Name KEY Date_______________ Period______

Worksheet 2.2—Limits & Continuity

Give simplified, exact values for all answers. **No Calculator is Permitted.**

I. Multiple Choice

B 1. Suppose $\lim_{x \to 5} f(x) = 4$. Which of these statements must be true?

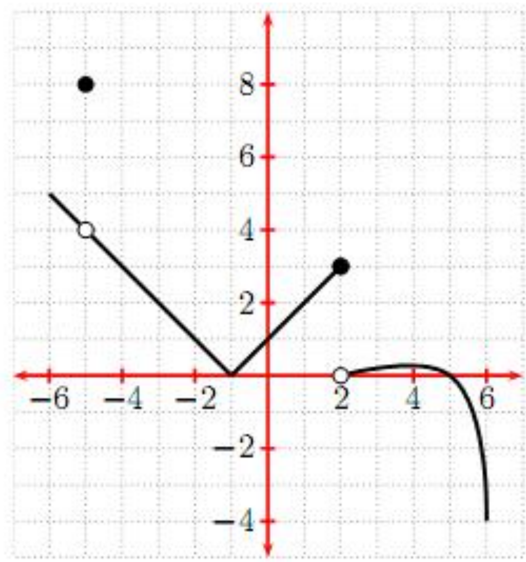
I. The range of f contains 4.

II. $f(5) = 4$

III. As x approaches 5 from the left, $f(x)$ approaches 4.

(A) II only (B) III only (C) I and III only (D) I, II, and III only (E) None of them

Use the graph of $f(x)$ at right to answer questions 2 through 6.



B 2. Determine $\lim_{x \to 2^-} f(x)$.

(A) 0 (B) 3 (C) −4 (D) $\frac{3}{2}$ (E) DNE

A 3. Determine $\lim_{x \to 2^+} f(x)$.

(A) 0 (B) 3 (C) −4 (D) $\frac{3}{2}$ (E) DNE

E 4. Determine $\lim_{x \to 2} f(x)$.

(A) 0 (B) 3 (C) −4 (D) $\frac{3}{2}$ (E) DNE

A 5. Determine $\lim_{x \to -5} f(x)$.

(A) 4 (B) 8 (C) 0 (D) 6 (E) DNE

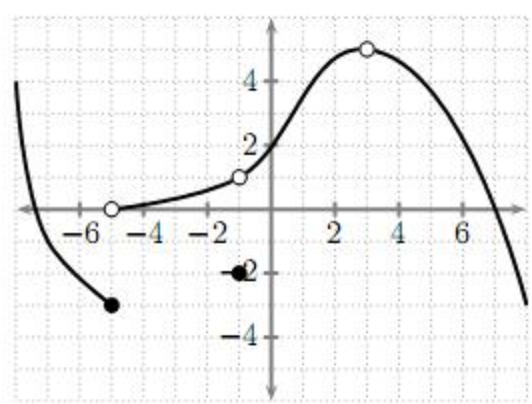
B 6. Determine $f(-5)$.

(A) 4 (B) 8 (C) 0 (D) 6 (E) DNE

D 7. The graph of a function $f(x)$ is shown at right. Use the graph to determine all the values of x at which f fails to be continuous on the open x-interval $x \in (-8, 8)$.

(A) −5 , 3 (B) −1, 3 (C) −5, −1

(D) −5, −1, 3 (E) f is continuous everywhere



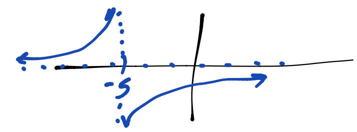
B 8. Evaluate $\lim\limits_{x\to\infty}\dfrac{-4x^2+3x^5+1}{-7x^5+x^2+2} = -\dfrac{3}{7}$ (Num. & Denom. grow at same rate.)

(A) 0 (B) $-\dfrac{3}{7}$ (C) $\dfrac{4}{7}$ (D) ∞ (E) $-\infty$

E 9. Evaluate $\lim\limits_{x\to-\infty}\dfrac{-5x^7-8x^5+2}{-3x^6-x^3+x^2} = -\infty$ (Top grows faster, plug "negative" into leading terms to determine sign.)

(A) 0 (B) $\dfrac{5}{3}$ (C) $-\dfrac{5}{3}$ (D) ∞ (E) $-\infty$

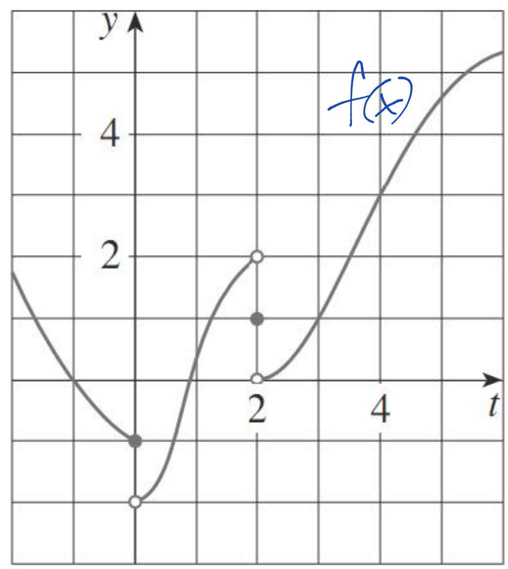
E 10. Evaluate $\lim\limits_{x\to-5^+}\dfrac{-2}{x+5}$ (you might want t sketch a graph first.)



(A) 0 (B) -2 (C) $-\dfrac{1}{5}$ (D) ∞ (E) $-\infty$

II. Short Answer

11. Given the graph at right, for each of the following, use the 3-step definition of continuity at a point to determine if the function is continuous at the indicated point. Be sure to show the analysis of all 3 steps, with correct notation, and a complete sentence at the end stating your conclusion with justification.



(a) at $x=0$

$\lim\limits_{x\to0^-} f(x) = -1$

$\lim\limits_{x\to0^+} f(x) = -2$

$f(0) = -1$

f(x) is not continuous at $x=0$ because $-1 \neq -2$.

(b) at $x=2$

$\lim\limits_{x\to2^-} f(x) = 2$

$\lim\limits_{x\to2^+} f(x) = 0$

$f(2) = 1$

f(x) is not continuous at $x=2$ because $2 \neq 0$.

(c) at $x=4$

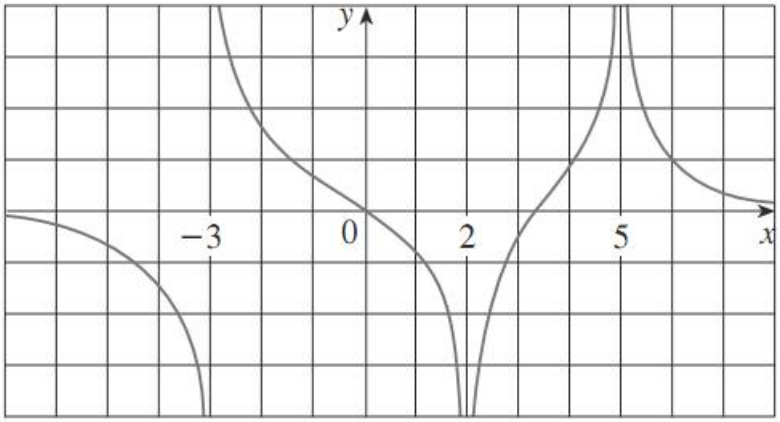
$\lim\limits_{x\to4^-} f(x) = 3$

$\lim\limits_{x\to4^+} f(x) = 3$

$f(4) = 3$

f(x) is continuous at $x=4$ because $3=3=3$.

12. For the function R whose graph is shown below, state the following:



(a) $\lim\limits_{x\to 2} R(x)$

$-\infty$ (or DNE) (Same behavior on both sides)

(b) $\lim\limits_{x\to 5} R(x)$

∞ (or DNE) (Same behavior on both sides)

(c) $\lim\limits_{x\to -3^-} R(x)$

$-\infty$

(d) $\lim\limits_{x\to -3^+} R(x)$

∞

(e) $\lim\limits_{x\to -3} R(x)$

DNE (Different behavior on either side)

13. For each function, $f(x)$, determine i) $\lim\limits_{x\to-\infty} f(x)$ ii) $\lim\limits_{x\to\infty} f(x)$ iii) any equations of HA's

(a) $f(x)=\dfrac{-5x+1}{x+7x^2+4}$

(i) $\lim\limits_{x\to-\infty} f(x)=0$ (Denom grows faster)

(ii) $\lim\limits_{x\to\infty} f(x)=0$

(iii) f has a HA @ $y=0$

(b) $f(x)=\dfrac{3x^6-2x^3+\pi}{8-9x^4}$

(i) $\lim\limits_{x\to-\infty} f(x)=-\infty$

(ii) $\lim\limits_{x\to\infty} f(x)=-\infty$

(iii) No HA's

(c) $f(x)=6x^2-100x-5000$

Parab. opening up

(i) $\lim\limits_{x\to-\infty} f(x)=\infty$

(ii) $\lim\limits_{x\to\infty} f(x)=\infty$

(iii) No HA's

(d) $f(x)=3-2x^3$

cubic function

(i) $\lim\limits_{x\to-\infty} f(x)=\infty$

(ii) $\lim\limits_{x\to\infty} f(x)=-\infty$

(iii) No HA's

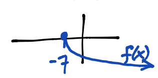
(e) $f(x)=\dfrac{12x+17-53x^2}{1x^2+22x-114}$

(i) $\lim\limits_{x\to-\infty} f(x)=-53$

(ii) $\lim\limits_{x\to\infty} f(x)=-53$

(iii) HA @ $y=-53$

(f) $f(x)=-4\sqrt{x+7}$



(i) $\lim\limits_{x\to-\infty} f(x)=$ DNE

(ii) $\lim\limits_{x\to\infty} f(x)=-\infty$

(iii) No HA's

15. Determine of the following functions are continuous at the indicated point. Show the work that leads to your answer using correct notation and justify your answer. Justify by sketching a graph of each functions.

(a) $f(x)=\begin{cases} 2x^2-3x, & x\le -1 \\ 2-4x, & x>-1 \end{cases}$ at $x=-1$

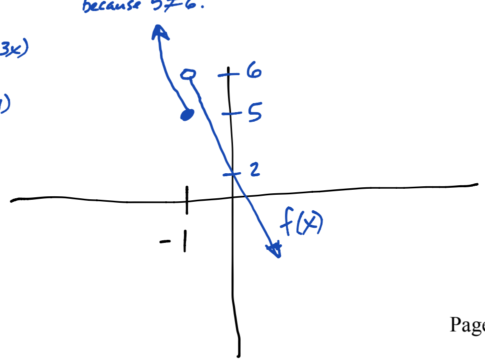
$x(2x-3)=0$
$x=0,\ x=\frac{3}{2}$
(Neither in interval)

$\lim\limits_{x\to -1^-} f(x)$
$=\lim\limits_{x\to -1^-}(2x^2-3x)$
$=2(-1)^2-3(-1)$
$=\boxed{5}$

$\lim\limits_{x\to -1^+} f(x)$
$=\lim\limits_{x\to -1^+}(2-4x)$
$=2-4(-1)$
$=\boxed{6}$

$f(-1)=5$

$f(x)$ is not continuous at $x=-1$ because $5\neq 6$.



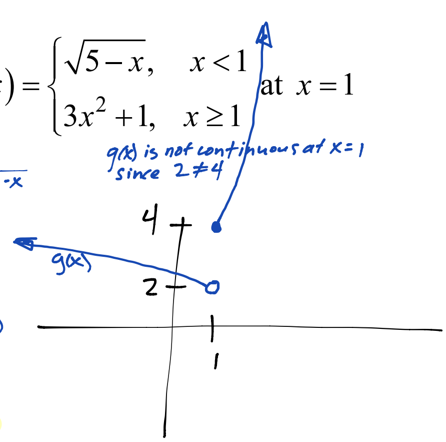
(b) $g(x)=\begin{cases} \sqrt{5-x}, & x<1 \\ 3x^2+1, & x\ge 1 \end{cases}$ at $x=1$

$\lim\limits_{x\to 1^-} g(x)$
$=\lim\limits_{x\to 1^-}\sqrt{5-x}$
$=\sqrt{5-1}$
$=\boxed{2}$

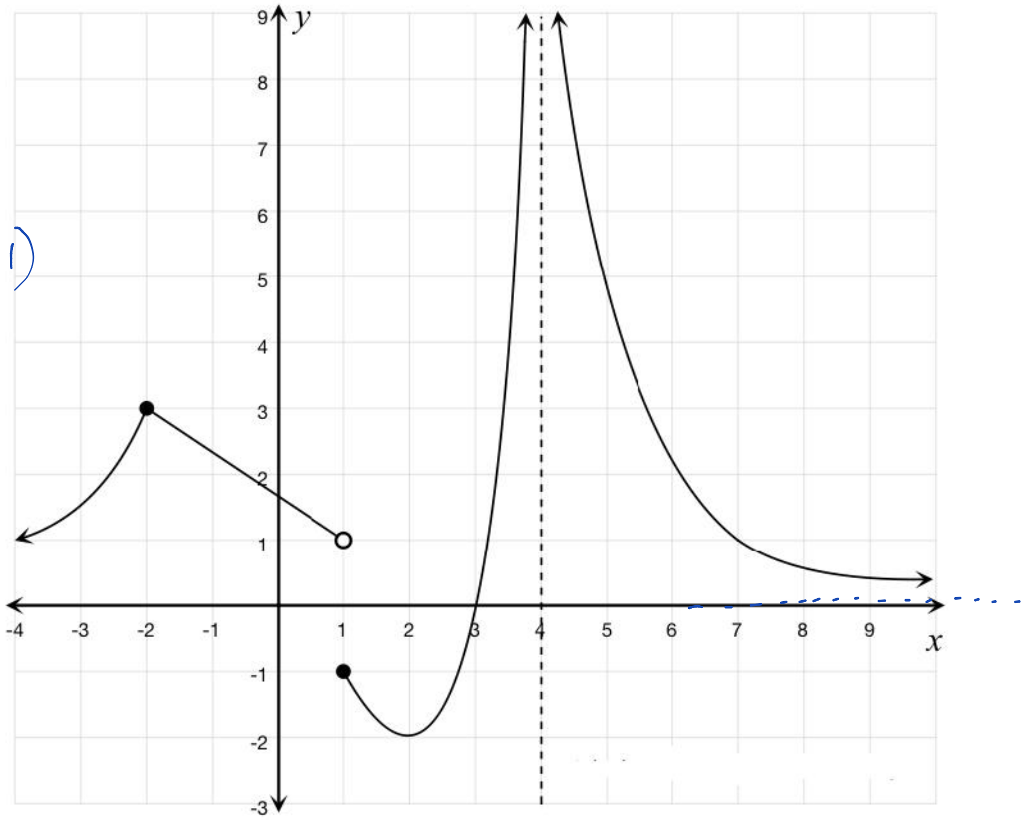
$\lim\limits_{x\to 1^+} g(x)$
$=\lim\limits_{x\to 1^+}(3x^2+1)$
$=3(1^2)+1$
$=\boxed{4}$

$g(1)=4$

$g(x)$ is not continuous at $x=1$ since $2\neq 4$



The graph of $y = f(x)$ is given below. Answer the following questions below.



15. $\lim_{x \to 1^-} f(x) =$ 1

16. $\lim_{x \to 1^+} f(x) =$ −1

17. $\lim_{x \to 1} f(x) =$ DNE ($1 \neq -1$)

18. $f(1) =$ −1

19. $\lim_{x \to 4^-} f(x) = \infty$

20. $\lim_{x \to 4^+} f(x) = \infty$

21. $\lim_{x \to 4} f(x) = \infty$

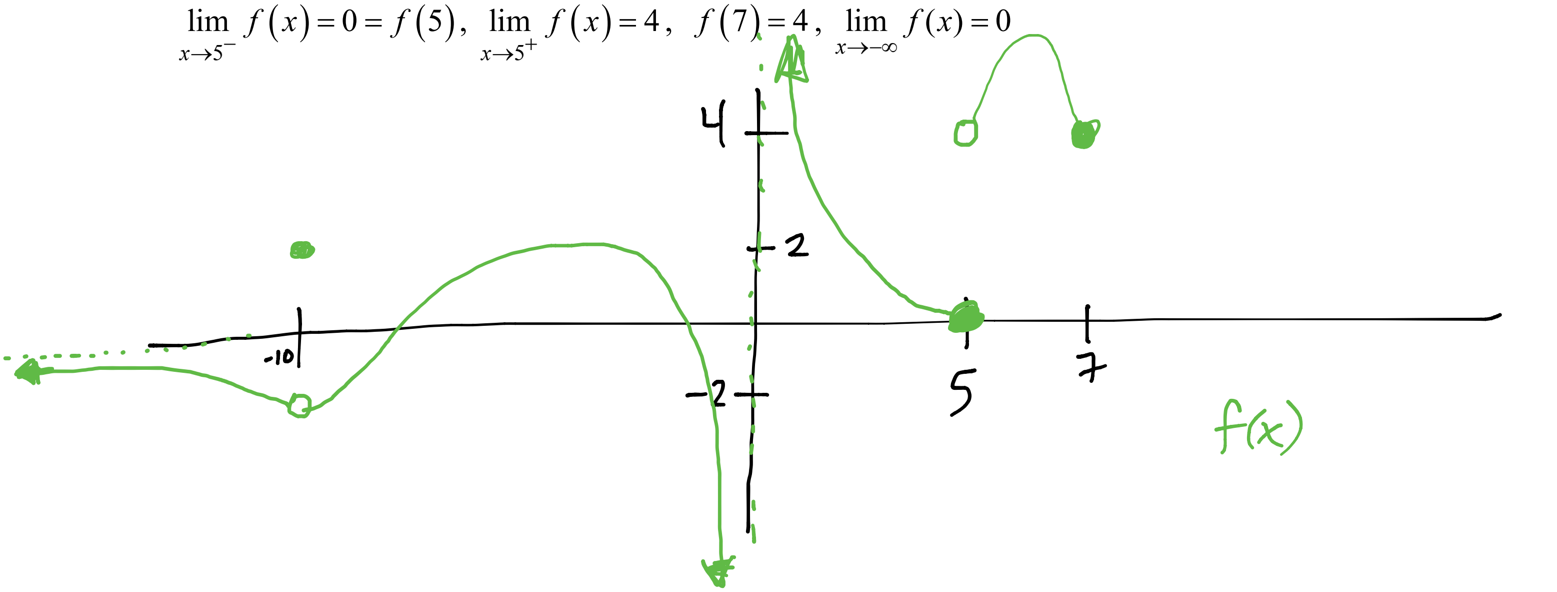
22. $f(4) =$ DNE

23. $\lim_{x \to \infty} f(x) = 0$

24. Draw the graph of a function $f(x)$ on the interval $-\infty < x \le 7$ with the following characteristics. (Answers will, and should, vary).

$\lim_{x \to -10^-} f(x) = -2 = \lim_{x \to -10^+} f(x)$, $f(-10) = 2$, $\lim_{x \to 0^-} f(x) = -\infty$, $\lim_{x \to 0^+} f(x) = \infty$

$\lim_{x \to 5^-} f(x) = 0 = f(5)$, $\lim_{x \to 5^+} f(x) = 4$, $f(7) = 4$, $\lim_{x \to -\infty} f(x) = 0$



$\boxed{\text{Answers will vary}}$