

Name _____ Date _____ Period _____

Worksheet 5.6—The Other Trig Functions

Show all work on a separate sheet of paper. A calculator is permitted. Report three decimals and units in all final answers.

Multiple Choice

1. The graph of $y = \cot x$ can be obtained by a horizontal shift of the graph of the function $y =$

(A) $-\tan(x + \pi)$ (B) $-\cot\left(x - \frac{\pi}{2}\right)$ (C) $\sec x$ (D) $\tan\left(x - \frac{\pi}{2}\right)$ (E) ~~none of these~~ E No of these

2. The graph of $y = \sec x$ **never** intersects the graph of $y =$

(A) x (B) x^2 (C) $\csc x$ (D) $\cos x$ (E) $\sin x$

3. If $k \neq 0$, what is the range of the function $y = k \csc x$?

(A) $[-k, k]$ (B) $(-k, k)$ (C) $(-\infty, -k) \cup (k, \infty)$ (D) $(-\infty, -k] \cup [k, \infty)$ (E) $(-\infty, -\frac{1}{k}] \cup [\frac{1}{k}, \infty)$

4. The function $y = \csc x$ has the same domain as the function $y =$

(A) $\sin x$ (B) $\tan x$ (C) $\cot x$ (D) $\sec x$ (E) $\csc 2x$

5. Consider the functions $f(x) = \tan \frac{\pi x}{4}$ and $g(x) = \frac{1}{2} \sec \frac{\pi x}{4}$ on the interval $(-1, 1)$. $g(x) = \frac{1}{2} \left(\frac{1}{\cos \frac{\pi x}{4}} \right)$

- I. Approximate the interval where $f < g$.

(A) $\left(-1, \frac{2}{3}\right)$ (B) $\left(-\frac{2}{3}, 1\right)$ (C) $\left(-1, \frac{4}{3}\right)$ (D) $\left(-\frac{4}{3}, 1\right)$ (E) None of these

- II. Approximate the interval where $2f < 2g$

(A) $\left(-1, \frac{4}{3}\right)$ (B) $\left(-1, \frac{2}{3}\right)$ (C) $\left(-\frac{4}{3}, 1\right)$ (D) $\left(-\frac{2}{3}, 1\right)$ (E) None of these

6. Use the Unit Circle to solve $\cot x = -\sqrt{3}$ on the interval $[-2\pi, 2\pi]$.

(A) $\frac{7\pi}{6}, \frac{\pi}{6}, -\frac{5\pi}{6}, -\frac{11\pi}{6}$ (B) $-\frac{4\pi}{3}, -\frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{3}$ (C) $-\frac{7\pi}{6}, -\frac{\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}$ (D) $\frac{4\pi}{3}, \frac{\pi}{3}, -\frac{2\pi}{3}, -\frac{5\pi}{3}$

7. Use the Unit Circle to solve $\csc x = \frac{2\sqrt{3}}{3}$ on the interval $[-2\pi, 2\pi]$. $\csc x = \frac{2}{\sqrt{3}}$ $\sin \frac{\sqrt{3}}{2}$

(A) $-\frac{4\pi}{3}, \frac{2\pi}{3}$ (B) $-\frac{2\pi}{3}, \frac{4\pi}{3}$ (C) $-\frac{4\pi}{3}, -\frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{3}$ (D) $-\frac{5\pi}{3}, -\frac{2\pi}{3}, \frac{\pi}{3}, \frac{4\pi}{3}$ (E) None of these

8. What is the period of the function $f(\theta) = \cot \frac{\pi\theta}{8}$? $\cot \frac{\pi}{8} \theta$ $B = \frac{\pi}{8}$ $P = \frac{2\pi}{B} = \frac{2\pi}{\frac{\pi}{8}} = 8$

(A) $P = 8$ (B) $P = 16/\pi$ (C) $P = 8/\pi$ (D) $P = 16$ (E) the function is not periodic

9. What is the period of $y = 2 \sec\left(\pi - \frac{7}{2}\pi x\right)$? $\cancel{\text{Ans: } \frac{1}{2}\pi(x-1)}$

$$\beta = -\frac{1}{2}\pi \quad P = \frac{2\pi}{|\frac{7}{2}\pi|} = \frac{4}{7}$$

- (A) $P = \frac{4}{7}$ (B) $P = \frac{7}{4}$ (C) $P = \frac{7}{2}$ (D) $P = 7$ (E) $P = \frac{2}{7}$

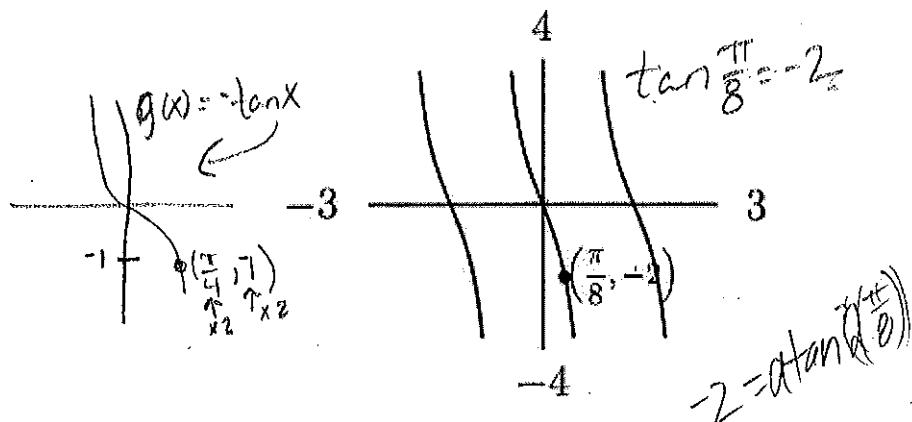
Short Answer

10. The graph at right is for $f(x) = a \tan bx$

- (a) Find the value of b .
(b) Find the value of a

$$a = -2$$

$$b = 2$$



For 11 through 16, match the trigonometric function with one of the graphs from I through VI.

11. $f(x) = \tan\left(x + \frac{\pi}{4}\right)$ IV

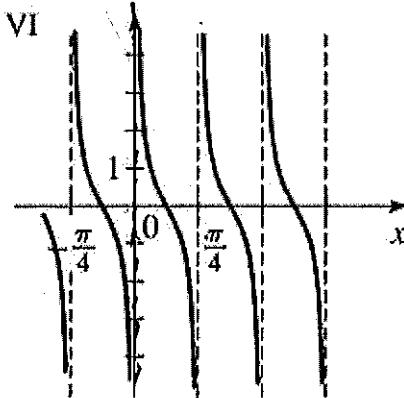
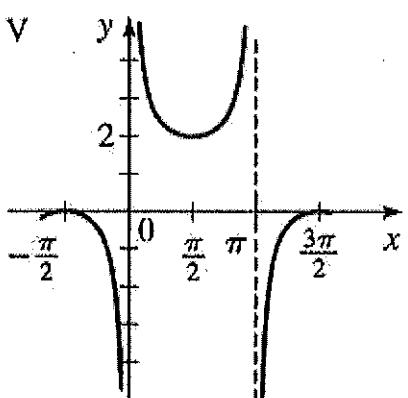
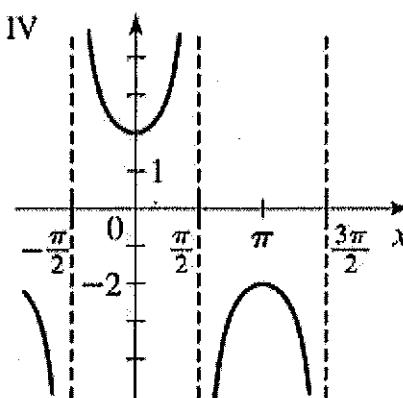
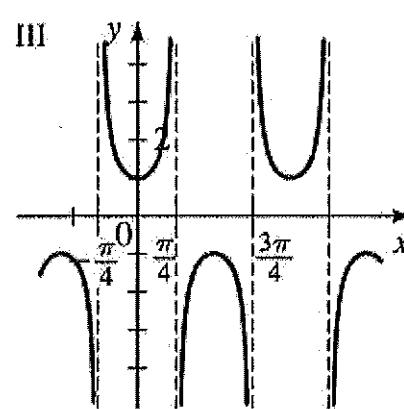
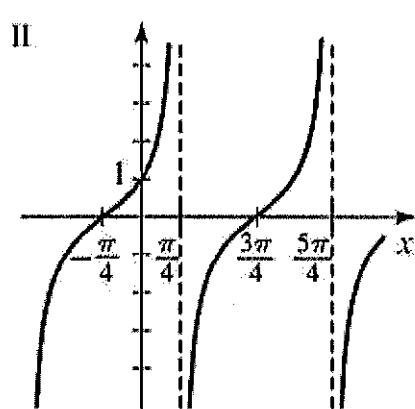
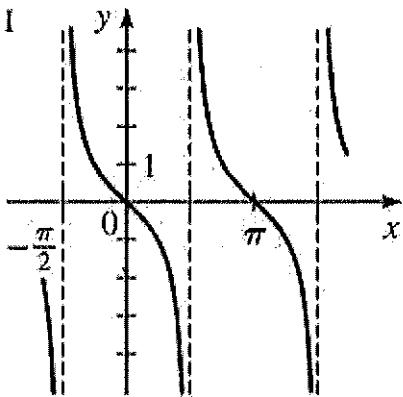
12. $f(x) = \sec 2x$ III

13. $f(x) = \cot x$ VII

14. $f(x) = -\tan x$ I

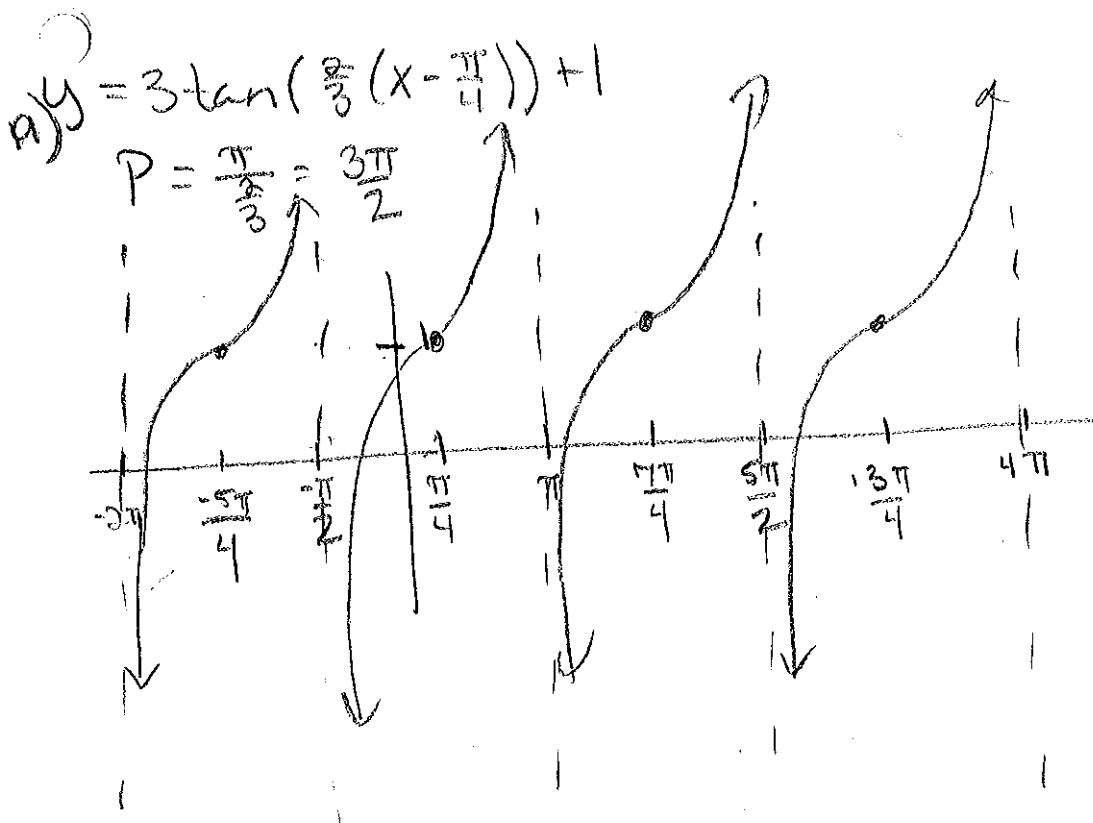
15. $f(x) = 2 \sec x$ V

16. $f(x) = 1 + \csc x$ II



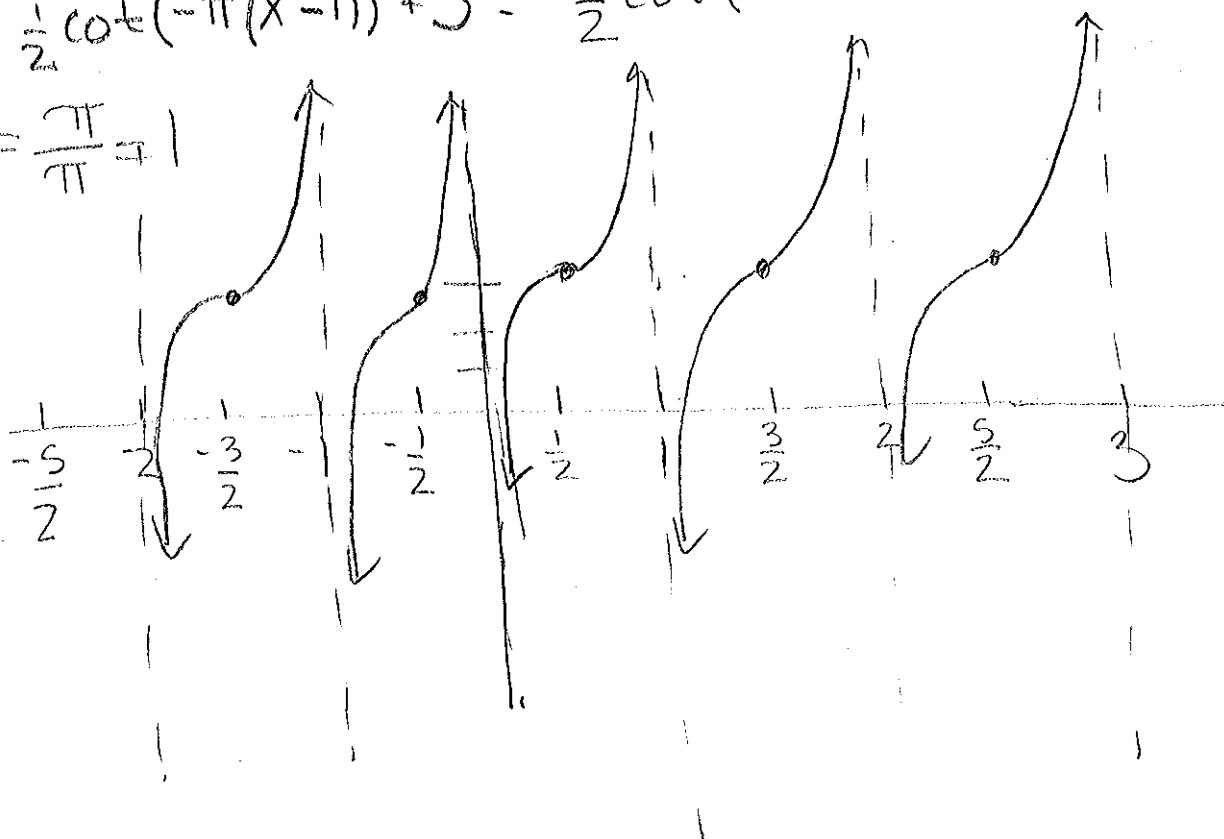
$$19. y = 3 \tan\left(\frac{2}{3}x - \frac{\pi}{6}\right) + 1$$

$$20. y = \frac{1}{2} \cot(\pi - \pi x) + 3$$



b) $y = \frac{1}{2} \cot(-\pi(x - 1)) + 3 = -\frac{1}{2} \cot(\pi(x - 1)) + 3$

$$P = \frac{\pi}{\pi} = 1$$



3

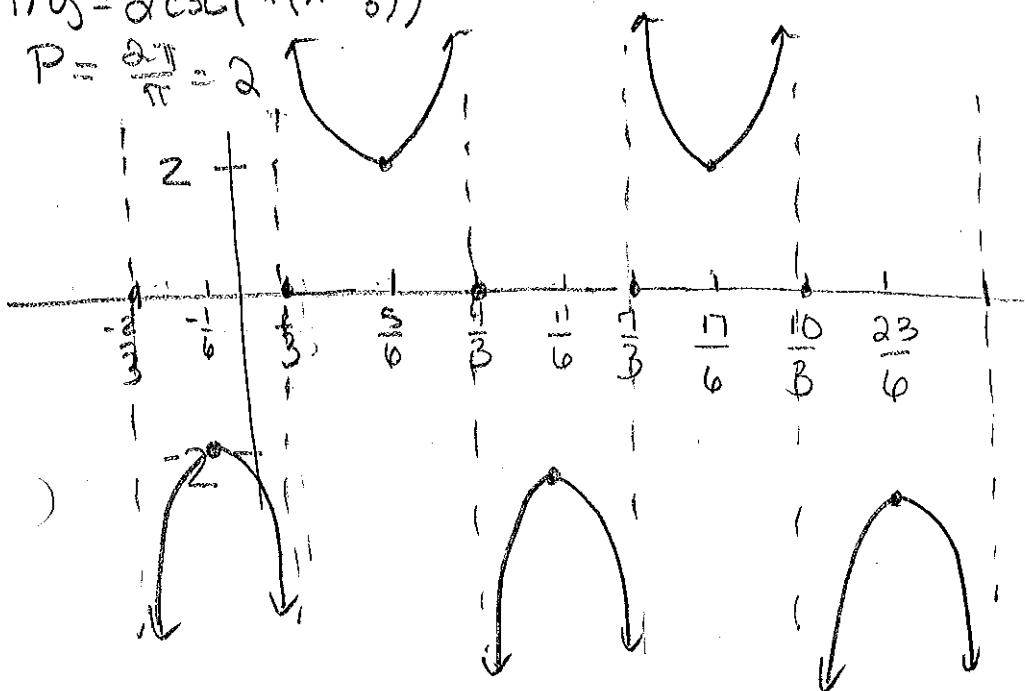
For 17-19 find the period, then sketch at least two cycles of the function. ***Then*** write an equivalent equation of the graph in terms of each function's cofunction.

$$17. y = 2 \csc\left(\pi x - \frac{\pi}{3}\right)$$

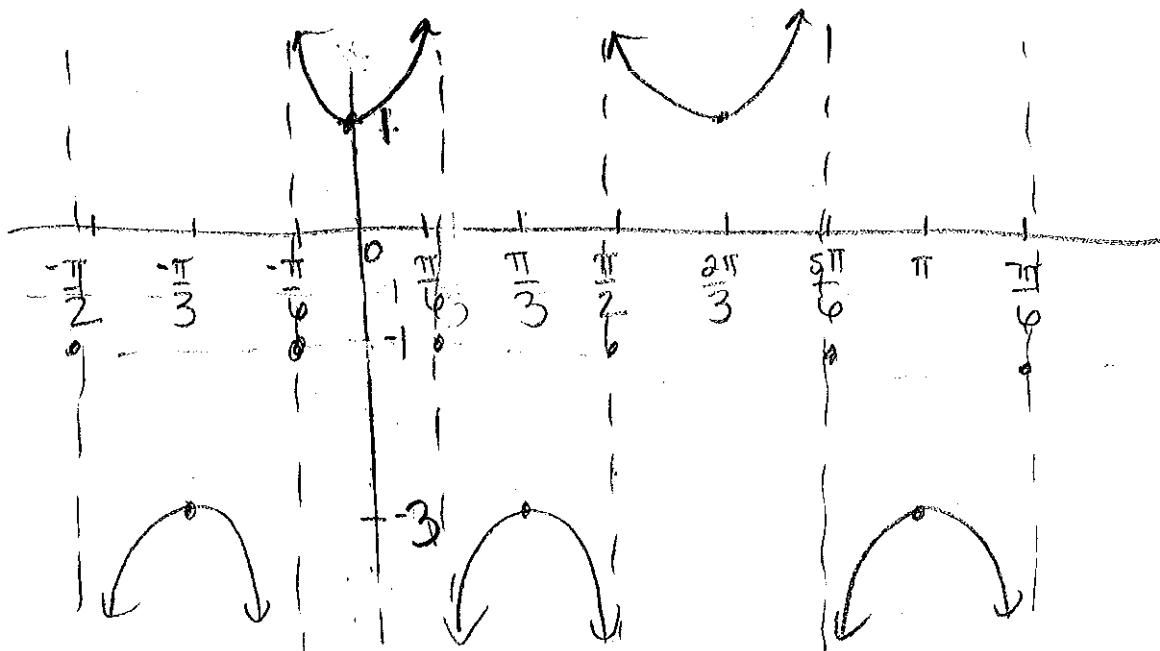
$$18. y = 2 \csc\left(3x + \frac{\pi}{2}\right) - 1$$

17) $y = 2 \csc\left(\pi(x - \frac{1}{3})\right)$

$$P = \frac{2\pi}{\pi} = 2$$



18) $2 \csc\left(3\left(x + \frac{\pi}{6}\right)\right) - 1 \quad P = \frac{2\pi}{3}$



21. (Calculator Permitted) On a day when the sun passes directly overhead at noon, a six-foot-tall man casts a shadow of length $S(t) = 6 \left| \cot \frac{\pi}{12} t \right|$ where S is measured in feet and t is the number of hours since 6 A.M.

- (a) Find the length of the shadow at 8:00 A.M., noon, 2:00 P.M., and 5:45 P.M. Show your set-ups.

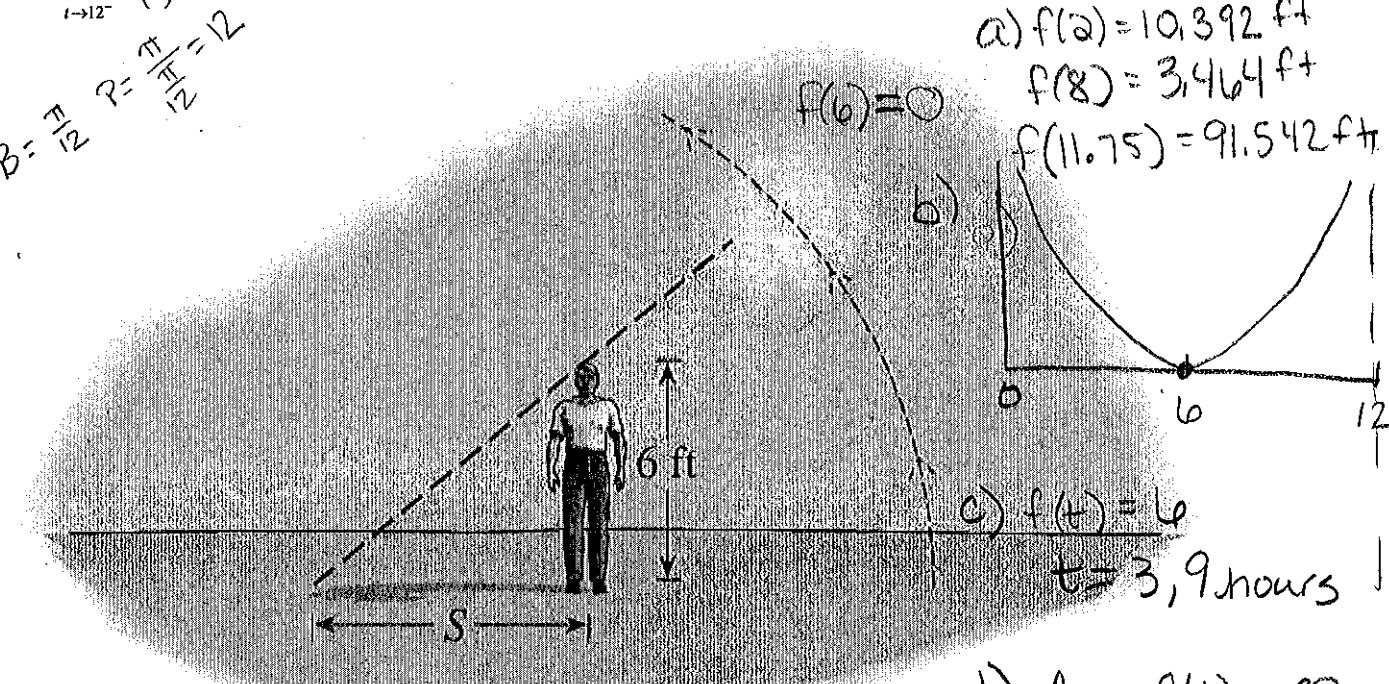
- (b) Sketch a graph of the function S for $0 < t < 12$.

- (c) From the graph determine the values of t at which the length of the shadow equals the man's height. To what time of day does each of these values correspond?

- (d) Explain what happens to the shadow as the time approaches 6 P.M., that is, explain the meaning of

$$\lim_{t \rightarrow 12^-} S(t)$$

$$B: \frac{\pi}{12} t = \frac{\pi}{2} \Rightarrow t = \frac{12}{\pi} = 3.9$$



$$d) \lim_{x \rightarrow 12^-} S(t) = \infty$$