Lesson 2—Skills 1-5

Skill 1: Absolute Value

The absolute value of x, denoted $|x|$, is simply the distance of x from zero. For any real number k,

1. If $|x| = k$ and $k > 0$, then $x = k$ or $x = -k$
2. If $|x| < k$ and $k > 0$, then $-k < x < k$
3. If $|x| > k$ and $k > 0$, then $x < -k$ or $x > k$
4. $|x| < 6 \Leftrightarrow x^2 < 36 \Leftrightarrow -6 < x < 6$
5. $|x| > 6 \Leftrightarrow x^2 > 36 \Leftrightarrow x < -6$ or $x > 6$
6. $|x - 5| = |5 - x|$

Example 1:

(a) If $|x| = 7$, what is the value of x?

(b) If $|x - 3| = 4$, what is the value of x?

(c) If $|x + 4| < 8$, what is the value of x?

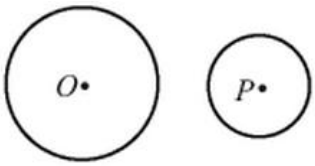
(d) If $|x + 5| > 6$, what is the value of x?

(e) If $-9 < x < 3$, express the interval using absolute value.

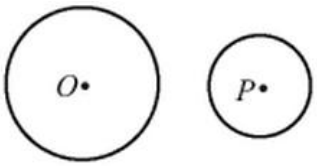
Skill 2: Ratio of Similar Figures

If the ratio of lengths is $a:b$, then
the ratio of areas is $a^2:b^2$, and
the ratio of volumes is $a^3:b^3$

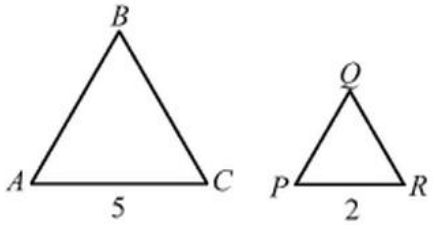
Example 2:



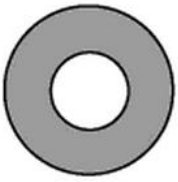
(a) In the figure above, if the ratio of the diameter of circle O to the diameter of circle P is 5:3, what is the ratio of the area of the circle O to the area of circle P ?



(b) In the figures above, if the ratio of the circumference of circle O to the circumference of circle P is 4:3, what is the ratio of the area of circle O to the area of circle P?



(c) The figure above shows two similar triangles with a side 5 and a side 2 respectively. If the area of ΔABC is 30, what is the area of ΔPQR ?



(d) In the figure above, the radius of the larger circle is $\frac{5}{2}$ times the radius of the smaller circle. What fraction of the larger is the shaded region?

Skill 3: Combined Range

If $5 \le A \le 10$ and $2 \le B \le 5$, then the following are true . . .

1. $7 \le A + B \le 15$
2. $10 \le A \times B \le 50$
3. $0 \le A - B \le 8$
4. $1 \le \frac{A}{B} \le 5$

**Smallest Value $\le$ Combined Range $\le$ Largest Value

Example 3:

(a) Given $2 \le P \le 8$ and $1 \le Q \le 4$. By how much is the maximum value of $\frac{P}{Q}$ greater than the minimum value of $\frac{P}{Q}$?

(b) If $-2 \le A \le 2$ and $-6 \le B \le -2$ and $C = (A - B)^2$, what is the smallest value of C?

(c) If $1 \le P \le 6$ and $3 \le Q \le 10$, what is the smallest value of $P \times Q$?

Skill 4: Classifying a Group into Two Different Ways

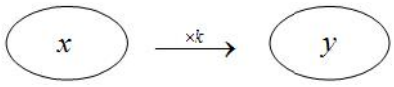
Example 4:

In a certain reading group organized of only senior and junior students, $\frac{3}{5}$ of the students are boys, and the ratio of seniors to juniors is $4:5$. If $\frac{2}{3}$ of the girls are seniors, what fraction of the boys are juniors?

*Making a chart here will help

	BOYS	GIRLS	
SENIORS	A	B	$\frac{4}{9}$
JUNIORS	C	D	$\frac{5}{9}$
	$\frac{3}{5}$	$\frac{2}{5}$	

Skill 5: Direct Variation

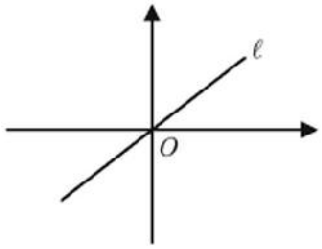


$$y = kx$$

or

$$\frac{y}{x} = \frac{y_1}{x_1} = \frac{y_2}{x_2} = \cdots = k \quad \text{(Constant of Proportionality)}$$

In the xy-plane, $y = mx$, where m is slope as well as the constant of proportionality, but the y-intercept must be zero.



Example 5:

(a) The value y changes directly proportional to the value of x. If $y = 15$ when $x = 5$, what is the value of y when $x = 12.5$.

(b) A group of workers can harvest all the grapes from 10 square meters of a vineyard in $\frac{1}{3}$ minutes. At his rate, how many minutes will the group need to harvest all the grapes from 300 square meters of this vineyard?

(c) To make an orange dye, 5 parts of red dye are mixed with 3 parts of yellow dye. To make a green dye, 4 parts of blue dye are mixed with 2 parts of yellow dye. If equal amounts of green and orange are mixed, what fraction of the new mixture is yellow dye?