① $1 + \frac{\cot A}{\csc A} - \sin^2 A = \cos A(\cos A + 1)$

$1 - \sin^2 A + \frac{\cos A}{\sin A} \cdot \sin A$

$\cos^2 A + \cos A$

$\cos A(\cos A + 1)$

– done

② $\tan B + \cot B = \csc B \sec B$

$\frac{\sin B}{\cos B} + \frac{\cos B}{\sin B}$

$\frac{\sin^2 B + \cos^2 B}{\sin B \cos B}$

$\frac{1}{\sin B \cos B}$

$\csc B \sec B$

– done

③ $\frac{\cot \phi}{\sec \phi} = \csc \phi - \sin \phi$

$\frac{1}{\sin \phi} - \frac{\sin \phi}{1}$

$\frac{1 - \sin^2 \phi}{\sin \phi}$

$\frac{\cos^2 \phi}{\sin \phi}$

$\frac{\cos \phi}{\sin \phi} \cdot \cos \phi$

$\cot \phi \cdot \frac{1}{\sec \phi}$

$\frac{\cot \phi}{\sec \phi}$ – done

④ $\frac{\sec \beta}{1 + \cos \beta} = \csc^2 \beta (\sec \beta - 1)$

$\left(\frac{1 - \cos \beta}{1 - \cos \beta}\right) \frac{\sec \beta}{1 + \cos \beta}$

$\frac{\sec \beta - 1}{1 - \cos^2 \beta}$

$\frac{\sec \beta - 1}{\sin^2 \beta}$

$\csc^2 \beta (\sec \beta - 1)$ – done

⑤ $(\tan \alpha - \sec \alpha)^2 = \frac{1 - \sin \alpha}{1 + \sin \alpha}$

$\tan^2 \alpha - 2 \tan \alpha \sec \alpha + \sec^2 \alpha$

$\frac{\sin^2 \alpha}{\cos^2 \alpha} - 2 \frac{\sin \alpha}{\cos \alpha} \cdot \frac{1}{\cos \alpha} + \frac{1}{\cos^2 \alpha}$

$\frac{\sin^2 \alpha - 2 \sin \alpha + 1}{\cos^2 \alpha}$

$\frac{(\sin \alpha - 1)^2}{(1 - \sin^2 \alpha)}$

$\frac{-(1 - \sin \alpha)(-1)(1 - \sin \alpha)}{(1 - \sin \alpha)(1 + \sin \alpha)}$

$\frac{1 - \sin \alpha}{1 + \sin \alpha}$

– done

⑥ $\sin^4 \psi - \cos^4 \psi = 1 - 2\cos^2 \psi$

$(\sin^2 \psi - \cos^2 \psi)(\sin^2 \psi + \cos^2 \psi)$

$(\sin^2 \psi - \cos^2 \psi)(1)$

$(1 - \cos^2 \psi) - \cos^2 \psi$

$1 - 2\cos^2 \psi$

– done

(7) $\sec^4\delta - 2\sec^2\delta\tan^2\delta + \tan^4\delta = 1$

$(\sec^2\delta - \tan^2\delta)^2$

$((1+\tan^2\delta) - \tan^2\delta)^2$

1^2

1

– done

(8) $\sqrt[3]{\tan^2 x - \sec^2 x} = -1$

$\sqrt[3]{\tan^2 x - (1+\tan^2 x)}$

$\sqrt[3]{\tan^2 x - 1 - \tan^2 x}$

$\sqrt[3]{-1}$

-1

– done

(9) $\dfrac{1}{\csc y + \cot y} = \dfrac{1-\cos y}{\sin y}$

$\dfrac{1}{\frac{1}{\sin y} + \frac{\cos y}{\sin y}}$

$\dfrac{1}{\frac{1+\cos y}{\sin y}}$

$\left(\dfrac{1-\cos y}{1-\cos y}\right)\dfrac{\sin y}{1+\cos y}$

$\dfrac{\sin y(1-\cos y)}{1-\cos^2 y}$

$\dfrac{\sin y(1-\cos y)}{\sin^2 y}$

$\dfrac{1-\cos y}{\sin y}$

– done

(10) $(\sec x+\tan x)^3(\sec x-\tan x)^4 = \dfrac{1-\sin x}{\cos x}$

$[(\sec x+\tan x)(\sec x-\tan x)]^3(\sec x-\tan x)$

$(\sec^2 x - \tan^2 x)^3\left(\dfrac{1}{\cos x} - \dfrac{\sin x}{\cos x}\right)$

$(1^3)\left(\dfrac{1-\sin x}{\cos x}\right)$

$\dfrac{1-\sin x}{\cos x}$

– done

(11) $\dfrac{\sin K - 1}{\cos K} = \tan K - \sec K$

$\dfrac{\sin K}{\cos K} - \dfrac{1}{\cos K}$

$\dfrac{\sin K - 1}{\cos K}$

– done

(12) $\dfrac{\sec w}{\sin w} - \dfrac{\sin w}{\cos w} = \cot w$

$\dfrac{\cos w \sec w - \sin^2 w}{\sin w \cos w}$

$\dfrac{1-\sin^2 w}{\sin w \cos w}$

$\dfrac{\cos^2 w}{\sin w \cos w}$

$\dfrac{\cos w}{\sin w}$

$\cot w$

– done

(13) $\frac{\sec x}{1+\cos x} = \csc^2 x(\sec x - 1)$

$\left(\frac{1-\cos x}{1-\cos x}\right)\left(\frac{\sec x}{1+\cos x}\right)$

$\frac{\sec x - 1}{1-\cos^2 x}$

$\frac{\sec x - 1}{\sin^2 x}$

$\csc^2 x(\sec x - 1)$

(14) $\tan x - 1 = \frac{\sec x - \csc x}{\csc x}$

$\frac{\sec x}{\csc x} - \frac{\csc x}{\csc x}$

$\frac{1}{\cos x} \cdot \frac{\sin x}{1} - 1$

$\frac{\sin x}{\cos x} - 1$

$\tan x - 1$

- done

(15) $\csc x(\sin x + \cos x) = \cot x + 1$

$\csc x \sin x + \csc x \cos x$

$1 + \frac{1}{\sin x} \cdot \cos x$

$1 + \cot x$

$\cot x + 1$

- done

(16) $\frac{\sin y + \tan y}{1+\sec y} = \sin y$

$\frac{\sin y + \frac{\sin y}{\cos y}}{1 + \frac{1}{\cos y}}\left(\frac{\cos y}{\cos y}\right)$

$\frac{\sin y \cos y + \sin y}{\cos y + 1}$

$\frac{\sin y(\cos y + 1)}{(\cos y + 1)}$

$\sin y$

- done

(17) $\tan^2 x \sin^2 x = \tan^2 x - \sin^2 x$

$\frac{\sin^2 x}{\cos^2 x} - \sin^2 x$

$\frac{\sin^2 x - \cos^2 x \sin^2 x}{\cos^2 x}$

$\frac{\sin^2 x(1 - \cos^2 x)}{\cos^2 x}$

$\frac{\sin^2 x}{\cos^2 x} \cdot (\sin^2 x)$

$\tan^2 x \sin^2 x$

- done

(18) $\frac{\cos x + 1}{\sin^3 x} = \frac{\csc x}{1-\cos x}\left(\frac{1+\cos x}{1+\cos x}\right)$

$\frac{(\cos x + 1)\left(\frac{1}{\sin x}\right)}{1 - \cos^2 x}$

$\frac{(\cos x + 1)\left(\frac{1}{\sin x}\right)}{\sin^2 x}\left(\frac{\sin x}{\sin x}\right)$

$\frac{\cos x + 1}{\sin^3 x}$

- done

(19) $\dfrac{\tan\Delta}{\sec\Delta} + \dfrac{\cot\Delta}{\csc\Delta} = \sin\Delta + \cos\Delta$

$\dfrac{\frac{\sin\Delta}{\cos\Delta}}{\frac{1}{\cos\Delta}}\left(\dfrac{\cos\Delta}{\cos\Delta}\right) + \dfrac{\frac{\cos\Delta}{\sin\Delta}}{\frac{1}{\sin\Delta}}\left(\dfrac{\sin\Delta}{\sin\Delta}\right)$

$\dfrac{\sin\Delta}{1} + \dfrac{\cos\Delta}{1}$

$\sin\Delta + \cos\Delta$ — done

(20) $\dfrac{\sin x - \cos x}{\sin x + \cos x} = \dfrac{2\sin^2 x - 1}{1 + 2\sin x \cos x}$

$\left(\dfrac{\sin x + \cos x}{\sin x + \cos x}\right)\left(\dfrac{\sin x - \cos x}{\sin x + \cos x}\right)$

$\dfrac{\sin^2 x - \cos^2 x}{\sin^2 x + 2\sin x\cos x + \cos^2 x}$

$\dfrac{\sin^2 x - (1 - \sin^2 x)}{1 + 2\sin x\cos x}$

$\dfrac{2\sin^2 x - 1}{1 + 2\sin x\cos x}$ — done

(21) $(\sin x + \cos x)^2 \tan x = \tan x + 2\sin^2 x$

$(\sin^2 x + 2\sin x\cos x + \cos^2 x)\tan x$

$(1 + 2\sin x\cos x)\tan x$

$\tan x + 2\sin x\cos x \tan x$

$\tan x + 2\sin x\cos x\left(\dfrac{\sin x}{\cos x}\right)$

$\tan x + 2\sin^2 x$ — done

(22) $\dfrac{\csc^2 P - 1}{\csc P + 1} = \dfrac{1}{\sin P} - 1$

$\dfrac{(\csc P - 1)(\csc P + 1)}{(\csc P + 1)}$

$\csc P - 1$

$\dfrac{1}{\sin P} - 1$ — done

(23) $\dfrac{\sec^2 x - 6\tan x + 7}{\sec^2 x - 5} = \dfrac{\tan x - 4}{\tan x + 2}$

$\dfrac{1 + \tan^2 x - 6\tan x + 7}{1 + \tan^2 x - 5}$

$\dfrac{\tan^2 x - 6\tan x + 8}{\tan^2 x - 4}$

$\dfrac{(\tan x - 4)(\tan x - 2)}{(\tan x + 2)(\tan x - 2)}$

$\dfrac{\tan x - 4}{\tan x + 2}$ — done