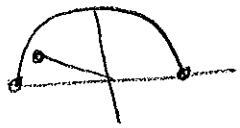
Name_____ Date_____ Period_____

Worksheet 5.7—Inverse Trig Functions

Show all work on a separate sheet of paper. No calculator is permitted unless specified otherwise. Unless otherwise stated, report three decimals and units in all final answers.

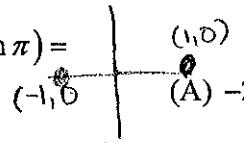
Multiple Choice

1. $\text{Cos}^{-1}\left(-\frac{\sqrt{3}}{2}\right) =$ 

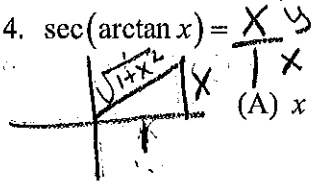
(A) $-\frac{7\pi}{6}$ (B) $\frac{7\pi}{6}$ (C) $-\frac{\pi}{6}$ (D) $-\frac{11\pi}{6}$ (E) $\frac{5\pi}{6}$

2. $\text{Arcsin}\left(-\frac{1}{2}\right) =$ 

(A) $-\frac{7\pi}{6}$ (B) $\frac{7\pi}{6}$ (C) $-\frac{\pi}{6}$ (D) $-\frac{11\pi}{6}$ (E) $\frac{5\pi}{6}$

3. $\arcsin(\sin\pi) =$ 

(A) -2π (B) $-\pi$ (C) π (D) 0 (E) 2π

4. $\sec(\arctan x) =$ 

(A) x (B) $\csc x$ (C) $\sqrt{1+x^2}$ (D) $\sqrt{1-x^2}$ (E) $\frac{\sqrt{1-x^2}}{x}$

5. The range of the function $f(x) = \arcsin x$ is

(A) $(-\infty,\infty)$ (B) $(-1,1)$ (C) $[-1,1]$ (D) $[0,\pi]$ (E) $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

6. The range of the function $f(x) = \arccos x$ is

(A) $(-\infty,\infty)$ (B) $(-1,1)$ (C) $[-1,1]$ (D) $[0,\pi]$ (E) $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

7. The range of the function $f(x) = \arctan x$ is

(A) $(-\infty,\infty)$ (B) $[-1,1]$ (C) $[0,\pi]$ (D) $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ (E) None of these

$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

8. Find the exact value of each expression, if it is defined. Give your answers in radians in the interval $[0,2\pi)$.

(a) $\text{Sin}^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ (b) $\text{Arccos}\left(-\frac{\sqrt{3}}{2}\right)$ (c) $\text{Tan}^{-1}\sqrt{3}$ (d) $\text{Arcsin}\sqrt{3}$ (e) $\text{Cos}^{-1}(-1)$

a) $= -\frac{\pi}{3}$ b) $\frac{5\pi}{6}$ c) $= \frac{\pi}{3}$ d) DNE e) π

9. Use a calculator to find an approximate value of each expression correct to 5 decimal places, if it is defined. Give your answers in decimal degrees in the interval $[0^\circ, 360^\circ)$.

(a) $\sin^{-1}(0.13844)$ (b) $\arccos(-0.92761)$ (c) $\tan^{-1}(26.23110)$

a) 7.957° 158.065° c) 87.816°

10. The following facts to find an approximate value (using a calculator) of each expression correct to 5 decimal places, if it is defined. Give your answers in radians in the interval $[0,2\pi)$.

$$\csc\theta = \frac{1}{\sin\theta},\ \sec\theta = \frac{1}{\cos\theta},\ \text{and}\ \cot\theta = \frac{1}{\tan\theta}$$

(Hint: rewrite each inverse trig function as a trig function, then express each in terms of their reciprocals, then resolve for θ.)

(a) $\theta = \csc^{-1}(10.13844)$ (b) $\theta = \text{arcsec}(-1.92761)$ (c) $\theta = \cot^{-1}(26.23110)$

$\csc\theta = (10.13844)$ $\sec\theta = (-1.92761)$ $\cot\theta = 26.23110$

$\sin\theta = \frac{1}{10.13844}$ $\cos\theta = \frac{-1}{1.92761}$ $\tan\theta = \frac{1}{26.23110}$

$\theta = \sin^{-1}\left(\frac{1}{10.13844}\right)$ $\theta = \cos^{-1}\left(\frac{-1}{1.92761}\right)$ $\theta = \tan^{-1}\left(\frac{1}{26.23110}\right)$

Finish on calculator

11. Find the **exact value** of each expression if it is defined.

(a) $\sin\left(\sin^{-1}\frac{1}{4}\right)$ (b) $\tan(\arctan 3)$ (c) $\cos\left(\cos^{-1}3\right)$ (d) $\cos^{-1}(\cos 3)$

(e) $\arcsin\left(\sin\left(-\frac{\pi}{7}\right)\right)$ (f) $\sin^{-1}\left(\sin\left(\frac{4\pi}{7}\right)\right)$ (g) $\tan^{-1}\left(2\sin\frac{2\pi}{3}\right)$ (h) $\arccos\left(\sqrt{3}\sin\frac{11\pi}{6}\right)$

a) $\frac{1}{4}$ b. 3 c) DNE d) 3

e) $-\frac{\pi}{7}$ f $\frac{3\pi}{7}$ g. $\tan^{-1}\left(2\left(\frac{\sqrt{3}}{2}\right)\right)$ h) $\arccos\left(\sqrt{3}\left(\frac{1}{2}\right)\right)$

$\tan^{-1}(\sqrt{3})$

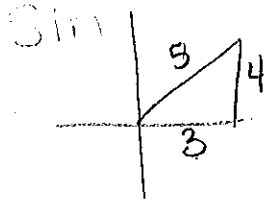
$=\frac{\pi}{3}$

$\arccos\frac{\sqrt{3}}{2}$

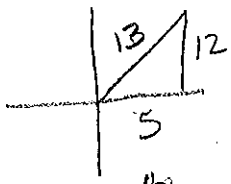
$=\frac{\pi}{6}$

12. Evaluate each expression by sketching a triangle and finding the missing side of the triangle.

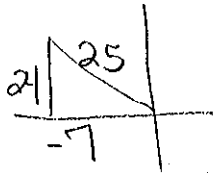
(a) $\sin\left(\arccos\frac{3}{5}\right)$ (b) $\sin\left(\tan^{-1}\frac{12}{5}\right)$ (c) $\csc\left(\cos^{-1}\left(-\frac{7}{25}\right)\right)$



$\sin\theta = \frac{4}{5}$



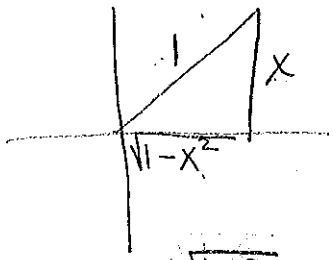
$\sin\theta = \frac{12}{13}$



$\csc\theta = \left(\frac{25}{24}\right)$

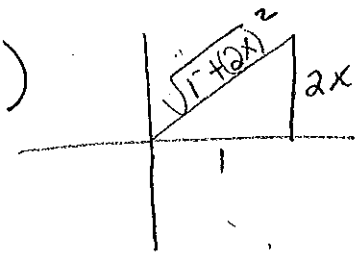
13. Rewrite each expression as an algebraic expression in x.

(a) $\cos\left(\sin^{-1}x\right)$ (b) $\sec(\arctan 2x)$



$\sin\theta = \sqrt{1-x^2}$

b)

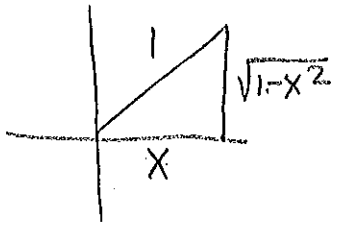


$\sec\theta = \sqrt{(1+4x^2}$

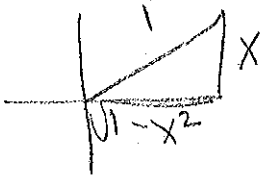
14. Rewrite each of the following into a composition of a tring and inverse trig function. Give two equivalent compostitions for each.

(a) $\frac{\sqrt{1-x^2}}{x}$ (b) $\frac{1}{2}\sqrt{4-9x^2}$

a)

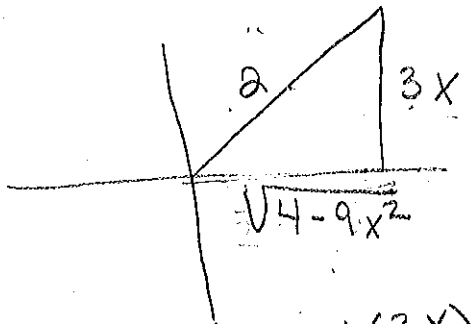


$\tan(\arccos x)$

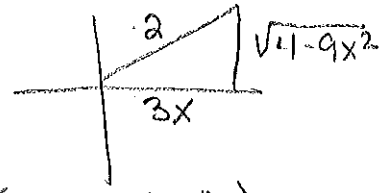


$\cot(\arcsin x)$

b)



$\cos\left(\sin^{-1}\left(\frac{3x}{2}\right)\right)$



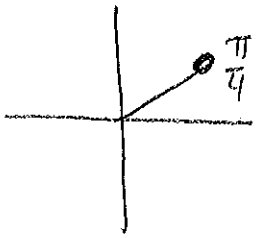
$\sin\left(\cos^{-1}\left(\frac{3x}{2}\right)\right)$

15. Using a graphing calculator, (a) find all the solutions to the following equation correct to three decimal places, then (b) find the exact solution using your knowledge of the unit circle.

$$\text{Arcsin}x - \text{Arccos}x = 0$$

$x = .707$

$x = \frac{\sqrt{2}}{2}$



$\text{Arcsin}x = \text{Arccos}x$