Name_____ Date_____ Period_____

Worksheet 5.4—Sinusoids

Show all work on a separate sheet of paper. All answers must be given as either simplified, exact answers. A calculator **is not permitted** unless otherwise stated.

Multiple Choice

1. A sinusoid with an amplitude of 4 has a minimum value of 5. Its maximum value is
(A) 7 (B) 9 (C) 11 (D) 13 (E) 15

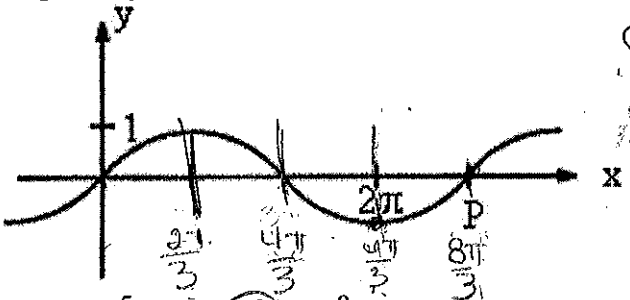
2. The graph of $y=f(x)$ is a sinusoid with a period of 45 passing through the point $(6,0)$. Which of the following can be determined from the given information?
I. $f(0)$ II. $f(6)$ III. $f(96)$
(A) I only (B) II only (C) I and III only (D) II and III only (E) I, II, and III

3. The period of the function $f(x)=210\sin(420x+840)$ is
(A) $\frac{\pi}{840}$ (B) $\frac{\pi}{420}$ (C) $\frac{\pi}{210}$ (D) $\frac{210}{\pi}$ (E) $\frac{420}{\pi}$

4. The number of solutions to the equation $\sin(2000x)=\frac{3}{7}$ in the interval $[0,2\pi]$ is
(A) 1000 (B) 2000 (C) 4000 (D) 6000 (E) 8000

5. As frequency increases, the period does what?
(A) decreases (B) increases (C) remains the same (D) doubles (E) ends

6. The graph of the sinusoidal function shown below has a minimum at $x=2\pi$ and an x-intercept at $x=0$. The x-intercept at point P is given by



(A) $x=\frac{9\pi}{4}$ (B) $x=\frac{5\pi}{2}$ (C) $x=\frac{8\pi}{3}$ (D) $x=3\pi$ (E) $x=4\pi$

7. Determine the period of $y=8+6\cos\frac{2\pi x}{15}$
(A) $\frac{2}{15}$ (B) $\frac{15}{2}$ (C) 15 (D) 30 (E) $\frac{2\pi}{15}$

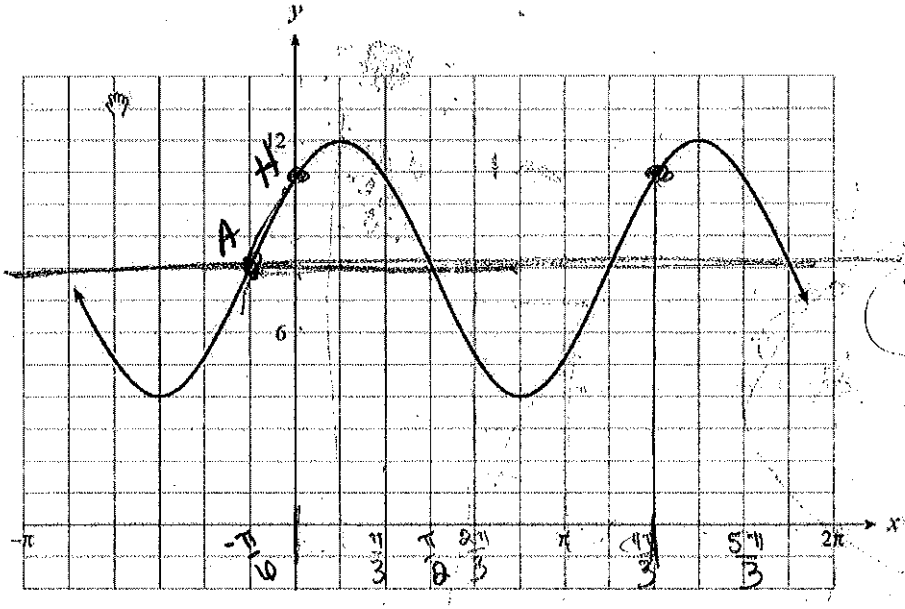
8. Determine the range of the function $f(x)=4-2\sin 3x$.

(A) $[-6,2]$ (B) $[-2,2]$ (C) $[0,4]$ (D) $[-4,4]$ (E) $[2,6]$

9. Determine the amplitude of $g(x)=4-5\sin\pi(x-3)$.

(A) -5 (B) 5 (C) 3 (D) 4 (E) -3

10. Which equation represents the sinusoid graphed below?



(A) $y=4\sin\frac{4}{3}\left(x+\frac{\pi}{6}\right)+8$ (B) $y=4\cos\frac{3}{2}\left(x+\frac{\pi}{6}\right)+8$ (C) $y=4\sin\frac{3}{2}\left(x-\frac{\pi}{6}\right)+8$

(D) $y=4\cos\frac{4}{3}\left(x-\frac{\pi}{6}\right)+8$ (E) $y=4\sin\frac{3}{2}\left(x+\frac{\pi}{6}\right)+8$

11. Determine the range of the function $y=b\cos ax-2b$, where $a>0$, $b>0$.

(A) $\{y|b\le y\le 3b\}$ (B) $\{y|-3b\le y\le -b\}$ (C) $\{y|b-a\le y\le b+a\}$

(D) $\{y|2b-a\le y\le 2b+a\}$ (E) $\{y|-2b-a\le y\le -2b+a\}$

12. Determine the general solution for $\sin 2x=-\frac{1}{2}$ for any integer n

(A) $\frac{7\pi}{12}+2\pi n,\ \frac{11\pi}{12}+2\pi n$ (B) $\frac{7\pi}{12}+\pi n,\ \frac{11\pi}{12}+\pi n$ (C) $\frac{13\pi}{12}+2\pi n,\ \frac{21\pi}{12}+2\pi n$

(D) $\frac{13\pi}{12}+\pi n,\ \frac{21\pi}{12}+\pi n$ (E) $\frac{\pi}{12}+\pi n,\ \frac{5\pi}{12}+\pi n$

13. What is the measure of the central angle of a circle with radius 4 inches subtended by an arc of length 3π inches?

(A) $\frac{3\pi}{4}$ (B) $\frac{4}{3\pi}$ (C) $\frac{3\pi}{2}$ (D) $\frac{2}{3\pi}$ (E) 3π

14. Determine the phase shift of the function $f(x)=-3\cos\left(\frac{x}{3}-\pi\right)-3$

(A) $\frac{\pi}{3}$ to the right (B) π to the right (C) π to the left (D) 3π to the right (E) 6π to the right

15. A sinusoidal equation of the form $y=A\sin\left(B(x-C)\right)+D$ has a maximum value of 14 and a minimum vale of -6. The equation of the sinusoidal axis is at $y=$

(A) 10 (B) -8 (C) 8 (D) -2 (E) 4

16. The graph of $y=\cos\left(x+\frac{\pi}{2}\right)$ is identical to the graph of

(A) $y=-\sin x$ (B) $y=\sin x$ (C) $y=-\cos x$ (D) $y=\cos\left(x-\frac{\pi}{2}\right)$ (E) $y=\sin(x-\pi)$

17. The y-intercept fo the graph represented by $f(x)=-3\cos\left(kx+\frac{\pi}{2}\right)-b$ is

(A) $-b$ (B) $3-b$ (C) $\frac{3-b}{k}$ (D) $\frac{-3-b}{k}$ (E) 0

18. A graph of a sinusoidal function has its first positive maximum value at $\left(\frac{\pi}{2},112\right)$ and its first positive minimum value at $\left(\frac{3\pi}{2},28\right)$. The equation of this sinusoid is

(A) $y=42\cos\left(x-\frac{\pi}{2}\right)+28$ (B) $y=42\cos(x-\pi)+70$ (C) $y=42\cos\left(x-\frac{\pi}{2}\right)+70$

(D) $y=42\cos\left(x-\frac{3\pi}{2}\right)+70$ (E) $y=42\cos\left(x-\frac{3\pi}{2}\right)+28$

19. An appropriate window setting on your calculator $x:[x\min,x\max,x\ \text{scl}]$, $y:[y\min,y\max,y\ \text{scl}]$ for the graph of $y=20.1\sin\left(\frac{2\pi}{300}(x-265)\right)+6.2$ is

(A) $x:[0,17000,5000]$, $y:[-20,30,10]$ (B) $x:[-265,0,50]$, $y:[0,12.4,1]$

(C) $x:[0,600,100]$, $y:[-15,30,5]$ (D) $x:\left[0,2\pi,\frac{\pi}{2}\right]$, $y:[-20,30,5]$

20. Determine the number of cycles in 2π for the graph of $y=-4\cos(\pi x+3\pi)-2$

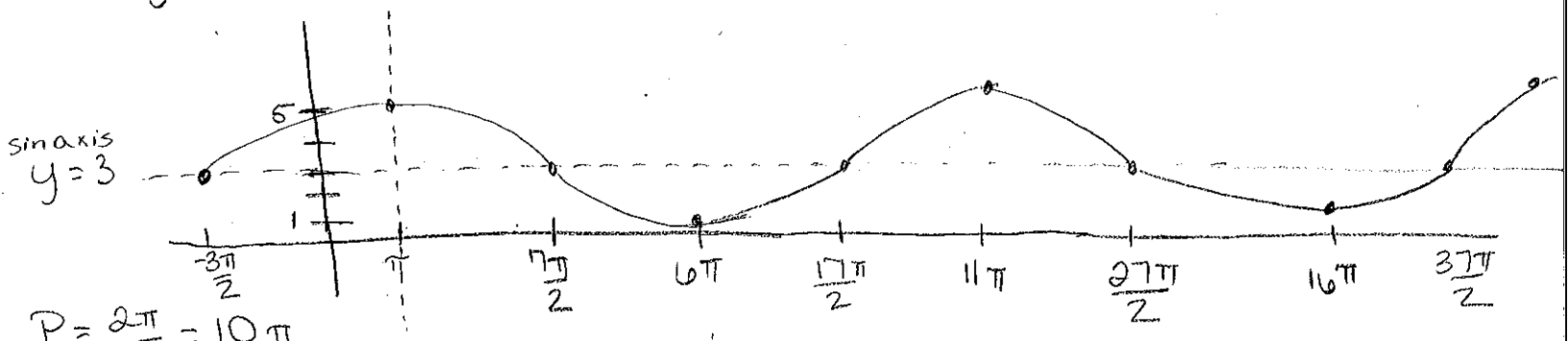
(A) π (B) 3π (C) $\frac{2}{3}$ (D) 2 (E) $\frac{2\pi}{3}$

Short Answer

21. For each of the following, sketch at least two cycles. Be sure to show and label maximum values, minimum values, sinusoidal axis, and all (**and only**) the critical values on the x-axis. Be sure to show at least one critical value on either side of the y-axis.

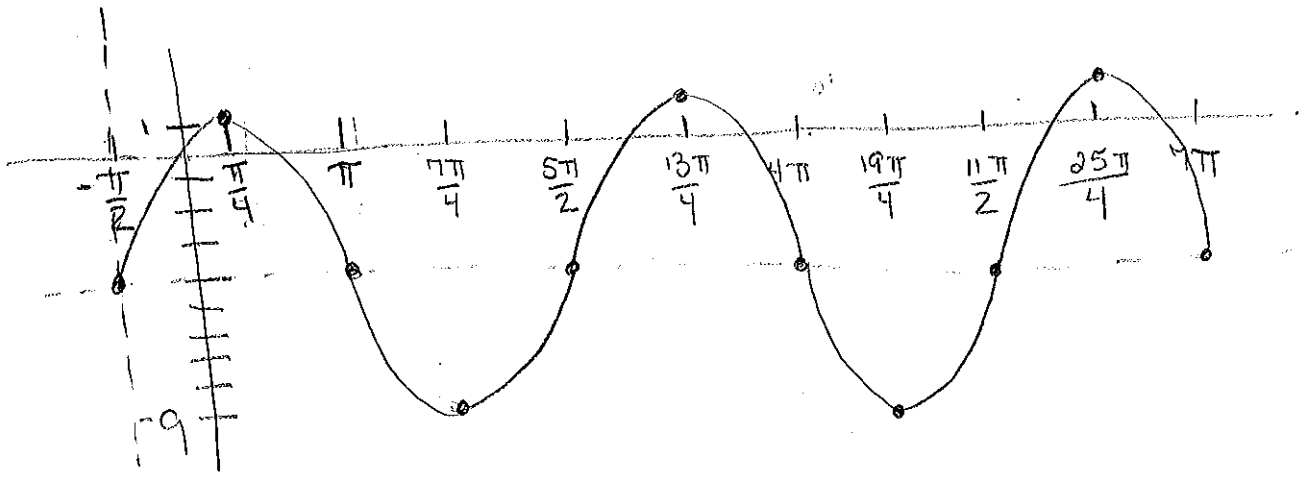
(a) $y = 3 + 2\cos\frac{1}{5}(x - \pi)$ (b) $y = 5\sin\frac{2}{3}\left(x + \frac{\pi}{2}\right) - 4$

a) $y = 2\cos\frac{1}{5}(x - \pi) + 3$



$P = \frac{2\pi}{\frac{1}{5}} = 10\pi$

b) $y = 5\sin\frac{2}{3}\left(x + \frac{\pi}{2}\right) - 4$

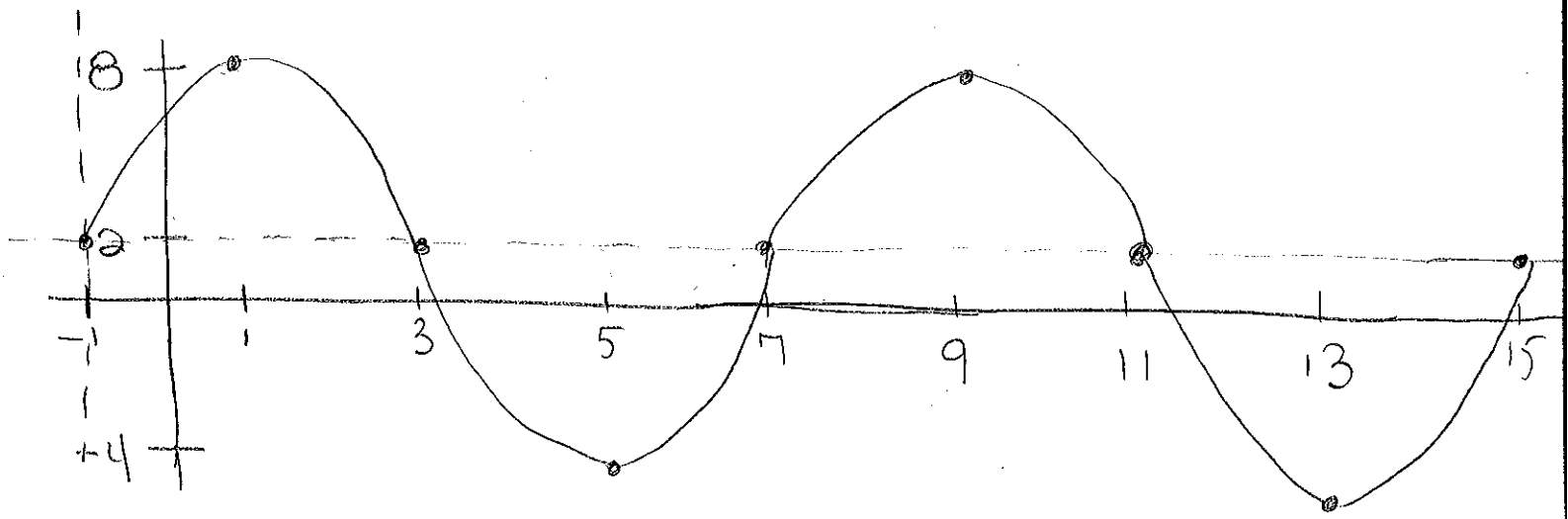


sin axis $y = -4$

$P = \frac{2\pi}{\frac{2}{3}} = 2\pi \cdot \frac{3}{2} = 3\pi$

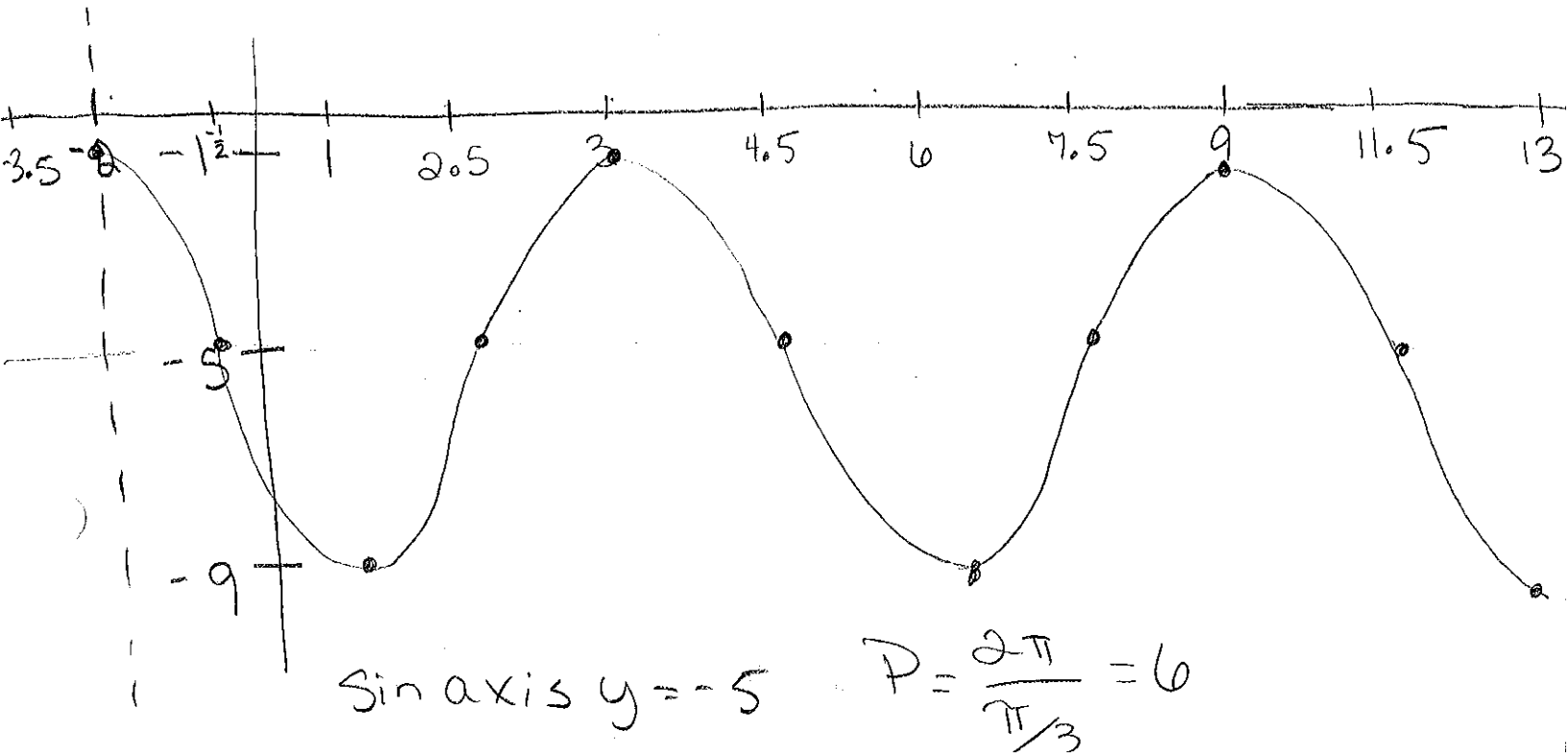
(c) $y=6\sin\left(\frac{\pi}{4}x-\frac{\pi}{4}\right)+2$ (d) $y=-5+4\cos\frac{\pi}{3}(x+2)$

c) $y=6\sin\left(\frac{\pi}{4}(x-1)\right)+2$



sinaxis $y=2$ $P=\frac{2\pi}{\frac{\pi}{4}}=2\pi\cdot\frac{4}{\pi}=8$

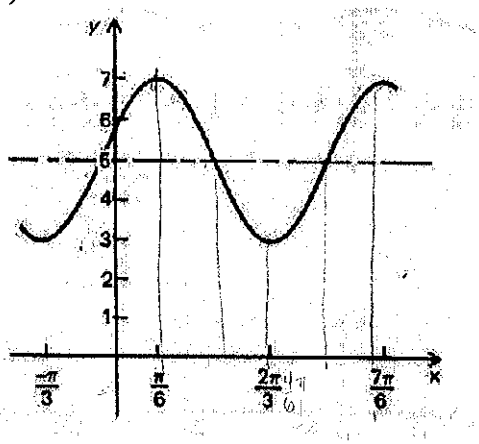
d) $y=4\cos\frac{\pi}{3}(x+2)-5$



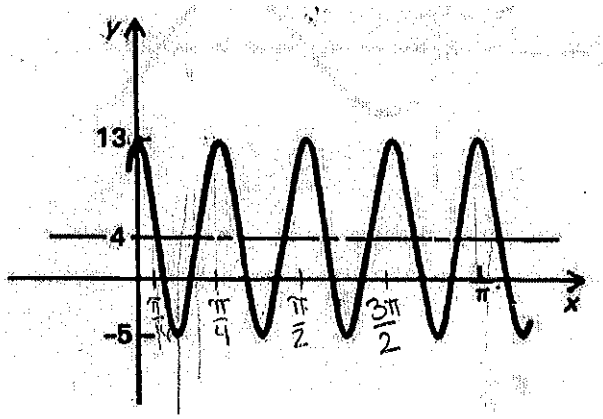
sin axis $y=-5$ $P=\frac{2\pi}{\pi/3}=6$

22. For each of the following graphs, write at least two equations of the sinusoid shown in standard transformation form, one in terms of sine, and one in terms of cosine, one of which has a **negative** value of A.

(a)



(b)



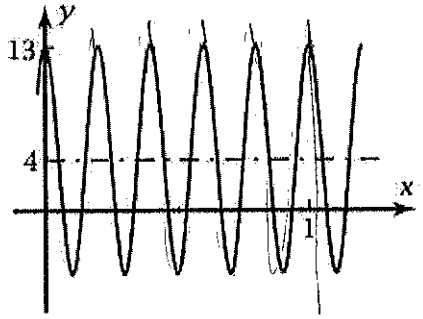
$|A| = 2$
$|B| = 2$
$C =$ depends
$D = 5$
$T = \pi$
$\pi = \frac{2\pi}{|B|}$

$y = 2\cos(2(x - \frac{\pi}{6})) + 5$
$y = -2\sin(2(x - \frac{5\pi}{12})) + 5$

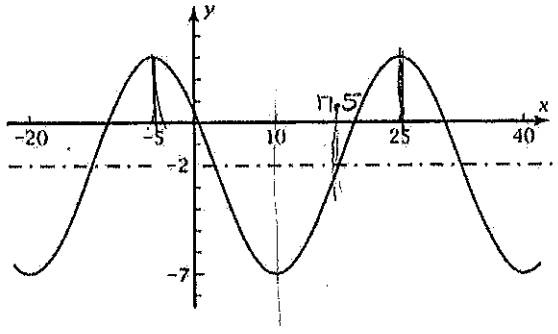
$|A| = 9$
$|B| = 8$
$C =$
$D = 4$
$P =$ $4P = \pi$ $P = \frac{\pi}{4}$ $\frac{\pi}{4} = \frac{2\pi}{|B|}$ $B\pi = 8\pi$ $B = 8$

$y = 9\cos(8x) + 4$
$y = -9\sin(8(x - \frac{\pi}{16})) + 4$

(c)



(d)



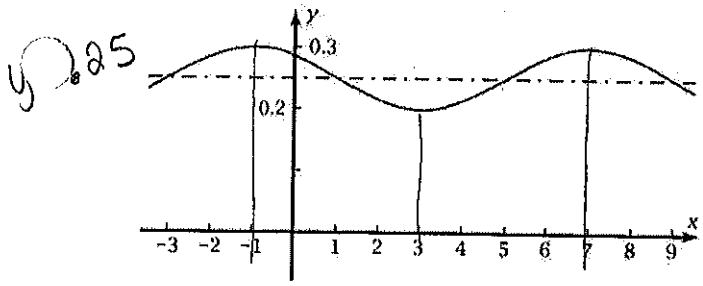
$|A| = 9$
$|B| = 10\pi$
$D = 4$
$P = \frac{1}{5}$
$\frac{1}{5} = \frac{2\pi}{B}$
$B = 10\pi$

$y = 9\cos(10\pi(x))+4$
$y = -9\sin(10\pi(x - \frac{1}{20})) + 4$

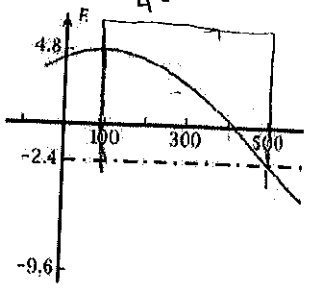
$|A| = 5$
$|B| = \frac{\pi}{15}$
$D = -2$
$P = 30$
$30 = \frac{2\pi}{B}$ $\frac{\pi}{15} = B$

$y = -5\cos(\frac{\pi}{15}(x - 10)) - 2$
$y = 5\sin(\frac{\pi}{15}(x - 17.5)) - 2$

(e)



(f) $\frac{1}{4}$ of Period



$|A| = .05$
$|B| = \frac{\pi}{4}$
$D = .25$
$P = 8$

$8B = 2\pi$
$B = \frac{\pi}{4}$

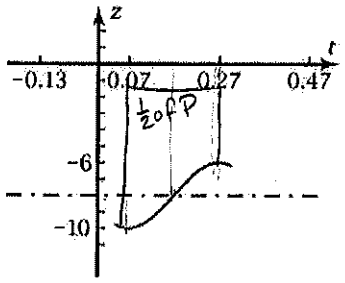
$y = -.05\cos\left(\frac{\pi}{4}(x-3)\right) + .25$
$y = .05\sin\left(\frac{\pi}{4}(x-5)\right) + .25$

$|A| = 7.2$
$|B| = \frac{\pi}{400}$
$D = -2.4$
$P = 1600$

$1600B = 2\pi$
$B = \frac{\pi}{800}$

$y = 7.2\cos\left(\frac{\pi}{800}(x-100)\right) - 2.4$
$y = -7.2\sin\left(\frac{\pi}{800}(x-500)\right) - 2.4$

(g)



$|A| = 2$
$|B| = 5\pi$
$D = -8$
$P = .4$

$\frac{.4B}{.4} = \frac{2\pi}{.4}$
$B = 5\pi$

$y = -2\cos(5\pi(x-0.07)) - 8$
$y = 2\sin(5\pi(x-0.17)) - 8$