Name_____ Date_____ Period_____
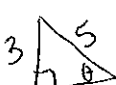
Worksheet 5.3—Circular Trig Functions

Show all work on a separate sheet of paper. All answers must be given as either simplified, exact answers. A calculator **is not permitted** unless otherwise stated.

Multiple Choice

1. Which of the following trig functions is undefined?

    (A) $\sin 30°$ (B) $\tan 0$ (C) $\cos\dfrac{\pi}{2}$ (D) $\csc 90°$ (E) $\sec\dfrac{3\pi}{2}$

2. If θ is the smallest angle in a 3-4-5 right triangle, then $\sin\theta =$

    (A) $\dfrac{3}{5}$ (B) $\dfrac{3}{4}$ (C) $\dfrac{4}{5}$ (D) $\dfrac{5}{4}$ (E) $\dfrac{5}{3}$

3. If a non-horizontal line has a slope of $\sin\theta$ for some θ, then the line will be perpendicular to a line with a slope of

    (A) $\cos\theta$ (B) $-\cos\theta$ (C) $\csc\theta$ (D) $-\csc\theta$ (E) $-\sin\theta$
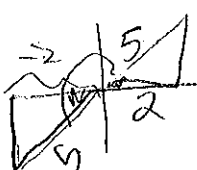
4. Which of the following trig **ratios** could NOT be π?

    (A) $\tan\theta$ (B) $\cos\theta$ (C) $\cot\theta$ (D) $\sec\theta$ (E) $\csc\theta$

5. If $\sin\theta = 0.4$, then $\sin(-\theta) + \csc\theta =$

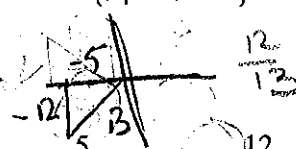   (A) -0.15 (B) 0 (C) 0.15 (D) 0.65 (E) 2.1

6. If $\cos\theta = 0.4$, then $\cos(\theta + \pi) =$

    (A) -0.6 (B) -0.4 (C) 0.4 (D) 0.6 (E) 3.54

7. The range of the function $f(x) = (\sin\theta)^2 + (\cos\theta)^2$ is

    (A) $\{y \mid y = 1\}$ (B) $\{y \mid -1 \le y \le 1\}$ (C) $\{y \mid 0 \le y \le 1\}$ (D) $\{y \mid 0 \le y \le 2\}$ (E) $\{y \mid y \ge 0\}$
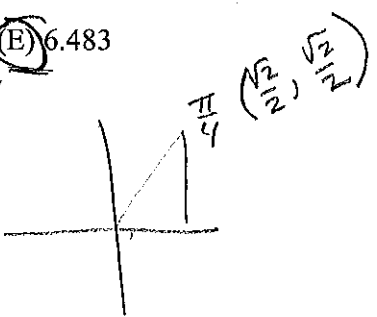
8. If $\sec\theta = -\dfrac{13}{5}$ and $\tan\theta > 0$, then $\sin\theta =$

    (A) $-\dfrac{12}{13}$ (B) $-\dfrac{5}{12}$ (C) $\dfrac{5}{13}$ (D) $\dfrac{5}{12}$ (E) $\dfrac{12}{13}$

9. (Calculator Permitted) Evaluate $\sec 30$

    (A) 0.5 (B) -1.012 (C) undefined   (D) 1.547 (E) 6.483

10. Evaluate $\cos\dfrac{57\pi}{4}$

    (A) $\dfrac{\sqrt{2}}{2}$ (B) $-\dfrac{\sqrt{2}}{2}$ (C) 1 (D) -1 (E) 0
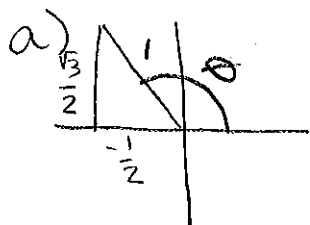
11. For each of the following angles θ, draw the associated reference triangle, then find the reference angle. Then find the values for $\cos\theta$, $\sin\theta$, and $\tan\theta$.

(a) $\theta = \dfrac{2\pi}{3}$ 　　(b) $\theta = \dfrac{5\pi}{4}$ 　　(c) $\theta = \dfrac{5\pi}{6}$ 　　(d) $\theta = \dfrac{7\theta}{4}$ 　　(e) $\theta = \dfrac{3\pi}{2}$

a)

$\dfrac{\sqrt{3}}{2}$ 　 1 　 θ

$-\dfrac{1}{2}$

$\theta_{ref} = 60°$

$\theta = 120°$

$\theta_{ref} = \dfrac{\pi}{3}$

$\cos\theta = -\dfrac{1}{2}$

$\sin\theta = \dfrac{\sqrt{3}}{2}$

$\tan\theta = -\sqrt{3}$

b)

$\theta_{ref} = \dfrac{\pi}{4}$

$\cos\theta = -\dfrac{\sqrt{2}}{2}$

$\sin\theta = -\dfrac{\sqrt{2}}{2}$

$\tan\theta = 1$

c) $\theta_{ref} = \dfrac{\pi}{6}$

$\cos\theta = -\dfrac{\sqrt{3}}{2}$

$\sin\theta = \dfrac{1}{2}$

$\tan\theta = -\dfrac{1}{\sqrt{3}} = -\dfrac{\sqrt{3}}{3}$

d) $\theta = \dfrac{7\pi}{4}$

$\theta_{ref} = \dfrac{\pi}{4}$

$\cos\theta = \dfrac{\sqrt{2}}{2}$

$\sin\theta = -\dfrac{\sqrt{2}}{2}$

$\tan\theta = -1$

e) $\theta = \dfrac{3\pi}{2}$

$\theta_{ref} = $ nRRA.

$\cos\theta = 0$

$\sin\theta = -1$

$\tan\theta = $ DNE

12. Evaluate the following from the Unit Circle.

(a) $\sin 245°$ (b) $\cos\dfrac{5\pi}{3}$ (c) $\tan 150°$ (d) $\csc\dfrac{\pi}{3}$ (e) $\sec\dfrac{\pi}{2}$ (f) $\cot 0$

a) $\sin 240° = \dfrac{-\sqrt{3}}{2}$ b) $\cos\dfrac{5\pi}{3} = \dfrac{1}{2}$ c) $\tan 150° = \dfrac{-1}{\sqrt{3}} = \dfrac{-\sqrt{3}}{3}$

d) $\csc\dfrac{\pi}{3} = \dfrac{2}{\sqrt{3}} = \dfrac{2\sqrt{3}}{3}$ e) $\sec\dfrac{\pi}{2} = \dfrac{1}{0} = DNE$ f) $\cot 0 = \dfrac{1}{0} = DNE$

13. (Calculator Permitted) Find the exact value of each of the following trig ratios of angles that are coterminal with unit circle angles.

(a) $\sin\left(\dfrac{29131\pi}{4}\right)$ (b) $\sec\left(\dfrac{674523\pi}{6}\right)$ (c) $\csc\left(\dfrac{201152010\pi}{3}\right)$ (d) $\cot\left(\dfrac{897513\pi}{6}\right)$

(e) $\cos\left(\dfrac{-8675309\pi}{3}\right)$ (f) $\tan\left(\dfrac{643281359\pi}{4}\right)$

a) $\sin\dfrac{3\pi}{4} = \dfrac{\sqrt{2}}{2}$ b) $\sec\left(\dfrac{\pi}{2}\right) = DNE$ c) $\csc 0 = DNE$

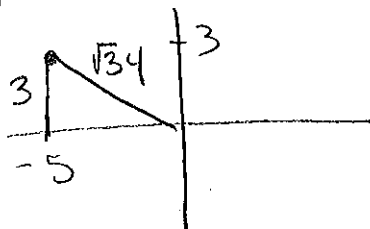D) $\cot\dfrac{3\pi}{2} = 0$ e) $\cos -\dfrac{5\pi}{3} =$ f) $\tan\dfrac{7\pi}{4} = -1$

$\dfrac{5\pi}{3}$

$\cos\dfrac{\pi}{3} = \dfrac{1}{2}$

14. If the terminal side of θ passes through the given point, find the simplified, exact values of all six trig functions of θ. THEN, find θ and θ_{ref} if $0° < \theta < 360°$.

(a) $(-5,3)$ (b) $(-4,-5)$

a)

$$\cos\theta = \frac{-5}{\sqrt{34}} = \frac{-5\sqrt{34}}{34}$$

$$\sin\theta = \frac{3}{\sqrt{34}} \quad \frac{3\sqrt{34}}{34}$$
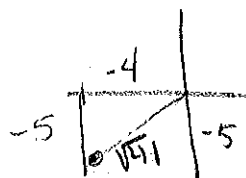
$$\tan\theta = \frac{-3}{5}$$

$$\sec\theta = \frac{-\sqrt{34}}{5}$$

$$\csc\theta = \frac{\sqrt{34}}{3}$$

$$\cot\theta = \frac{-5}{3}$$

$$\theta_{ref} = 30.963° \quad \theta = 149.036°$$

b)

$$\cos\theta = \frac{-4}{\sqrt{41}} = \frac{-4\sqrt{41}}{41}$$

$$\sin\theta = \frac{-5}{\sqrt{41}} = \frac{-5\sqrt{41}}{41}$$

$$\tan\theta = \frac{5}{4}$$

$$\sec\theta = \frac{-\sqrt{41}}{4}$$

$$\csc\theta = \frac{-\sqrt{41}}{5}$$

$$\cot\theta = \frac{4}{5}$$

$$\theta_{ref} = 51.340° \quad \theta = 231.340°$$

15. If $\cot\theta$ is undefined and $\sec\theta < 0$, find the exact values of all six trig functions of θ. THEN, find θ and θ_{ref} if $0 < \theta < 2\pi$.

$$\theta = \pi$$

$$\sin\theta = 0 \qquad \csc\theta = DNE$$
$$\cos\theta = -1 \qquad \sec\theta = -1$$
$$\tan\theta = 0 \qquad \cot\theta = DNE$$

16. Solve the following equations for $0 \le \theta < 2\pi$ using your Unit Circle knowledge. (There might be more than one solution for each)

(a) $\sin \theta = -\dfrac{1}{2}$ (b) $\tan \theta = -1$ (c) $\cot \theta = \dfrac{\sqrt{3}}{3}$ (d) $\sec \theta = 2$

a) $\theta = \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$

b) $\theta = \dfrac{3\pi}{4}, \dfrac{7\pi}{4}$

c) $\tan \theta = \dfrac{3}{\sqrt{3}} \cdot \left(\dfrac{\sqrt{3}}{\sqrt{3}}\right)$

$\tan \theta = \dfrac{\sqrt{3}}{1} \cdot \dfrac{\frac{1}{2}}{\frac{1}{2}}$

$\tan \theta = \dfrac{\sqrt{3}/2}{1/2} = \dfrac{y}{x}$

$\left(\dfrac{1}{2}, \dfrac{\sqrt{3}}{2}\right) \rightarrow \dfrac{\pi}{3}$

$\theta = \dfrac{\pi}{3}, \dfrac{4\pi}{3}$

d) $\cos \theta = \dfrac{1}{2}$

$\theta = \dfrac{\pi}{3}, \dfrac{5\pi}{3}$

17. For $\theta \in [-2\pi, 4\pi]$, solve the equation $\tan \theta = 1$ if $\sin \theta < 0$

$\theta = \dfrac{-3\pi}{4}, \dfrac{5\pi}{4}, \dfrac{13\pi}{4}$

18. (Calculator Permitted) Evaluate the following to 3 decimals.

(a) $\sin 257°13''$

$$\boxed{-.974}$$

(b) $\cos 13$

$$\boxed{= .907}$$

(c) $\cot(-190.3°)$

$$= \frac{1}{\tan(-190.3°)}$$

$$\boxed{= -5.502}$$

(d) $\sec\dfrac{25\pi}{7}$

$$= \frac{1}{\cos\left(\frac{25\pi}{7}\right)}$$

$$\boxed{= 4.493}$$

(e) $\csc\dfrac{5\pi}{6}°$

$$= \frac{1}{\sin\left(\frac{5\pi}{6}°\right)}$$

$$\boxed{= 21.892}$$