0$ & $x-1\neq 0$

$x\geq -3$ $x\neq 1$

So Domain: $x\geq -3, x\neq 1$

(d) Find the domain of $h(x)=4-2\sqrt{5-3x}$

$5-3x\geq 0$

$-3x\geq -5$

$x\leq \frac{5}{3}$