Lesson 2—Skills 1-5

Skill 1: Absolute Value

The absolute value of x, denoted $|x|$, is simply the distance of x from zero. For any real number k,

1. If $|x| = k$ and $k > 0$, then $x = k$ or $x = -k$
2. If $|x| < k$ and $k > 0$, then $-k < x < k$
3. If $|x| > k$ and $k > 0$, then $x < -k$ or $x > k$
4. $|x| < 6 \Leftrightarrow x^2 < 36 \Leftrightarrow -6 < x < 6$
5. $|x| > 6 \Leftrightarrow x^2 > 36 \Leftrightarrow x < -6$ or $x > 6$
6. $|x - 5| = |5 - x|$

✗ Drop absolute value:

one is plus, one is minus

Example 1:

(a) If $|x| = 7$, what is the value of x?

$+x = 7$ or $-x = 7$

$x = 7$ or $x = -7$

(b) If $|x - 3| = 4$, what is the value of x?

$+(x-3) = 4$ or $-(x-3) = 4$

$x - 3 = 4$ $\quad$ $x - 3 = -4$

$x = 7$ or $x = -1$

(c) If $|x + 4| < 8$, what is the value of x?

$x + 4 < 8$ or $-(x+4) < 8$

$x < 4$ or $x + 4 > -8$

$x > -12$

$-12 < x < 4$

(d) If $|x + 5| > 6$, what is the value of x?

$x + 5 > 6$ $\quad$ $-(x+5) > 6$

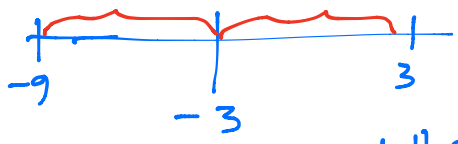
$x > 1$ $\quad$ $x + 5 < -6$

$x < -11$

$x < -11$ or $x > 1$

(e) If $-9 < x < 3$, express the interval using absolute value.

midpt of -9 & 3 $= \frac{-9+3}{2} = \frac{-6}{2} = -3$



where $c = -3$, $d = 6$

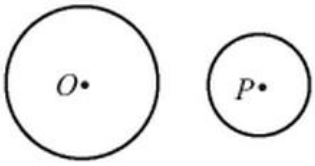
$|x - c| < d$

$|x + 3| < 6$

Skill 2: Ratio of Similar Figures

If the ratio of lengths is $a:b$, then
the ratio of areas is $a^2:b^2$, and
the ratio of volumes is $a^3:b^3$

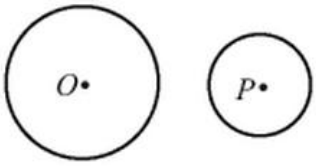
Example 2:



(a) In the figure above, if the ratio of the diameter of circle O to the diameter of circle P is 5:3, what is the ratio of the area of the circle O to the area of circle P?

Length → 5:3

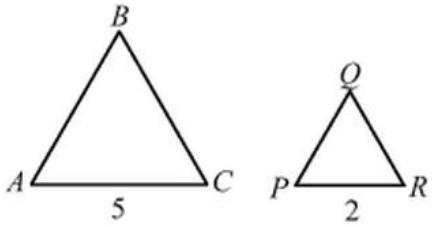
Area → $5^2:3^2 = 25:9$



(b) In the figures above, if the ratio of the circumference of circle O to the circumference of circle P is 4:3, what is the ratio of the area of circle O to the area of circle P?

Length → 4:3

Area → $4^2:3^2 = 16:9$



(c) The figure above shows two similar triangles with a side 5 and a side 2 respectively. If the area of ΔABC is 30, what is the area of ΔPQR?

Big : Small → Length → $\frac{5}{2}$ "x"

Area → $\frac{25}{4} = \frac{30}{x}$

$x = \left(\frac{30}{1}\right)\left(\frac{4}{25}\right)$

$x = \frac{6}{5}(4) = \frac{24}{5}$

(d) In the figure above, the radius of the larger circle is $\frac{5}{2}$ times the radius of the smaller circle. What fraction of the larger is the shaded region?

Length: 5:2

Area: 25:4

Shaded : Total → $\frac{25-4}{25} = \frac{21}{25}$

Skill 3: Combined Range

If $5 \le A \le 10$ and $2 \le B \le 5$, then the following are true . . .

1. $7 \le A + B \le 15$
2. $10 \le A \times B \le 50$
3. $0 \le A - B \le 8$
4. $1 \le \frac{A}{B} \le 5$

**Smallest Value ≤ Combined Range ≤ Largest Value

Example 3:

(a) Given $2 \le P \le 8$ and $1 \le Q \le 4$. By how much is the maximum value of $\frac{P}{Q}$ greater than the minimum value of $\frac{P}{Q}$?

$\frac{P}{Q}$: $\frac{2}{1}, \frac{2}{4}, \frac{8}{1}, \frac{8}{4} = 2, \frac{1}{2}, 8, 2$

So $8 - \frac{1}{2} =$ $\frac{15}{2}$ or 7.5

(b) If $-2 \le A \le 2$ and $-6 \le B \le -2$ and $C = (A - B)^2$, what is the smallest value of C?

$A-B$: $-2+6, -2+2, 2+6, 2+2$

$= 4, 0, 8, 4$

$(A-B)^2$: $16, 0, 64, 16$

Smallest is 0

(c) If $1 \le P \le 6$ and $3 \le Q \le 10$, what is the smallest value of $P \times Q$?

PQ: $1(3), 1(10), 6(3), 6(10)$

$= 3, 10, 18, 60$

Smallest is 3

Skill 4: Classifying a Group into Two Different Ways

Example 4:

In a certain reading group organized of only senior and junior students, $\frac{3}{5}$ of the students are boys, and the ratio of seniors to juniors is $4:5$. If $\frac{2}{3}$ of the girls are seniors, what fraction of the boys are juniors?

wana $\frac{C}{3/5}$

*Making a chart here will help

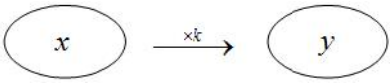
	BOYS	GIRLS	
SENIORS	$A = \frac{8}{45}$	$B = \frac{4}{15}$	$\frac{4}{9}$
JUNIORS	$C = \frac{19}{45}$	D	$\frac{5}{9}$
	$\frac{3}{5}$	$\frac{2}{5}$	

$\frac{2}{3}\left(\frac{2}{5}\right) = B = \frac{4}{15}$, $A + \frac{4}{15} = \frac{4}{9}$, $A = \frac{4}{9} - \frac{4}{15} = \frac{20}{45} - \frac{12}{45} = \frac{8}{45}$

$C + \frac{8}{45} = \frac{3}{5}$, $C = \frac{3}{5} - \frac{8}{45} = \frac{27}{45} - \frac{8}{45} = \frac{19}{45}$

$\frac{19/45}{3/5} = \frac{19}{45}\left(\frac{5}{3}\right) =$ $\frac{19}{27}$

Skill 5: Direct Variation

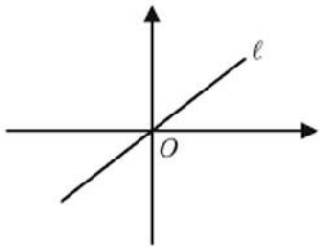


$y = kx$

or

$$\frac{y}{x} = \frac{y_1}{x_1} = \frac{y_2}{x_2} = \cdots = k \quad \text{(Constant of Proportionality)}$$

In the xy-plane, $y = mx$, where m is slope as well as the constant of proportionality, but the y-intercept must be zero.



Example 5:

(a) The value y changes directly proportional to the value of x. If $y = 15$ when $x = 5$, what is the value of y when $x = 12.5$.

$y = kx$, $k = \frac{y_1}{x_1} = \frac{y_2}{x_2}$

$\frac{15}{5} = \frac{y}{12.5}$

$y = 12.5(3)$

$y = 37.5$

(b) A group of workers can harvest all the grapes from 10 square meters of a vineyard in $\frac{1}{3}$ minutes. At his rate, how many minutes will the group need to harvest all the grapes from 300 square meters of this vineyard?

$K = \frac{10 \text{ meter}^2}{\frac{1}{3} \text{ min}} = \frac{300 \text{m}^2}{x}$

$30x = 300$

$x = 10 \text{ min}$

(c) To make an orange dye, 5 parts of red dye are mixed with 3 parts of yellow dye. To make a green dye, 4 parts of blue dye are mixed with 2 parts of yellow dye. If equal amounts of green and orange are mixed, what fraction of the new mixture is yellow dye?

Orange: $5R + 3y = 8$ parts } ×3 LCM of 8 & 6 is 24
Green: $4B + 2y = 6$ parts } ×4

Orange: $15R + 9y = 24$ parts } equal parts
Green: $16B + 8y = 24$ parts

Final: $15R + 16B + 17y = 48$ parts

mixture. 17R + 16D + 15J

So yellow proportion is $\frac{17}{48}$