Precal Matters WS 6.1 KEY

① (A) $\cos\left(\frac{\pi}{2}-x\right)=\sin x$, (B) $\cos\left(x-\frac{\pi}{2}\right)=\cos\left(-\left(\frac{\pi}{2}-x\right)\right)=\cos\left(\frac{\pi}{2}-x\right)$ $=\sin x$

D

(C) $\sqrt{1-\cos^2 x}=\sqrt{\sin^2 x}=\sin x$ (D) $\tan x\sec x=\frac{\sin x}{\cos x}\left(\frac{1}{\cos x}\right)=\frac{\sin x}{\cos^2 x}\neq\sin x$

(E) $-\sin(-x)=-(-\sin x)=\sin x$ D

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② 4 trig functions (A) ODD. (all but $\sin x, \csc x$) only 2 are even, all are periodic only 2 are continuous, only 2 are bounded

A

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③ only 2 are monotonic ($\tan x, \cot x$) C None are 1:1, 4 are odd, 4 are discontinuous 4 are unbounded

C

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④ $(\sec y+1)(\sec y-1)=\sec^2 y-1=\tan^2 y$

C

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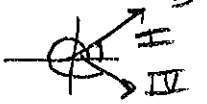
⑤ $3\cos^2 x+\cos x-2=0$

$(3\cos x-2)(\cos x+1)=0$

$\cos x=\frac{2}{3}$, $\cos x=-1$ $\longrightarrow$ 3 solutions

$x=\pi$

D



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⑥ (a) $\sin\theta=0.45$, $\cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta=$ 0.45

(b) $\cot(-\theta)=7.89$

$\tan\left(\theta-\frac{\pi}{2}\right)=\tan\left(-\left(\frac{\pi}{2}-\theta\right)\right)$

$=-\tan\left(\frac{\pi}{2}-\theta\right)=-\cot\theta$

$=\cot(-\theta)=$ 7.89

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⑦ (a) $\tan x\cos x$ $\frac{\sin x}{\cos x}\cdot\cos x$ $\sin x$

(b) $\sec\beta\sin\left(\frac{\pi}{2}-\beta\right)$ $\left(\frac{1}{\cos\beta}\right)\frac{\cos\beta}{1}$ 1

(c) $\frac{1-\cos^2 K}{\sin K}$ $\frac{\sin^2 K}{\sin K}$ $\sin K$

(d) $\frac{\sin^2 u+\tan^2 u+\cos^2 u}{\sec u}$ $\frac{1+\tan^2 u}{\sec u}=\frac{\sec^2 u}{\sec u}$ $\sec u$

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(8) (a) $-\sec(-x)\cos(-x)$

$-\sec(x)\cos(x)$

$-\left(\frac{1}{\cos x}\right)\left(\frac{\cos x}{1}\right)$

$\boxed{-1}$

(b) $\cot(-\Omega)\cot\left(\Omega-\frac{\pi}{2}\right)$

$-\cot(\Omega)\cot\left(-\left(\frac{\pi}{2}-\Omega\right)\right)$

$(-\cot\Omega)\left(-\cot\left(\frac{\pi}{2}-\Omega\right)\right)$

$\cot\Omega\cdot\tan\Omega$

$\boxed{1}$

(c) $\sin^2(-\alpha)+\cos^2(-\alpha)$

$(\sin(-\alpha))^2+(\cos(-\alpha))^2$

$(-\sin\alpha)^2+(\cos\alpha)^2$

$\sin^2\alpha+\cos^2\alpha$

$\boxed{1}$

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(9) (a) $\dfrac{\tan\left(\frac{\pi}{2}-x\right)\csc x}{1+\cot^2 x}$

$\dfrac{\cot x\cdot\csc x}{\csc^2 x}$

$\dfrac{\cot x}{\csc x}$

$\dfrac{\cos x}{\sin x}\cdot\dfrac{\sin x}{1}$

$\boxed{\cos x}$

(b) $\dfrac{1+\tan\mu}{1+\cot\mu}$

$\dfrac{1+\frac{\sin\mu}{\cos\mu}}{1+\frac{\cos\mu}{\sin\mu}}\left(\dfrac{\cos\mu\sin\mu}{\cos\mu\sin\mu}\right)$

$\dfrac{\cos\mu\sin\mu+\sin^2\mu}{\cos\mu\sin\mu+\cos^2\mu}$

$\dfrac{\sin\mu(\cos\mu+\sin\mu)}{\cos\mu(\sin\mu+\cos\mu)}$

$\dfrac{\sin\mu}{\cos\mu}$

$\boxed{\tan\mu}$

(c) $\left(\sec^2\phi+\csc^2\phi\right)-\left(\tan^2\phi+\cot^2\phi\right)$

$1+\tan^2\phi+1+\cot^2\phi-\tan^2\phi-\cot^2\phi$

$1+1$

$\boxed{2}$

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(10) (a) $\sin x(\tan x+\cot x)$

$\sin x\left(\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\right)$

$\dfrac{\sin^2 x}{\cos x}+\dfrac{\cos x}{1}\left(\dfrac{\cos x}{\cos x}\right)$

$\dfrac{\sin^2 x+\cos^2 x}{\cos x}$

$\dfrac{1}{\cos x}$

$\boxed{\sec x}$

(b) $\dfrac{(\sec\psi-\tan\psi)(\sec\psi+\tan\psi)}{\sec\psi}$

$\left(\sec^2\psi-\tan^2\psi\right)\left(\frac{1}{\sec\psi}\right)$

$\left(\dfrac{1}{\cos^2\psi}-\dfrac{\sin^2\psi}{\cos^2\psi}\right)\cos\psi$

$\left(\dfrac{1-\sin^2\psi}{\cos^2\psi}\right)\cos\psi$

$\left(\dfrac{\cos^2\psi}{\cos^2\psi}\right)(\cos\psi)$

$\boxed{\cos\psi}$

(c) $\dfrac{\sec^2 A\csc A}{\sec^2 A+\csc^2 A}$

$\dfrac{\frac{1}{\cos^2 A}\cdot\frac{1}{\sin A}}{\frac{1}{\cos^2 A}+\frac{1}{\sin^2 A}}\left(\dfrac{\cos^2 A\sin^2 A}{\cos^2 A\sin^2 A}\right)$

$\dfrac{\sin A}{\sin^2 A+\cos^2 A}$

$\dfrac{\sin A}{1}$

$\boxed{\sin A}$

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(11) (a) $\frac{1}{\sin^2 x} + \frac{\sec^2 x}{\tan^2 x}$

$\frac{1}{\sin^2 x} + \sec^2 x \cdot \cot^2 x$

$\frac{1}{\sin^2 x} + \frac{\sec^2 x \cdot \cos^2 x}{\sin^2 x}$

$\frac{1 + \left(\frac{1}{\cos^2 x}\right)\cos^2 x}{\sin^2 x}$

$\frac{1+1}{\sin^2 x}$

$2\csc^2 x$

(b) $\frac{1}{\sec x - 1} - \frac{1}{\sec x + 1}$

$\frac{(\sec x + 1) - (\sec x - 1)}{(\sec x - 1)(\sec x + 1)}$

$\frac{\sec x + 1 - \sec x + 1}{\sec^2 x - 1}$

$\frac{2}{\tan^2 x}$

$2\cot^2 x$

(c) $\frac{\sin x}{1 - \cos x} + \frac{1 - \cos x}{\sin x}$

$\frac{\sin^2 x + (1 - \cos x)^2}{\sin x (1 - \cos x)}$

$\frac{\sin^2 x + 1 - 2\cos x + \cos^2 x}{\sin x (1 - \cos x)}$

$\frac{(\sin^2 x + \cos^2 x) + 1 - 2\cos x}{\sin x (1 - \cos x)}$

$\frac{2 - 2\cos x}{\sin x (1 - \cos x)}$

$\frac{2(1 - \cos x)}{\sin x (1 - \cos x)}$

$2\csc x$

(12) (a) $1 - 2\sin x + (1 - \cos^2 x)$

$1 - 2\sin x + \sin^2 x$

$(1 - \sin x)^2$

(b) $4\tan^2 x - \frac{4}{\cot x} + \sin x \csc x$

$4\tan^2 x - 4\tan x + 1$

$(2\tan x - 1)^2$

(c) $\sec^2 x - \sec x + \tan^2 x$

$\sec^2 x - \sec x + \sec^2 x - 1$

$2\sec^2 x - \sec x - 1$

$(2\sec x + 1)(\sec x - 1)$

(13) (a) $\frac{1 - \sin^2 x}{1 - \sin x}$

$\frac{(1 - \sin x)(1 + \sin x)}{(1 - \sin x)}$

$1 + \sin x$

(b) $\frac{1 - \tan^2 \beta}{\tan \beta + 1}$

$\frac{(1 - \tan \beta)(1 + \tan \beta)}{(1 + \tan \beta)}$

$1 - \tan \beta$

(c) $\frac{\tan^2 \alpha}{\sec \alpha + 1}$

$\frac{\sec^2 \alpha - 1}{\sec \alpha + 1}$

$\frac{(\sec \alpha + 1)(\sec \alpha - 1)}{(\sec \alpha + 1)}$

$\sec \alpha - 1$

(14) (a) $\cos^3 x + \sin^2 x \cos x$

$\cos x(\cos^2 x + \sin^2 x)$

$\cos x \cdot 1$

$\boxed{\cos x}$

(b) $\tan A \cos A \csc A$

$\frac{\sin A}{\cos A}\left(\frac{\cos A}{1}\right)\left(\frac{1}{\sin A}\right)$

$\frac{1}{1}$

$\boxed{1}$

(c) $\dfrac{\cos\theta}{\sec\theta + \tan\theta}$

$\dfrac{\cos\theta}{\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}}\left(\frac{\cos\theta}{\cos\theta}\right)$

$\dfrac{\cos^2\theta}{1+\sin\theta}$

$\dfrac{1-\sin^2\theta}{1+\sin\theta}$

$\dfrac{(1+\sin\theta)(1-\sin\theta)}{(1+\sin\theta)}$

$\boxed{1-\sin\theta}$

(d) $\dfrac{\tan^2 x + 2}{\sec^2 x} - 1$

$\dfrac{\sec^2 x - 1 + 2}{\sec^2 x} - \dfrac{\sec^2 x}{\sec^2 x}$

$\dfrac{\sec^2 x + 1 - \sec^2 x}{\sec^2 x}$

$\frac{1}{\sec^2 x}$

$\boxed{\cos^2 x}$

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(15) (a) $\dfrac{\tan y}{\csc y} = \sec y - \cos y$, for $y = \frac{\pi}{4}$

$\dfrac{\tan\frac{\pi}{4}}{\csc\frac{\pi}{4}}$ | $\sec\frac{\pi}{4} - \cos\frac{\pi}{4}$

$\frac{1}{\sqrt{2}}$ | $\frac{\sqrt{2}}{1} - \frac{\sqrt{2}}{2}$

$\frac{\sqrt{2}}{2}$ | $\frac{2\sqrt{2} - \sqrt{2}}{2}$

$\frac{\sqrt{2}}{2}$ ✓

(b) $\cot(-x)\cos(-x) + \sin(-x) = -\csc x$, $x = \frac{\pi}{4}$

$\cot\left(-\frac{\pi}{4}\right)\cos\left(-\frac{\pi}{4}\right) + \sin\left(-\frac{\pi}{4}\right)$ | $-\csc\frac{\pi}{4}$

$\cot\left(\frac{7\pi}{4}\right)\cos\left(\frac{7\pi}{4}\right) + \sin\left(\frac{7\pi}{4}\right)$ | $-\sqrt{2}$

$(-1)\left(\frac{\sqrt{2}}{2}\right) - \frac{\sqrt{2}}{2}$

$\frac{-2\sqrt{2}}{2}$

$-\sqrt{2}$ ✓

(c) $\dfrac{1+\sec^2 x}{1+\tan^2 x} = 1 + \cos^2 x$, $x = \frac{\pi}{4}$

$\dfrac{1+\left(\sec\frac{\pi}{4}\right)^2}{1+\left(\tan\frac{\pi}{4}\right)^2}$ | $1+\left(\cos\frac{\pi}{4}\right)^2$

$\dfrac{1+(\sqrt{2})^2}{1+(1)^2}$ | $1+\left(\frac{\sqrt{2}}{2}\right)^2$

$\frac{3}{2}$ | $1+\frac{2}{4}$

$\frac{3}{2}$ ✓

(16) (a) $2\cos x \sin x = \cos x$

$2\cos x \sin x - \cos x = 0$

$\cos x(2\sin x - 1) = 0$

$\cos x = 0, \ \sin x = \frac{1}{2}$

$x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$

(b) $\sqrt{2}\tan x \cos x = \tan x$

$\tan x(\sqrt{2}\cos x - 1) = 0$

$\tan x = 0, \ \cos x = \frac{\sqrt{2}}{2}$

$x = 0, \pi, \frac{\pi}{4}, \frac{7\pi}{4}$

(c) $2\tan^2 x = 6$

$\tan x = \pm\sqrt{3}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

(17) (a) $4\cos^2 x - 4\cos x = -1$

$4\cos^2 x - 4\cos x + 1 = 0$

$(2\cos x - 1)^2 = 0$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

$x = \frac{\pi}{3} + 2\pi n$

$x = \frac{5\pi}{3} + 2\pi n$

(b) $3\sin t = 2\cos^2 t$

$2(1 - \sin^2 t) - 3\sin t = 0$

$-2\sin^2 t - 3\sin t + 2 = 0$

$2\sin^2 t + 3\sin t - 2 = 0$

$(2\sin t - 1)(\sin t + 2) = 0$

$\sin t = \frac{1}{2}, \ \sin t = -2$ (No soln)

$t = \frac{\pi}{6}, \frac{5\pi}{6}$

$t = \frac{\pi}{6} + 2\pi n$

$t = \frac{5\pi}{6} + 2\pi n$

(c) $\cos(\sin x) = 1$

$\sin x = \cos^{-1}(1)$

$\sin x = 0$

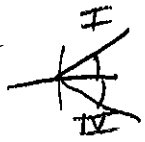
$x = 0 + 2\pi n$

$x = \pi + 2\pi n$

or

$x = 2\pi n$

(18) (a) $\cos x = 1.77$



$x = \cos^{-1} 1.77$

but $1.77 > 1$

so

No Solution

(b) $\cot x = 0.2$

$\tan x = \frac{1}{0.2} = 5$

$x = \tan^{-1} 5$

$x = 1.373 \text{ rad} + \pi n$

(c) $\sin^2 x = 0.9$

$\sin x = \pm\sqrt{0.9}$

$x = 1.249 + 2\pi n$

$x = (\pi - 1.249\ldots) + 2\pi n$

$= 1.892 + 2\pi n$

so

$x = \pm 1.249 + 2\pi n$

$x = \pm 1.892 + 2\pi n$