Name KEY ______________________ Date ______________________ Period ______

Worksheet 9.4—Conic Sections: Parabolas

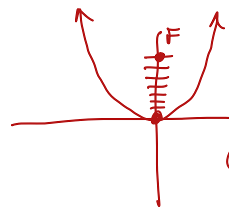
Show all work. No calculator is permitted, unless explicitly stated.

* I use p instead of c here
$(x-h)^2 = 4c(y-k)$
OR $(x-h)^2 = 4p(y-k)$
Both are acceptable and are the focal length.

Multiple Choice

A 1. Find the standard equation of the parabola with the vertex at the origin and a focus at (0,7).

(A) $x^2 = 28y$ (B) $x^2 = 7y$ (C) $x^2 = -7y$
(D) $y^2 = 28x$ (E) $y^2 = 7x$

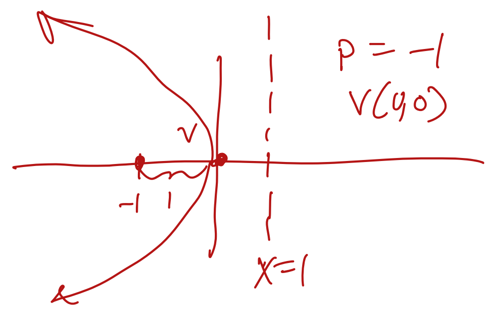


$p = 7$
$V(0,0)$
$(x-0)^2 = 4(7)(y-0)$
$x^2 = 28y$

E 2. Find the standard equation of the parabola with a vertex at the origin and a directrix of $x = 1$.

(A) $x^2 = -4y$ (B) $x^2 = 4y$ (C) $x^2 = y$
(D) $y^2 = x$ (E) $y^2 = -4x$

$(y-0)^2 = 4(-1)(x-0)$
$y^2 = -4x$



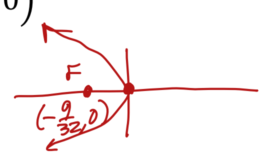
$p = -1$
$V(0,0)$

D 3. Find the vertex and focus of the parabola $y^2 = -\frac{9}{8}x$.

$V(0,0)$

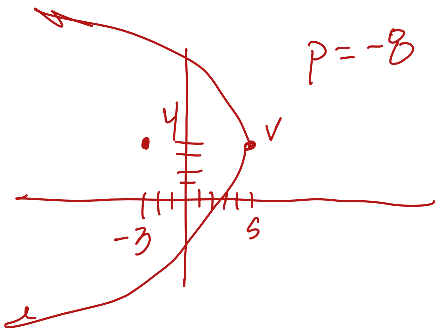
(A) Vertex: $\left(0, -\frac{5}{4}\right)$, Focus: $\left(-\frac{9}{8}, -\frac{9}{8}\right)$ (B) Vertex: $(0,0)$, Focus: $\left(0, -\frac{9}{8}\right)$
(C) Vertex: $(0,0)$, Focus: $\left(-\frac{9}{8}, 0\right)$ (D) Vertex: $(0,0)$, Focus: $\left(-\frac{9}{32}, 0\right)$

$4p = -\frac{9}{8}$
$p = -\frac{9}{32}$



A 4. Find the equation of the parabola with a vertex at $(5,4)$ & focus at $(-3,4)$.

(A) $(y-4)^2 = -32(x-5)$ (B) $(y-4)^2 = 32(x-5)$ (C) $(y+4)^2 = 32(x+5)$
(D) $(y+4)^2 = -32(x-5)$ (E) $(y-4)^2 = 8(x-5)$



$p=-8$ $(y-4)^2 = -32(x-5)$

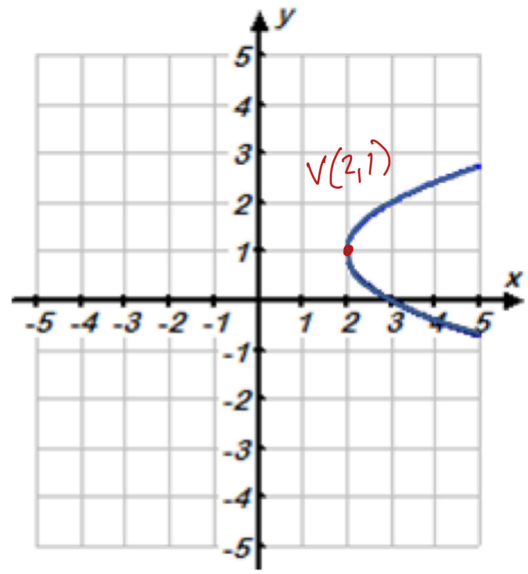
A 5. Which equation below describes the parabola shown in the graph?

(A) $(x-2) = (y-1)^2$

(B) $(x+2) = (y+1)^2$

(C) $(y-2) = (x-1)^2$

(D) $(y+2) = (x+1)^2$



D 6. What is the vertex of the parabola $x = y^2 - 6y + 3$?

(A) $(3,-3)$ (B) $(-3,-3)$
(C) $(6,-3)$ (D) $(-6,3)$

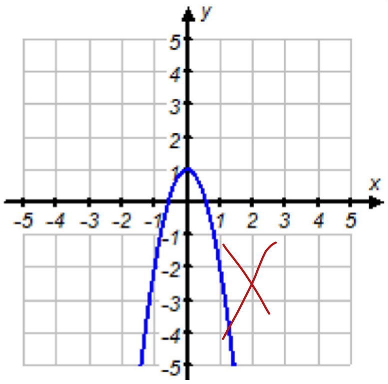
$9 + x - 3 = (y^2 - 6y + 9)$
$(x+6) = (y-3)^2$
$(y-3)^2 = 1\cdot(x+6)$

$V(-6,3)$

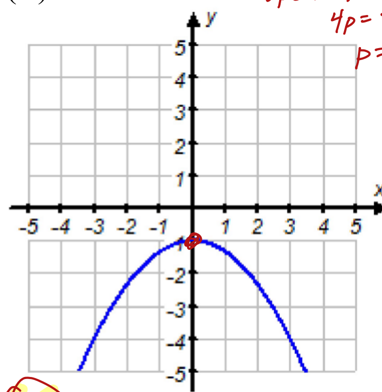
D 7. Which graph below describes the equation $-3x^2 = y + 1$

$(x-0)^2 = -\frac{1}{3}(y+1)$ V(0, −1) opens down $4p = -\frac{1}{3}$ $p = -\frac{1}{12}$ ← narrow parab since small focal length

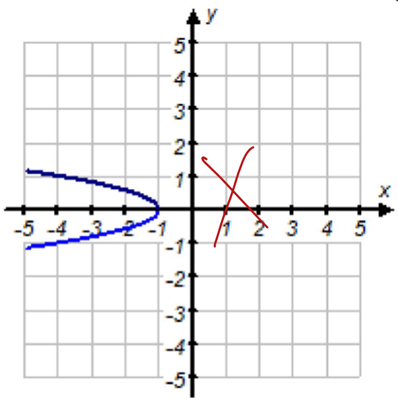
(A)



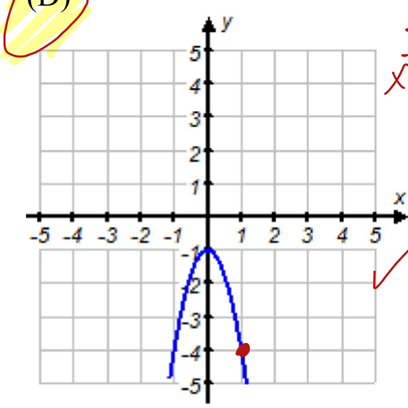
(B)



(C)



(D)



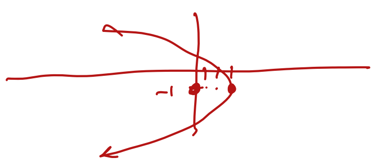
Test a pt: $x=1$: $-3 = y+1$, $y = -4$

B 8. Find the focus of the parabola $(x-3) = -\frac{1}{12}(y+1)^2$.

put into standard form: $(y+1)^2 = -12(x-3)$

V(3, −1), opens Left

(A) $(6,-1)$
(B) $(0,-1)$
(C) $(3,2)$
(D) $(3,-4)$

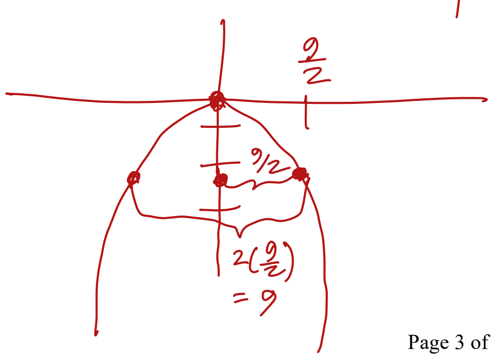


$4p = -12$
$p = -3$
F(0, −1)

D 9. The length of the latus rectum of $x^2 = -9y$ is equal to

V(0,0), opens down $4p = -9$ $p = \frac{-9}{4}$

(A) 3 units
(B) -3 units
(C) $\frac{9}{4}$ units
(D) 9 units



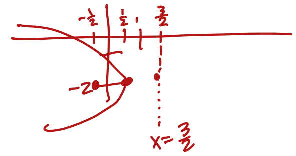
when $y = -\frac{9}{4}$: $x^2 = -9\left(-\frac{9}{4}\right)$
$x^2 = \frac{81}{4}$
$x = \frac{9}{2}$
$2x = 9$

C ___ 10. The equation of the directrix of the parabola $y^2 + 4y + 4x + 2 = 0$ is

(A) $x = 1$ (B) $x = -1$

(C) $x = \frac{3}{2}$ (D) $x = -\frac{3}{2}$

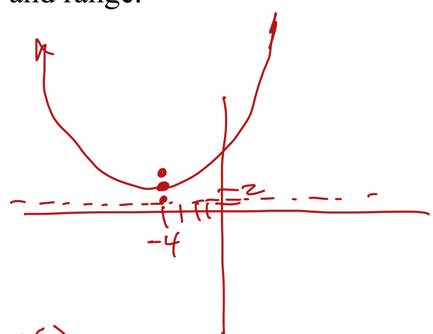
$y^2+4y+4 = -2+4-4x$
$(y+2)^2 = -4x+2$
$(y+2)^2 = -4(x-\frac{1}{2})$
$V(\frac{1}{2}, -2)$
$4p = -4$
$p = -1$
Left



Short Answer

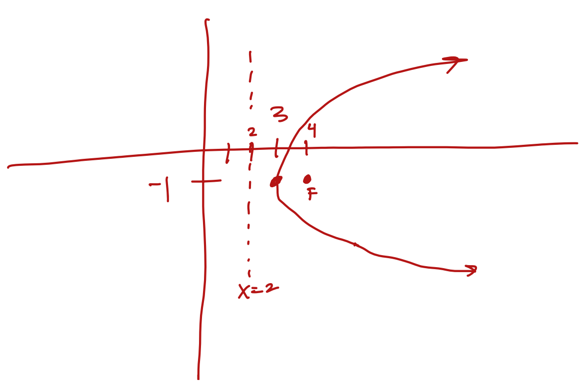
11. Write $y - 24x = 3x^2 + 50$ in standard form. Sketch the graph. Identify the vertex, focus, axis of symmetry, eccentricity, directrix, domain, and range.

$3x^2 + 24x = y - 50$
$3(x^2+8x+16) = y - 50 + 48$
$3(x+4)^2 = y - 2$
$(x+4)^2 = \frac{1}{3}(y-2)$
Vertex @ $(-4, 2)$
$4p = \frac{1}{3}$
$p = \frac{1}{12}$ opens up
Focus @ $(-4, 2+\frac{1}{12}) = (-4, \frac{25}{12})$
Directrix @ $y = 2 - \frac{1}{12} \Rightarrow y = \frac{23}{12}$
$e = 1$
Axis of symmetry @ $x = -4$
D: $\mathbb{R}$, R: $y \in [2, \infty)$

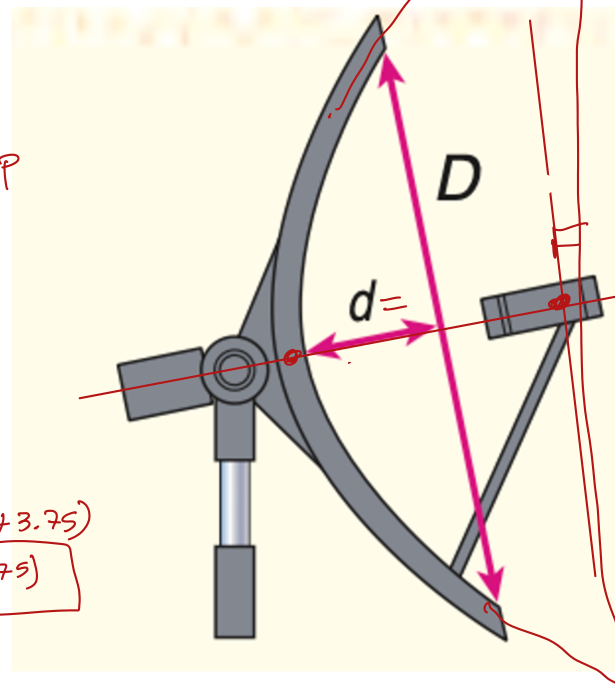


12. Write $2y + 13 = 4x - y^2$ in standard form. Sketch the graph. Identify the vertex, focus, axis of symmetry, eccentricity, directrix, domain, and range.

$y^2 + 2y = 4x - 13$
$y^2 + 2y + 1 = 4x - 13 + 1$
$(y+1)^2 = 4x - 12$
$(y+1)^2 = 4(x-3)$
Vertex @ $(3, -1)$
$4p = 4$
$p = 1$ (opens right)
Focus @ $(4, -1)$
Directrix @ $x = 2$
Axis of symmetry: $y = -1$
$e = 1$
D: $x \in [3, \infty)$
R: $\mathbb{R}$



13. *Satellite TV* The important characteristics of a satellite dish are that the diameter D, depth d, and the ratio $\frac{f}{D}$, where f is the distance between the focus and the vertex (the focal length, what we called c in the lesson). A typical dish has the values $D = 60$ cm, $d = 6.25$ cm, and $\frac{f}{d} = 0.6$.



a. Use this information to write an equation that models a cross section of a satellite dish (as shown to the right). Assume that the focus is at the origin, and the parabola opens to the right.

Template: $(y-k)^2 = 4p(x-h)$

$(y-0)^2 = 4(3.75)(x+3.75)$

$y^2 = 15(x+3.75)$

$\frac{f}{d} = 0.6$

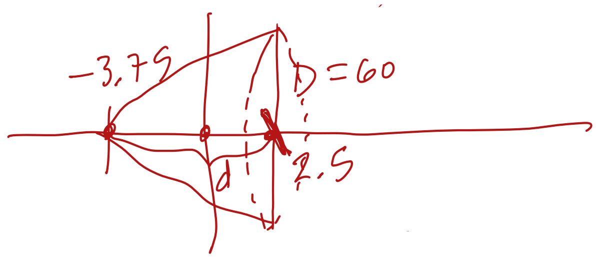
So $\frac{f}{6.25} = 0.6$

$f = 3.75 = c$ or p

So, Vertex @ $(-3.75, 0)$

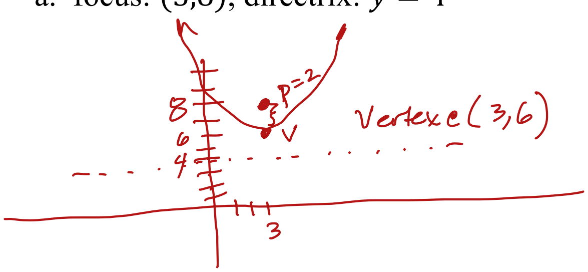
$6.25 - 3.75 = 2.5$

b. Graph the equation.



14. Write an equation for each parabola described below, then draw the graph.

a. focus: $(3,8)$, directrix: $y = 4$

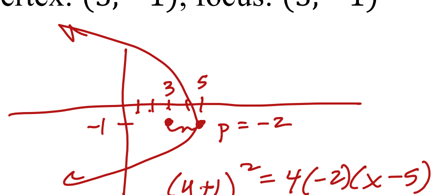


Vertex @ $(3,6)$

$(x-3)^2 = 4(2)(y-6)$

$(x-3)^2 = 8(y-6)$

b. vertex: $(5,-1)$, focus: $(3,-1)$



$p = -2$

$(y+1)^2 = 4(-2)(x-5)$

$(y+1)^2 = -8(x-5)$

15. For the general equation $x = 3y^2 + 4y + 1$

a. Put the equation into standard form.

$3y^2 + 4y = x - 1$

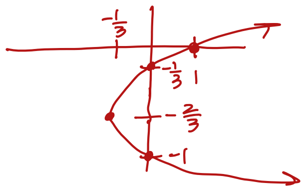
$3(y^2 + \frac{4}{3}y + \frac{4}{9}) = x - 1 + \frac{4}{3}$

$3(y + \frac{2}{3})^2 = x + \frac{1}{3}$

$(y + \frac{2}{3})^2 = \frac{1}{3}(x + \frac{1}{3})$

b. Draw the graph Vertex @ $(-\frac{1}{3}, -\frac{2}{3})$,

$4p = \frac{1}{3}$, $p = \frac{1}{12}$ (opens right)



c. Find the *x*-intercept(s) $(y + \frac{2}{3})^2 = \frac{1}{3}(x + \frac{1}{3})$

For x-ints, Let y=0:

$(\frac{2}{3})^2 = \frac{1}{3}(x + \frac{1}{3})$

$\frac{4}{9} \cdot 3 = x + \frac{1}{3}$

$\frac{4}{3} - \frac{1}{3} = x$

$x = 1$

x-int @ (1,0)

d. Find the *y*-intercept(s) $(y + \frac{2}{3})^2 = \frac{1}{3}(x + \frac{1}{3})$

For y-int, let x=0: $(y + \frac{2}{3})^2 = \frac{1}{3}(\frac{1}{3})$

$(y + \frac{2}{3})^2 = \frac{1}{9}$

$y + \frac{2}{3} = \pm \frac{1}{3}$

$y = -\frac{2}{3} + \frac{1}{3}$ or $y = -\frac{2}{3} - \frac{1}{3}$

$y = -\frac{1}{3}$ or $y = -1$

y-ints @ $(0, -\frac{1}{3})$ & $(0, -1)$

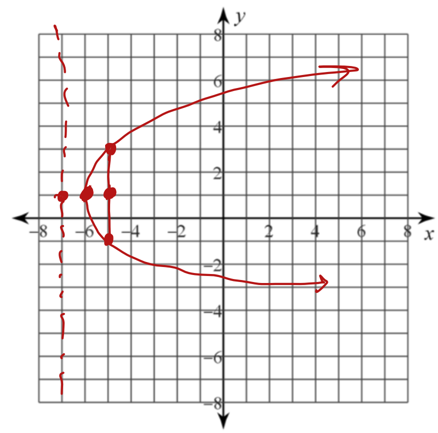
e. What is the equation for the axis of symmetry?

$y = -\frac{2}{3}$

f. What are the coordinates of the vertex?

Vertex @ $(-\frac{1}{3}, -\frac{2}{3})$

16. Write the general equation $-y^2 + 4x + 2y + 23 = 0$ in standard form. Determine the direction of the opening, the coordinates of the vertex, the focus, the equation of the directrix, domain, range, and the coordinates of the endpoints of the latus rectum of the parabola. Then sketch its graph.

$-y^2 + 4x + 2y + 23 = 0$	Standard Form:	
$y^2 - 2y + 1 = 4x + 23 + 1$ $(y-1)^2 = 4x + 24$ $(y-1)^2 = 4(x+6)$ $4p = 4$ $p = 1$ Latus Rectum: when $x = -5$: $(y-1)^2 = 4(1)$ $y - 1 = \pm 2$ $y = 1 + 2,\ y = 1 - 2$ $y = 3,\ y = -1$	$(y-1)^2 = 4(x+6)$ Direction of Opening: Right Vertex: $(-6, 1)$ Focus: $(-5, 1)$ Directrix: $x = -7$ Domain: $x \in [-6, \infty)$ Range: $\mathbb{R}$ Endpoints of Latus Rectum: $(-5,-1)$ & $(-5,3)$	

17. Determine an a, b, and c so that the parabola $y = ax^2 + bx + c$ passes through the points (0,9), (1,1), & (2,1).

@$(0,9)$: $a(0^2) + b(0) + c = 9$
@$(1,1)$: $a(1^2) + b(1) + c = 1$
@$(2,1)$: $a(2^2) + b(2) + c = 1$

$c = 9$
$a + b + c = 1$
$4a + 2b + c = 1$

$a + b + 9 = 1$
$4a + 2b + 9 = 1$

$a + b = -8$
$4a + 2b = -8$

$a = -b - 8$
$4a + 2b = -8$

$4(-b-8) + 2b = -8$
$-4b - 32 + 2b = -8$
$-2b = 24$
$b = -12$

So, $a = -(-12) - 8$
$a = 12 - 8$
$a = 4$

So, $y = 4x^2 - 12x + 9$

18. *Identifying Conics: Mixed Review*
Classify each general equation of a conic section as a circle, ellipse, hyperbola, or parabola. Complete the square then find the following information:
For circles, identify the center and radius.
For ellipses and hyperbolas, identify the center, vertices, and foci.
For parabolas, identify the vertex and focus.

a. $4x^2 - y^2 - 16x - 4 = 0$

Hypebola (x^2 & y^2 opp signs)

$a^2+b^2=c^2$

$c^2=25$

$c=5$

$4(x^2-4x+4)-y^2=4+16$

$4(x-2)^2-y^2=20$

$\frac{(x-2)^2}{5}-\frac{y^2}{20}=1$ (opens Horiz)

Center @ $(2,0)$

Vertices @ $(2-\sqrt{5},0)(2+\sqrt{5},0)$

Foci @ $(-3,0)(7,0)$

b. $2x^2 + 16x + 3y + 38 = 0$

Parabola (only one squared term)

$2(x^2+8x+16)=-3y-38+32$

$2(x+4)^2=-3y-6$

$2(x+4)^2=-3(y+2)$

$(x+4)^2=-\frac{3}{2}(y+2)$

$4p=-\frac{3}{2}$

$p=-\frac{3}{8}$ opens down

Vertex @ $(-4,-2)$, Focus $\left(-4,-2-\frac{3}{8}\right)=\left(-4,-\frac{19}{8}\right)$

c. $16x^2 + 9y^2 + 128x - 54y + 193 = 0$

Ellipse (x^2 & y^2 are same sign, but different values)

$16(x^2+8x+16)+9(y^2-6y+9)=-193+256+81$

$16(x+4)^2+9(y-3)^2=144$

$\frac{(x+4)^2}{9}+\frac{(y-3)^2}{16}=144$ (Vert)

$a^2-b^2=c^2$

$16-9=c^2$

$c=\sqrt{7}$

Center @ $(-4,3)$

Vertices: $(-4,-1)$ & $(-4,7)$

Foci: $(-4,3-\sqrt{7})$ & $(-4,3+\sqrt{7})$

d. $4x^2 + 4y^2 + 20x + 8y - 7 = 0$

Circle (x^2 & y^2 same sign & same value)

$4(x^2+5x+\frac{25}{4})+4(y^2+2y+1)=7+25+4$

$4(x+\frac{5}{2})^2+4(y+1)^2=36$

$(x+\frac{5}{2})^2+(y+1)^2=9$

Center @ $(-\frac{5}{2},-1)$

radius $=3$